

## Advanced Economic Growth, Problem Set 3

This problem set is due on or before the recitation on Friday, November 9

Please answer the following questions:

**Exercise 1** Recall the endogenous technology diffusion model discussed in the lecture, where the aggregate production function of economy  $j = 1, \dots, J$  at time  $t$  is given by

$$Y_j(t) = \frac{1}{1-\beta} \left[ \int_0^{N_j(t)} x_j(v, t)^{1-\beta} dv \right] L_j^\beta,$$

and the innovation possibilities frontier of economy  $j$  is

$$\dot{N}_j(t) = \eta_j \left( \frac{N(t)}{N_j(t)} \right)^\phi Z_j(t), \quad (1)$$

where  $N(t)$  represents world technology. Recall that in the baseline model

$$N(t) = \frac{1}{J} \sum_{j=1}^J N_j(t). \quad (2)$$

1. Suppose that the preferences of the representative household and country  $j$  is given by  $U_j = \int_0^\infty \exp(-\rho_j t) \left[ (c_j(t)^{1-\theta} - 1) / (1-\theta) \right] dt$ , where  $\rho_j$  differs across countries. Derive the appropriate transversality condition for the maximization problem of the representative household in each country and show that, provided that these transversality conditions hold, there exists a unique globally saddle-path stable world equilibrium where all countries grow at the same rate. Characterize this growth rate.

2. Now replace (2) with

$$N(t) = G(N_1(t), \dots, N_J(t)),$$

where  $G$  is increasing in all of its arguments and homogeneous of degree 1. Generalize the results in part 1 to this case and derive an equation that determines the world growth rate implicitly.

3. Derive an explicit equation for the world growth rate for the specific case in which  $N(t) = \max_j N_j(t)$ .

4. Show that if population grows at some constant rate  $n_j > 0$  in each country, there will not exist a steady-state equilibrium.

5. Now assume that population grows at the common rate  $n$  in all countries and modify the innovation possibilities frontier (1), so that  $\dot{N}_j(t) = \eta_j N(t)^\phi N_j(t)^{-\tilde{\phi}} Z_j(t)$ , where  $\tilde{\phi} > \phi$ . Show that a steady-state world equilibrium again exists in this case. Is it saddle-path stable?

**Exercise 2** Consider the model in inappropriate technologies due to differences in capital-labor ratios discussed in the lecture, where

$$Y = A \min \left\{ 1, \left( \frac{k}{k'} \right)^\gamma \right\} K^{1-\alpha} L^\alpha,$$

with  $k = K/L$  corresponding to the capital-labor ratio of the country in question, and  $A \min \{ 1, (k/k')^\gamma \}$  is the (total factor) productivity of technology designed to be used with capital-labor ratio  $k'$  when used instead with capital-labor ratio  $k$  (this total factor productivity may result from learning by doing or from external increasing returns). Suppose that the world consists of two countries with constant and equal populations, and constant savings rates  $s_1 > s_2$  (and the same rate of depreciation equal to  $\delta$ ). Suppose that  $k'$  in the above production function corresponds to the highest capital-labor ratio in any country experienced until then. There is no technological progress and both countries start with the same capital-labor ratio.

1. Characterize the steady-state world equilibrium (that is, the steady-state capital-labor ratios in both countries).
2. Characterize the output per capita dynamics in the two economies. How does an increase in  $\gamma$  affect these dynamics?
3. Show that the implied income per capita differences (in steady state) between the two countries are increasing in  $\gamma$ . Interpret this result.
4. Do you think this model provides a good/plausible mechanism for generating large income differences across countries? Substantiate your answer with theoretical or empirical arguments.

**Exercise 3** Consider the trade and specialization model with the AK technology discussed in the lectures, where terms of trade effects generated a stable world income distribution. Suppose now that production and allocation decisions within each country is made by a “country-specific social planner” (who maximizes the utility of the representative consumer within the country).

1. Show that the equilibrium allocation characterized in the lectures is no longer an equilibrium. Explain why.
2. Characterize the equilibrium in this case and show that all of the qualitative results derived in the lectures apply and provide the long-run growth rate of the world economy.
3. Show that world welfare is lower in this case than in the equilibrium in the lectures. Explain why.

**Exercise 4** Consider the Krugman product cycle model discussed in the lectures. Assume that new goods are created by technology firms in the North as in the expanding product variety model discussed in the lectures. Firms that invent new product varieties become monopolist suppliers until the good they have invented is copied by the South. The technology of production is the same as in the lecture, and assume that new goods can be produced by using final goods, with the technology  $\dot{N}(t) = \eta Z(t)$ , where  $Z(t)$  is final good spending. Imitation is still exogenous and takes place at the rate  $\iota$ . Once a good is imitated, it can be produced competitively in the South.

1. Show that for a good that is not copied by the South, the price will be

$$p(t, \nu) = \frac{\varepsilon}{\varepsilon - 1} w^n(t).$$

2. Characterize the static equilibrium for given levels of  $N^n(t)$  and  $N^o(t)$ .
3. Compute the net present value of a new product for a Northern firm.
4. Impose the free entry condition and derive the equilibrium rate of technological change for the world economy. Compute the world growth rate.
5. What is the effect of an increase in  $\iota$  on the equilibrium? Can an increase in  $\iota$  make the South worse-off? Explain the intuition for this result.