Costly search, information, and competition

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Homogenous good markets and price setting

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- 40 years ago, Varian wrote: "Economists have belatedly come to recognize that the "law of one price" is no law at all. Most retail markets are instead characterized by a rather large degree of price dispersion. The challenge to economic theory is to describe how such price dispersion can persist in markets where at least some consumers behave in a rational manner."

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  - key is lack of common knowledge that consumer can choose between at least two price quotes
  - this lack of common knowledge naturally arises even with free entry and low costs for the consumers to search, sellers to publicly post prices or information intermediaries to collect, advertise and distribute prices
  - see Stigler (1961), Diamond (1971), Rothschild (1973), Varian (1980), Burdett and Judd (1983), Stahl (1989, 1996), Baye and Morgan (2001) and many many more

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- No metric of how much price dispersion we can explain
- No results on level of prices (e.g., expected price) and welfare

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  2. Endogenous distribution of price quotes (e.g., simultaneous and sequential search, costly price posting, information intermediaries)

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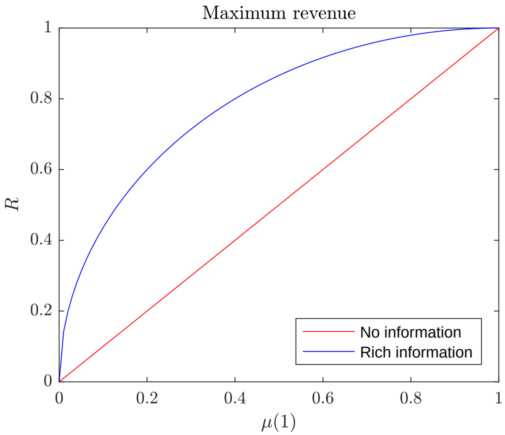
3. we show how to attain all bounds

Maximum Revenue



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    2. more subtly: under sequential search by consumers

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   3.3 relation to our prior work on auctions and informational robustness

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- two firms have 0 cost of production
- consumer collects a single (monopoly) quote with probability $\frac{1}{2}$, two (competitive) quotes with probability $\frac{1}{2}$
- more precisely, consumer gets quote from firm 1 only with probability $\frac{1}{4}$, firm 2 only with probability $\frac{1}{4}$, and both firms with probability $\frac{1}{4}$.

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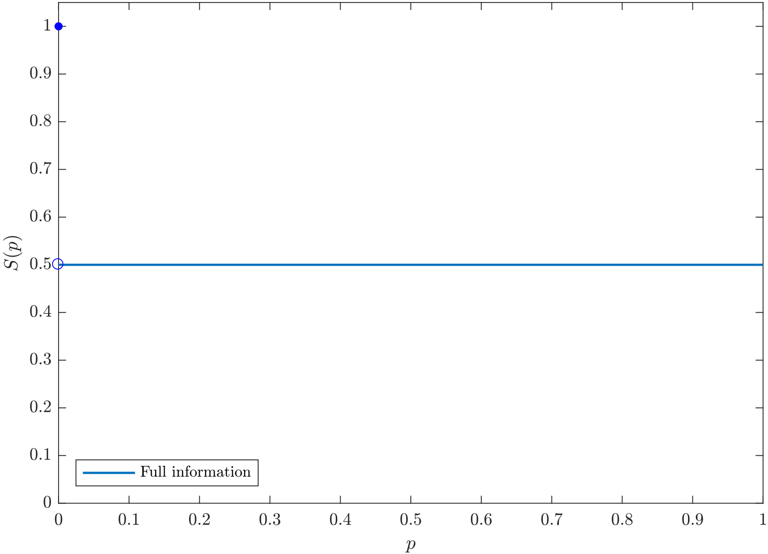
- figure 1 plots the equilibrium price distribution = probability that price is p or higher....

Full Information Price Distribution



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- this will be a general result about the no information case

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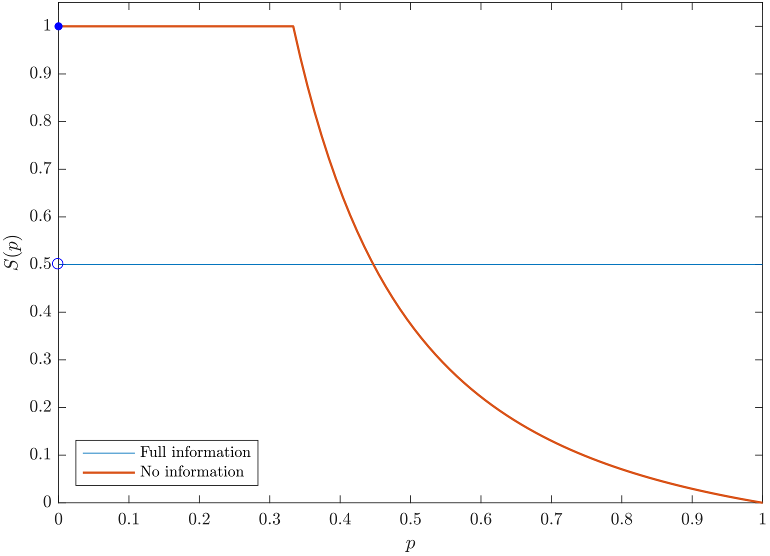

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  - condition (1) ensures that uninformed attaches a higher probability to facing monopoly price than uninformed firm

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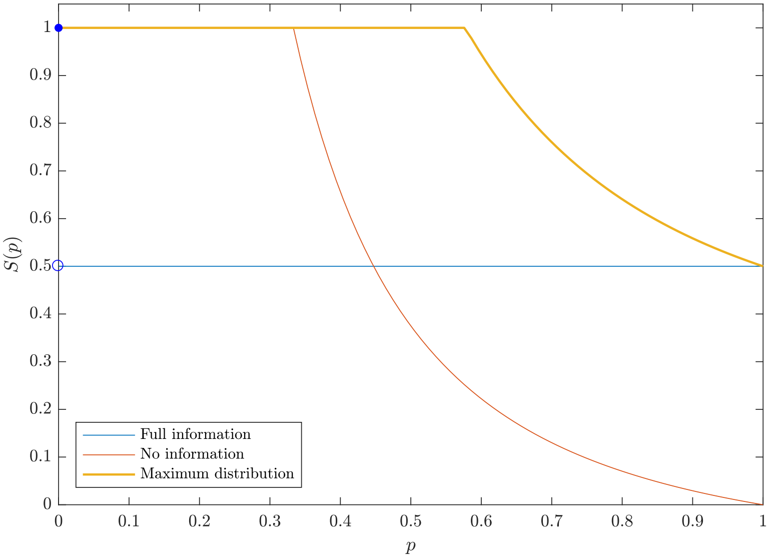
- result 1: analogous result for general distributions over number of quotes

Price Distributions



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- In each case, endogeneity of μ does not change pricing

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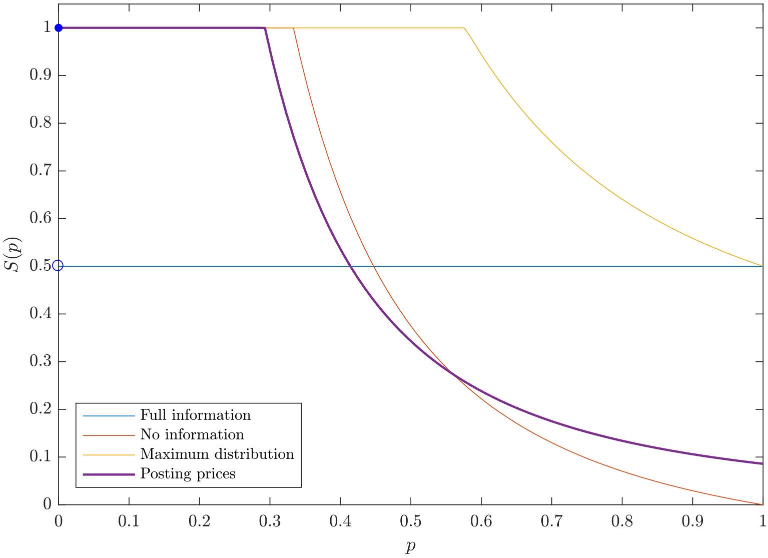

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  - advertising firms follow a symmetric mixed strategy $F(p) = \frac{1-p}{p}$

Endogenizing the Number of Quotes 3: Information Intermediaries

- like Baye and Morgan (2001), information intermediary charges firms pays a advertising fee $c > 0$ to have prices advertised on his site; consumer pays an access fee $\kappa \geq 0$ to observe
- if consumer accesses site, he buys at lowest price; if he does not access or if no price is posted, and he is randomly assigned to a firm
- there will be symmetric equilibrium where
  - firms are charged a fee $\frac{1}{2}$ and consumer is charged $\frac{1}{3}$;
  - firms randomize 50/50 between advertising and not advertising; so prob $\frac{1}{4}$ of two quotes
  - advertising firms follow a symmetric mixed strategy $F(p) = \frac{1-p}{p}$
  - non-advertising firms charge the monopoly price.

Endogenizing the Number of Quotes 4: Sequential Search

- like Stahl (1989), proportion $\frac{1}{2}$ of consumers observe one quote and can then choose (after observing the price) to pay $c > 0$ to get a second price; proportion $\frac{1}{2}$ of consumers ("shoppers") will always get two price quotes

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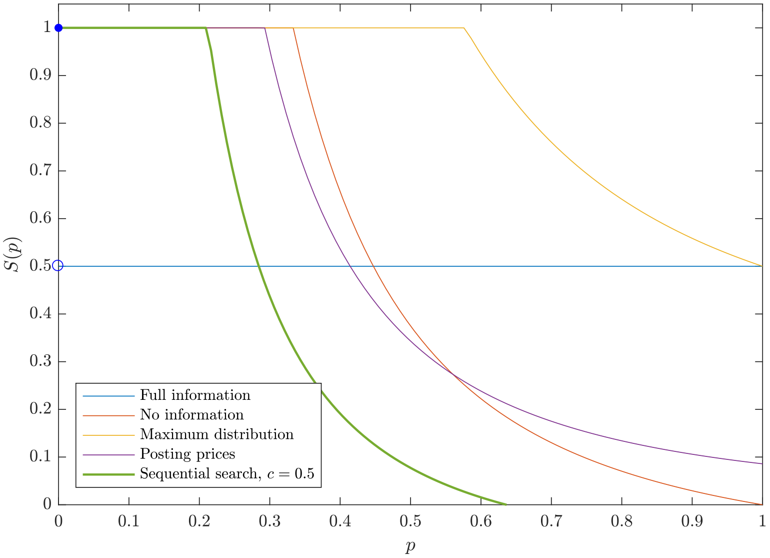
- see plot for $c = \frac{1}{2}$ and so $r \approx 0.74$

Price Distribution



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- Consumer buys from a firm with lowest price

$$p^* = \min_{k \in \tilde{N}} p_k$$

(Break ties uniformly)

Information & firms' strategies

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- For now, assume *symmetry*: distribution depends on the number of the firms, but not their identities

Strategies and Equilibrium

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- Firms want to maximize price times probability of sale
- F is an *equilibrium* if for all k and F'_k,

$$R_k(\sigma, F) \geq R_k(\sigma, F'_k, F_{-k})$$

Sale price distribution

- For a strategy F, let $S(p|n)$ denote the probability the sale price is at least p, conditional on n firms quoted:

$$S(p|n) = \int_{s \in S^n} \times_k F(p|s_k) \pi(ds|n)$$

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- Also let

$$S(p) = \sum_{n=1}^{N} \mu(n) S(p|n)$$

denote the ex ante distribution of the sale price

A constraint on sales

Theorem

In any equilibrium,

$$p \sum_{n=1}^{N} \mu(n) n \, S(p|n) \leq \int_{x=p}^{\infty} x S(dx).$$

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- Conditional on being active, there is a $1/n$ chance that firm k has the lowest price
- Hence, equilibrium surplus must be

$$\frac{1}{N}\sum_{n=1}^{N}\mu(n)\int_{x=0}^{1} xS(dx|n)$$

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- On the other hand, if the the equilibrium sale price is $x < p$, then the outcome is the same as it would have been in equilibrium (since firm k's price is unchanged as well)

Conclusion of proof

- A necessary condition for equilibrium is that firms wouldn't want to uniformly deviate down, i.e.,

$$\frac{1}{N}\sum_{n=1}^{N}\mu(n)\int_{x=0}^{v} xS(dx|n) \geq \frac{1}{N}\sum_{n=1}^{N}\mu(n)\left[\int_{x=0}^{p} xS(dx|n) + npS(p|n)\right.$$

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- Rearranging yields our result

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- Our inequality reduces to

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Proposition

There exists a highest price distribution $\overline{F}$ *that satisfies the uniform downward incentive constraints under no information. This distribution satisfies all of the constraints as equalities wherever* $\overline{F}(p) < 1$.

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- Moreover, the right-hand side has increased with F', so that F' must be feasible

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- Let $\mathcal{F}$ be the set of F's satisfying the uniform downward constraint
- Then the pointwise supremum of the F's, denoted $\overline{F}$, is finite and also satisfies the constraints, since

$$\begin{aligned} p\sum_{n=1}^{N}\mu(n)(n-1)\left(\overline{F}(p)\right)^n &= \sup_{F\in\mathcal{F}} p\sum_{n=1}^{N}\mu(n)(n-1)\left(F(p)\right)^n \\ &\le \sup_{F\in\mathcal{F}}\sum_{n=1}^{N}\mu(n)\int_{x=p}^{v}(F(x))^n dx \\ &= \sup_{F\in\mathcal{F}}\sum_{n=1}^{N}\mu(n)\int_{x=p}^{v}(\overline{F}(x))^n dx \end{aligned}$$

and hence is the largest element of $\mathcal{F}$ $\square$

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- Firms are also indifferent between charging monopoly price, proving (2) expected price

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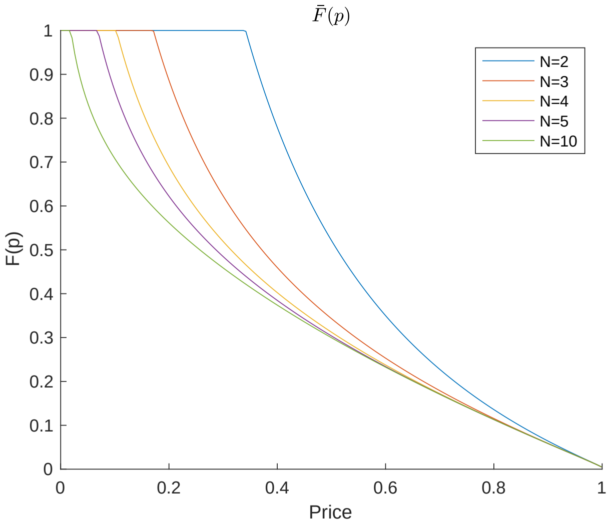

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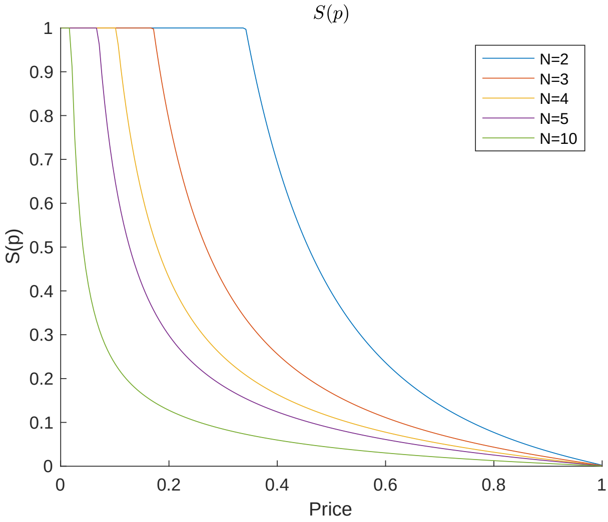

General information

- If firms get partial information about n, then the $S(p|n)$ can vary more flexibly

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- We again derive an upper bound, and now show that it is attained in an equilibrium for some information structure

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- The logic is somewhat different from the earlier proof, because we now have more flexibility in choosing the distributions

Maximizing $S(p)$

- Recall the incentive constraints:

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- Indeed, $S(p|1)$ only appears on the right, so can set $S(p|1) \equiv 1$

Monotonic solutions

- More generally, we can show that any feasible $S(p|n)$ is dominated by one that is *monotonic*:

$$S(p|n) > 0 \implies S(p|n') = 1 \text{ for all } n' < n$$

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- Once we restrict attention to monotonic solutions, we can use a similar trick as before to show that the set of $S(p)$ that can generated by $S(p|n)$ satisfying the uniform downward constraints is a complete semi-lattice

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$$p_n = p_{n-1} \left(\frac{T_{n-1}}{T_n} \right)^{\frac{n-1}{n}}$$

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- The distributions are

$$\overline{S}(p|n) = \frac{\left(\frac{p_n}{p}\right)^{\frac{n}{n-1}} - \left(\frac{p_n}{p_{n-1}}\right)^{\frac{n}{n-1}}}{1 - \left(\frac{p_n}{p_{n-1}}\right)^{\frac{n}{n-1}}}$$

Intuition for derivation

- Given that we have ordered supports, can rearrange the incentive constraint to

$$\mu(n)\left(\int_{x=p}^{v} S(x|n)dx - p(n-1)S(p|n)\right) = \sum_{k=1}^{n-1}\mu(k)(\int_{x=p}^{v} S(x|k)dx - p(k-1)S(p|k))$$

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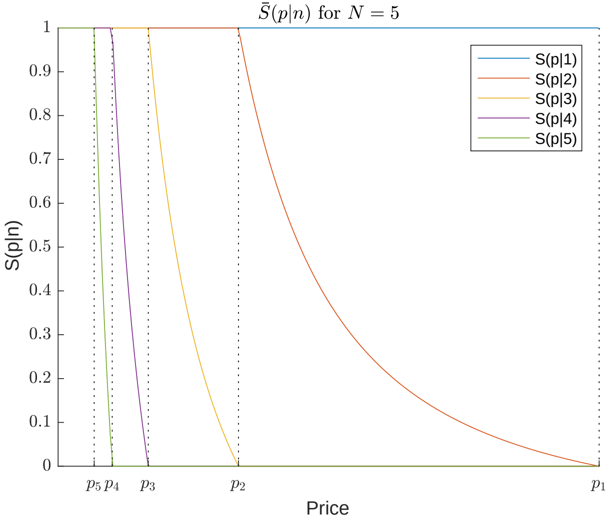
- $S(p|n)$ satisfies the boundary condition $S(p_{n-1}|n) = 0$, and p_n is defined through $S(p_n|n) = 1$

Uniform example revisited



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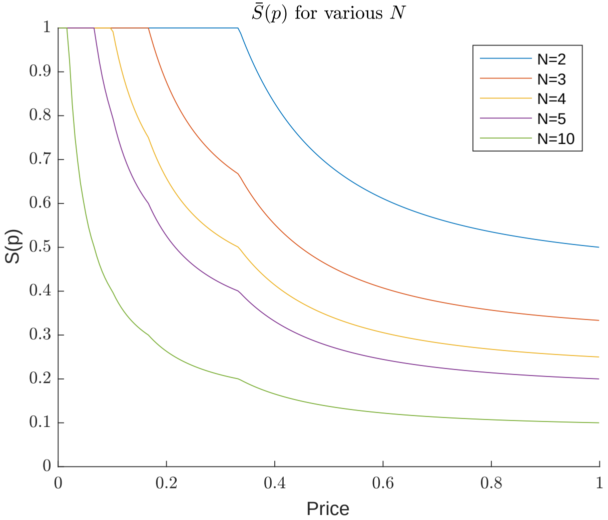
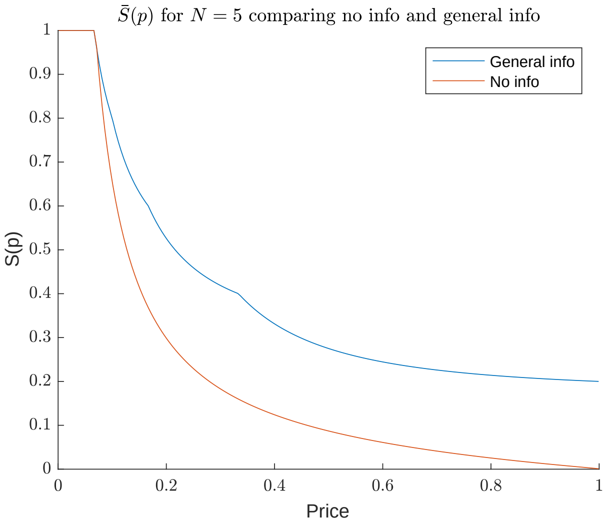


Uniform example revisited



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2. Prices are decreased (under FOSD) as the number of price quotes increases (under FOSD).
3. The expected price is under the maximum distribution of prices is in the interval $\left[\mu(1), \sqrt{\mu(1)(2-\mu(1))}\right]$

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- Firm receiving signal n follows a mixed strategy with support $[p_n, p_{n-1}]$
- Signal distributions can be chosen so that firm receiving signal n is indifferent between all signals in $[p_n, 1]$

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- This is upper bound on expected price if we shift distribution to $n \geq 3$, holding $\mu(1)$ fixed

More Formally: Firms' information

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- The firms get discrete signals in $S = \{1, \ldots, N\}$
- The distribution of signals is generated by first taking independent draws from $\alpha(\cdot|n)$ which has support on $\{1, \ldots, n\}$, and *throwing out signal profiles where no bidder gets a signal of n*

Detailed description

- The formulae are:

$$\alpha(k|n) = \frac{T_{k-1}p_{k-1}\left(\left(\frac{p_{k-1}}{p_k}\right)^{\frac{1}{k-1}} - 1\right)}{T_{n-1}p_{n-1}\left(\frac{p_{n-1}}{p_n}\right)^{\frac{1}{n-1}}}$$

and

$$\pi(s_{\tilde{N}}|n) = \frac{1}{1-(1-\alpha(n|n))^n} \times_{k\in\tilde{N}} \pi(s_k|n)$$

if $s_k = n$ for at least one $k \in \tilde{N}$, and $\pi(s|n) = 0$ otherwise

Strategies

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$$F(p|n) = \frac{\left(\frac{p_n}{p}\right)^{\frac{1}{n-1}} - \left(\frac{p_n}{p_{n-1}}\right)^{\frac{1}{n-1}}}{1 - \left(\frac{p_n}{p_{n-1}}\right)^{\frac{1}{n-1}}}$$

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- The proof that these strategies are an equilibrium shows that bidder surplus is quasiconcave in p conditional on s_k, and flat in $[p_{s_k}, p_{s_k-1}]$

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- The probability that the sale price is at least p when there are n active firms is

$$\begin{aligned}
&\frac{1}{1-(1-\alpha_n)^n}\sum_{k=1}^{n}\binom{n}{k}(\alpha_n F(p|n))^k(1-\alpha_n)^{n-k} \\
&= \frac{(1-\alpha_n+\alpha_n F(p|n))^n - 1}{1-(1-\alpha_n)^n} \\
&= S(p|n)
\end{aligned}$$

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- consider an arbitrary game in which, first the consumer, firms and other parties take actions that influence which which firms' prices the consumer will observes; and then the firms choose prices

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- this class of games embeds simultaneous search, costly price posting, information intermediaries

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- More formally, if $p(h_t)$ is the lowest price in history h_t, we would like to show that if $p(h_t) \leq p\left(h_t^{'}\right)$ and continuing to search is a best response at h_t for θ, then continuing to search is a strict best response at $h_t^{'}$ for θ.

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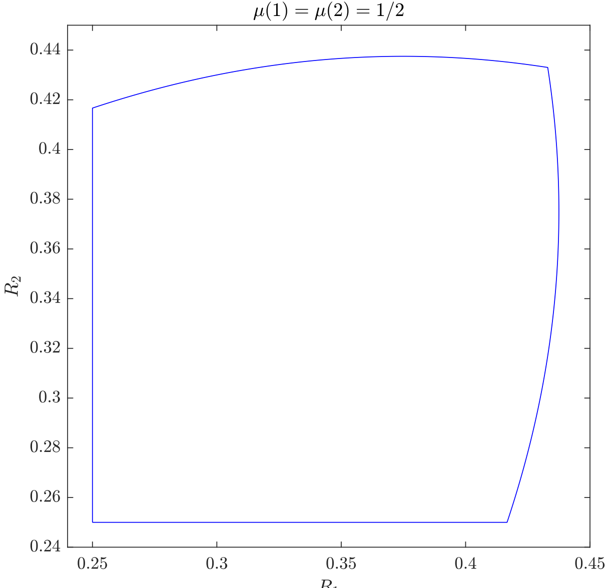
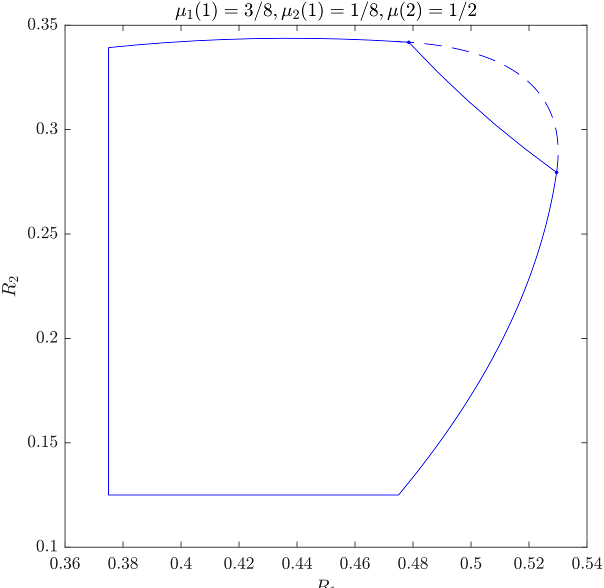
- today's problem is harder and the result is less general

Asymmetric Equilibria, Symmetric Distribution on *n*



Asymmetric Distribution and Equilibrium



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- the minimum expected price is the sum of the minimum expected cost and the firms' competitive rents

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- tight upper bound on price distribution, tight upper and lower bounds on expected prices

Further topics

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- Bergemann, Brooks and Morris (2017) solved for the (easier) case where bidders do not necessarily know their own value

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- Even in the symmetric case, symmetry is WLOG, since all equilibria have to be symmetric (Maskin and Riley 1983)

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- After seeing prices, consumer either chooses to buy at current lowest price, or continue searching $\sigma(\cdot|\theta, n_1, \ldots, n_t, \{p_k | k \in N_\tau, \tau \leq t\}) \in \Delta\left(\{0, 1, \ldots, N - \sum_{\tau \leq T} n_\tau\}\right)$

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- This is the most collusive search cost model, because it creates the weakest incentives for firms to cut prices

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- But for general information, lower prices may lead the consumer's behavior to shift in a way that makes the probability of a sale go *down*
- This would weaken incentives to deviate down, and support even higher prices than the ones we construct

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- True in simple sequential model
- But it is hard to rule out complex consumer response due to "leaked information" about signals and future prices

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- For example, what are the possible weighted sums of firms' profits, or profit and consumer surplus?
- A bit hard to think about consumer surplus, since we don't know search costs, but we can think about profits...

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- NB Would also be achieved under complete information

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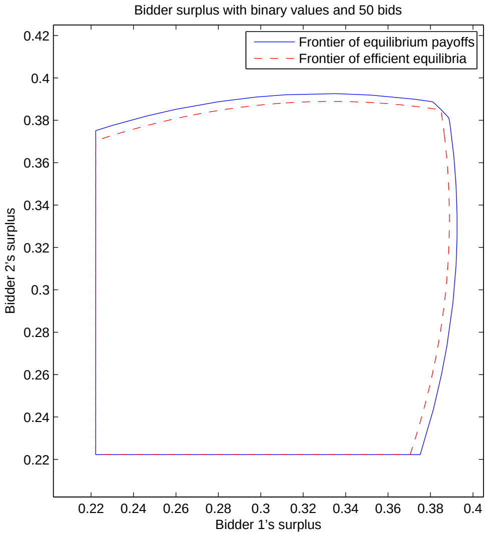
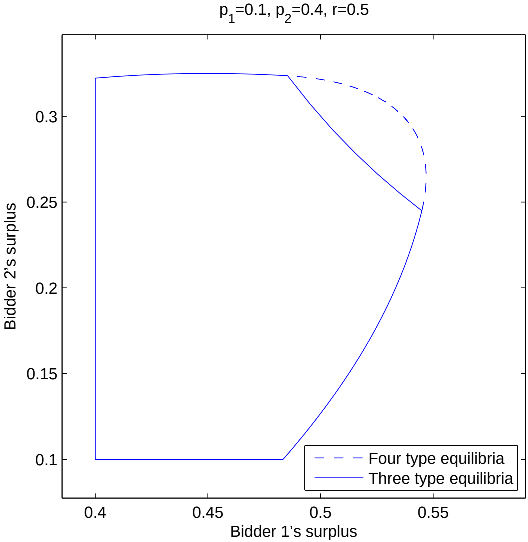
- (Mention connection with first-price auction?)

The set of firms' profits when $p_1 = p_2 = 2/9$, $p_0 = 1/9$



Asymmetric firms



Final words

Thank you!