

Information Design and the Market for Impressions

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Information and Economics

- The role of information - including asymmetric information - has been extensively studied since the 1970s
- Two developments of the last decade:
 - ① Many important new developments in the modelling of information in economic theory (e.g., *Information Design*)
 - ② Information has become increasingly central to the functioning of the modern economy via the importance of information on the internet (e.g., the *Market for Impressions* (Ad Views))

This Talk in Three Parts

- 1 Introduction to Information Design
- 2 Optimal Information Disclosure in Auctions
- 3 Relevance of Auction question to the Market for Impressions

Papers

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- 1 Survey: "**Information Design: A Unified Perspective**," *Journal of Economic Literature* (2019).
- 2 Auction Problem: "**Optimal Information Disclosure in Auctions**," conditionally accepted in *American Economic Review: Insights*, and to be circulated shortly.
- 3 Application to Market for Impressions: "**Selling Impressions: Efficiency versus Competition**"

Part 1: Information Design

- **Mechanism Design:**

- Fix an economic environment and information structure
- Design the rules of the game to get a desirable outcome

- **Information Design**

- Fix an economic environment and rules of the game
- Design an information structure to get a desirable outcome

Mechanism Design and Information Design

- **Mechanism Design:**

- Can compare particular mechanisms..
 - e.g., first price auctions versus second price auctions
- Can work with space of all mechanisms...
 - without loss of generality, let each agent's action space be his set of types... "revelation principle"
 - e.g., Myerson's optimal mechanism

- **Information Design**

- Can compare particular information structures
 - Linkage Principle: Milgrom-Weber 82
 - Information Sharing in Oligopoly: Novshek and Sonnenschein 82
- Can work with space of all information structures
 - without loss of generality, let each agent's type space be his set of actions..... "revelation principle"

Information Design

- There has always been work comparing parametric information structures
- Recent years has seen an explosion taking the "non-parametric" approach, i.e., allowing all information structures
- this includes the massive "Bayesian persuasion" literature (Kamenica-Genzkow 11) where the information is one economic agent or many non-strategic agents

Part 2: Information Design in Auctions

- consider classic problem of second price auction of single object to buyers with symmetric independent private values.....
-but suppose the seller controls how much each buyer knows about his private value (without knowing the private value herself)
- would the seller prefer full information (buyers to know their values perfectly), no information (buyers know nothing), or something in between?
 - with full information: **efficient allocation** but **information rents** - revenue is expectation of second highest value
 - with no information: **inefficiency** but **no information rent** - revenue is common ex ante expected value

Answer

- optimal information structure is something in between.....
 - in particular, low valuation buyers are told their values but high valuation buyers are pooled, i.e., just told that their value exceeds a critical threshold
- in fact, critical quantile where pooling starts depends only on the number of buyers (and is independent of the distribution of values)
- intuition: competition is lowest when there is a high winning value

Setting

- N bidders
- Private values symmetrically and independently distributed according to F
- A (symmetric) information structure generates a distribution of expected values G
- Blackwell & Girshick (1954) show: there exists a signal s that induces a distribution of expected valuations G from F if and if F is a mean preserving spread of G
- F is a mean preserving spread of G if

$$\int_v^\infty dF(t) \leq \int_v^\infty dG(t), \forall v \in \mathbb{R}_+$$

and

$$\int_0^\infty dF(t) = \int_0^\infty dG(t).$$

- if F is a mean preserving spread of G we write $F \prec G$

Revenue

- second-order statistic $w_{(2)}$ of N symmetrically and independently distributed random variables is

$$\mathbb{P}(w_{(2)} \leq t) = NG^{N-1}(t)(1 - G(t)) + G^N(t)$$

- expected revenue of seller:

$$R = \mathbb{E}[w_{(2)}] = \int_0^\infty t.d(NG^{N-1}(t)(1 - G(t)) + G^N(t))$$

- maximization problem:

$$R = \max_G \int_0^\infty t.d(NG^{N-1}(t)(1 - G(t)) + G^N(t))$$

subject to $F \prec G$.

- non-linear problem in optimization variable G
- neither convex nor concave program

Quantile Change of Variables

- denote by q_i a random variable that is uniformly distributed in $[0, 1]$ and

$$F^{-1}(q_i) = v_i.$$

- distribution function of quantile of second-highest valuation:

$$S_N(q) \triangleq Nq^{N-1}(1 - q) + q^N$$

- quantile distribution S_N is independent of the underlying distribution F
- just as quantile of any random variable is uniformly distributed, the quantile of second-order statistic of N random variables is distributed according to S_N for every distribution

Quantile Representation of Revenue

- revenue is expectation over quantiles using measure $S(q)$
- revenue given quantile of second-order statistic is G^{-1} :

$$\begin{aligned} \max_{G^{-1}} \int_0^1 S'(q)G^{-1}(q)dq \\ \text{subject to } G^{-1} \prec F^{-1} \end{aligned} \quad (\text{R})$$

- seller can choose any distribution of expected valuations whose quantile function G^{-1} is a mean-preserving spread of quantile function F^{-1}
- $F \prec G$ if and only if $G^{-1} \prec F^{-1}$
- objective linear in G^{-1}
- as I will discuss briefly below, this allows us to appeal to some recent general results to solve problem

Optimal Information Structure

Proposition (Optimal Information Structure)

Suppose that F is absolutely continuous, then the unique optimal symmetric information structure with N bidders is given by:

$$s(v_i) = \begin{cases} v_j & \text{if } q_i(v_i) \leq q_N^* \\ \mathbb{E}[v_j \mid F(v_j) \geq q] & \text{if } q_i(v_i) \geq q_N^* \end{cases}$$

where $q_N^* \in [0, 1)$ solves (for $N \geq 3$)

$$S_N(q) + S'_N(q)(1 - q) = S_N(1).$$

In particular, $q_2^* = 0$, q_N^* is increasing in N and $q_N^* \rightarrow \infty$ as $N \rightarrow \infty$. Note that q_N^* is independent of the distribution F .

Optimal Information Structure in Words

- reveal the valuation of all those bidders who have a valuation lower than some threshold determined by quantile q_N^*
- otherwise reveal no information beyond the fact that the valuation is above the threshold
- "upper censorship"

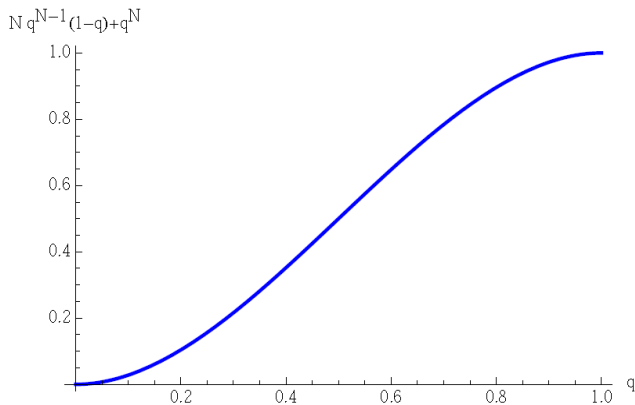
Intuition: Competition through Pooling

- bidders with lower valuations are likely to face nearby bids (even when bidders have full information)
- bidders with higher valuations are **unlikely** to face nearby bids when bidders have full information and so are like to earn high information rents (i.e., pay a price significantly below their value)
- so bidders with higher valuations are pooled so there is a significant probability that they face a bidder with exactly the same value

More Intuition: About Two Bidders with the High Value

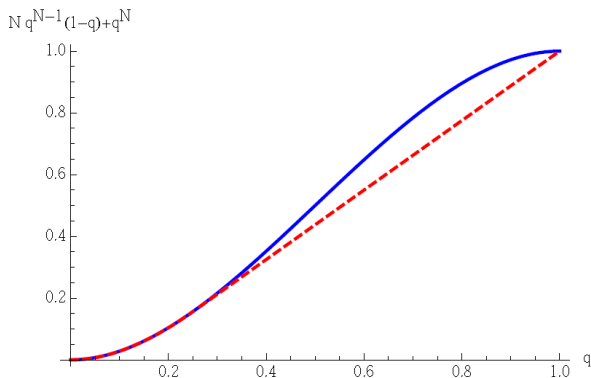
- For any N , the number of bidders with the high value will have a binomial distribution $B(N, 1 - q_N^*)$
- The expected number of bidders with the high value is $\rho_N^* = (1 - q_N^*) N$
- Can show ρ_N^* is around 2 for all N
 - $\rho_2^* = 2$
 - $\rho_3^* = 2.25$
 - ρ_N^* is decreasing for $N \geq 3$ and $\rho_N^* \rightarrow 1.73$ as $N \rightarrow \infty$

Second Order Statistic in Quantile Space



- graph of $S_3(q)$
- unique inflection point for all $N \geq 3$

Convex Hull of Second Order Statistic CDF



- convex hull of S_N is largest convex function below S_N
- problem reduces to finding q such that:

$$S(q) + S'(q)(1 - q) = S(1) = 1$$

- note that this was the characterization of q_N^*

Verification

- Kleiner et al (Proposition 2) characterizes the extreme points of the set of monotonic functions satisfying a "majorization" constraint.
- Recall that we transformed our maximization problem into one that was linear in G^{-1} subject to G^{-1} being a mean preserving spread of F^{-1}

Verification

- So re-writing Kleiner et al (Proposition 2) for our problem, we have

Proposition (Kleiner et al. Proposition 2)

Let G^{-1} be such that for some countable collection of intervals $\{[\underline{x}_i, \bar{x}_i) \mid i \in I\}$,

$$G^{-1}(q) = \begin{cases} F^{-1}(q) & q \notin \cup_{i \in I} [\underline{x}_i, \bar{x}_i) \\ \frac{\int_{\underline{x}}^{\bar{x}} F^{-1}(t) dt}{\bar{x}_i - \underline{x}_i} & q \in [\underline{x}_i, \bar{x}_i) \end{cases}$$

If $\text{conv } F$ is affine on $[x_i, x_i)$ for each $i \in I$ and if $\text{conv } F = F$ otherwise, then G solves the problem. Moreover, if F is strictly increasing the converse holds.

Part 3: Selling Impressions

- An "impression" in digital advertising is "a metric used to quantify the number of digital views or engagements of a piece of content, usually an advertisement, digital post, or a web page. Impressions are also referred to as an "ad view." They are used in online advertising, which often pays on a per-impression basis."
- Impressions are sold by publishers/intermediaries (sellers) to advertisers (buyers)
- Two-sided information: the publisher (seller) knows about the attributes of the viewer, each advertiser (buyer) knows which attributes he cares about
- So the publisher **can** control the information that the advertiser has about the value of the impression

Selling Impressions: Conflation

- So our model is relevant for the important economic decision about what information the seller reveals to buyers
- Addresses famous practical problem: how finely should buyers be allowed to target views? Accurate targetting creates monopoly power for buyer but efficient information flow.
- Levin and Milgrom (2010) argue that this is an example of more general "conflation" problem that is often swept under the rug: how broadly or narrowly are goods defined in the marketplace

Selling Impressions: A Model

- I will sketch a model that makes a tight connection between two sided information model and the model of auctions we just discussed
- impression/viewer is characterized by attribute $x = \{-1, 1\}^J$
- advertiser i is characterized by "preference" (who she cares about attributes) $y_i \in \{-1, 1\}^J$
 - assume attributes and preferences are independent and uniform and

$$v_i = u(x, y_i) = w \left(\frac{1}{\sqrt{J}} \sum_{j=1}^J y_{ij} x_j \right)$$

- as $J \rightarrow \infty$, can induce any distribution of values F
- captures idea of horizontal differentiation across viewers

Auto-Bidding

- Market for impressions broadly divided into auto-bidding and manual-bidding. Under auto-bidding, seller bids on buyer's behalf given buyer's instructions
- Our model of auto-bidding:
 - ① The publisher commits to a signal conditional on advertiser's reported preference and the viewer's attributes
 - ② The publisher commits to submitting advertiser optimal bid as a function of his reported preference and the publisher's signal
 - ③ Preferences and attributes are realized, preferences are reported to the advertiser, signals and bids are realized and the impression is allocated to the highest bidder at the second highest price

Auto-Bidding

Proposition (Truthful Reporting)

Advertisers have an incentive to truthfully report their preferences in the auto-bidding mechanism.

- Corollary: With those commitment powers, publisher's problem reduces to our main result

Summary

- ① I argued that richer modelling of information is a theme in modern economics
 - ① Theorists have become very interested in modelling it
 - ② Information plays an increasingly central role in the modern economy

Summary

- 1 I argued that richer modelling of information is a theme in modern economics
- 2 I described at a high level "information design"
 - useful analytically to focus on "pure information design" but ultimately this is part of broader mechanism literature

Summary

- 1 I argued that richer modelling of information is a theme in modern economics
- 2 I described at a high level "information design"
- 3 I described a clean illustration/application of the information design approach to auctions
 - There was a clean and intuitive insight: pool high valuation buyers in order to maintain competition at the top

Summary

- ① I argued that richer modelling of information is a theme in modern economics
- ② I described at a high level "information design"
- ③ I described a clean illustration/application of the information design approach to auctions
- ④ I discussed the market for impressions and argued that our auction result was relevant
 - At a high level, sellers clearly control information
 - Under assumptions, our auction result applies exactly