The Geneva Risk and Insurance Review, **2014**, **39**, (176–183)

www.palgrave-journals.com/grir/

Unravelling vs Unravelling: A Memo on Competitive Equilibriums and Trade in Insurance Markets

Nathaniel Hendren
Harvard University, 1805 Cambridge Street, Cambridge, MA 02138, U.S.A.
E-mail: nhendren@fas.harvard.edu

Both Akerlof (1970) and Rothschild and Stiglitz (1976) show that insurance markets may "unravel". This memo clarifies the distinction between these two notions of unravelling in the context of a binary loss model of insurance. I show that the two concepts are mutually exclusive occurrences. Moreover, I provide a regularity condition under which the two concepts are exhaustive of the set of possible occurrences in the model. Akerlof unravelling characterises when there are no gains to trade; Rothschild and Stiglitz unravelling shows that the standard notion of competition (pure strategy Nash equilibrium) is inadequate to describe the workings of insurance markets when there are gains to trade.
The Geneva Risk and Insurance Review (2014) **39,** 176–183. doi:10.1057/grir.2014.2

Keywords: insurance; unravelling; adverse selection; private information

Article submitted 13 August 2013; accepted 10 February 2014; published online September 2014

Introduction

Akerlof[1] and Rothschild and Stiglitz[2] have contributed greatly to the understanding of the potential problems posed by private information on the workings of insurance markets. Akerlof[1] shows how private information can lead to an *equilibrium of market unravelling*, so that the only unique equilibrium is one in which only the worst quality good (i.e. the "lemons") are traded. Rothschild and Stiglitz[2] show that private information can lead to an *unravelling of market equilibrium*, in which no (pure strategy) competitive equilibrium exists because insurance companies have the incentive to modify their contracts to cream skim the lower-risk agents from other firms.

Although the term *unravelling* has been used to describe both of these phenomena, the distinction between these two concepts is often unclear, arguably a result of each paper's different approach to modelling the environment. Akerlof[1] works in the

[1] Akerlof (1970).
[2] Rothschild and Stiglitz (1976).

context of a "supply and demand" environment with a fixed contract or asset (e.g. a used car), whereas Rothschild and Stiglitz[2] work in the context of endogenous contracts in a stylised environment with only two types (e.g. high and low types).

This memo develops a generalised binary loss insurance model that incorporates the forces highlighted in both Akerlof[1] and Rothschild and Stiglitz.[2] Using this unified model, I show that the *equilibrium of market unravelling* (in Akerlof) is a mutually exclusive occurrence from the *unravelling of market equilibrium* (in Rothschild and Stiglitz). Moreover, under the regularity condition that the type distribution either (a) contains a continuous interval or (b) includes $p = 1$, one of these two events must occur: either there is a Competitive (Nash) Equilibrium of no trade (Akerlof unravelling) or a Competitive (Nash) Equilibrium does not exist (Rothschild and Stiglitz unravelling). Thus, not only are these two concepts of unravelling different, but they are mutually exclusive and generically exhaustive of the potential occurrences in an insurance market with private information.

The mutual exclusivity result is more or less obvious in the canonical two-type binary loss model. The market unravels *à la* Rothschild and Stiglitz when the low type has an incentive to cross-subsidise the high type in order to obtain a more preferred allocation. This willingness of the good risk to subsidise the bad risk is precisely what ensures the market will not unravel *à la* Akerlof. Conversely, if the market unravels *à la* Akerlof, then the good risk is not willing to subsidise the bad risk, which implies an absence of the forces that drive non-existence in Rothschild and Stiglitz.

The intuition for the exhaustive result is also straightforward, but perhaps more difficult to see in the context of the stylised two-type model. When the support of the type distribution either (a) contains an interval or (b) contains the point $p = 1$, then trade necessarily involves cross-subsidisation of types.[3] But Rothschild and Stiglitz[2] show that a competitive (Nash) equilibrium cannot sustain such cross-subsidisation. Hence, if agents are willing to provide trade then the market unravels *à la* Rothschild and Stiglitz. In contrast, if no agents are willing to cross-subsidise the worse risks in the population, then there exists a unique Nash equilibrium at the endowment: no one on the margin is willing to pay the average cost of worse risks, and any potential contract (or menu of contracts) unravels *à la* Akerlof.[1]

The logic can be seen in the canonical two-type case. Here, the regularity condition requires one to assume that the bad risk will experience the loss with certainty. The only way for the low type (good risk) to obtain an allocation other than her endowment is to subsidise the high type (bad risk) away from her endowment. If the low type is willing to do so, the equilibrium unravels *à la* Rothschild and Stiglitz. If the low type is unwilling to do so, the equilibrium unravels *à la* Akerlof.

[3] As discussed below, Riley (1979) shows this is true in the case when the support contains an interval; I show below this is also the case when the support is discrete but includes the point $p = 1$.

In the two-type model, the assumption that the bad risk experiences the loss with certainty is clearly restrictive. However, for more general type distributions beyond the two-type case, the regularity condition is quite weak. Any distribution can be approximated quite well by distributions that have continuously distributed regions or by distributions with an arbitrarily small amount of mass at $p=1$. In this sense, the existence of pure strategy competitive equilibria of the type found by Rothschild and Stiglitz[2] that yield outcomes other than the endowment is a knife-edge result. This highlights the importance of recent and future work to aid in our understanding of how best to model competition in insurance markets.

Model

Agents have wealth w and face a potential loss of size l which occurs with probability p, which is distributed in the population according to the c.d.f. $F(p)$ with support Ψ.[4] In contrast to Rothschild and Stiglitz,[2] I do not impose any restrictions on $F(p)$.[5] It may be continuous, discrete or mixed. I let P denote the random variable with c.d.f. $F(p)$, so that realisations of P are denoted with lower-case p. Agents of type p have vNM preferences given by

$$pu(c_L)+(1-p)u(c_{NL}),$$

where u is increasing and strictly concave, c_L (c_{NL}) is consumption in the event of (no) loss. I define an allocation to be a set of consumption bundles, c_L and c_{NL}, for each type $p\in\Psi$, $A=\{c_L(p), c_{NL}(p)\}_{p\in\Psi}$.

I assume there exists a large set of risk-neutral insurance companies, J, which each can offer menus of contracts $A_j=\{c^j_L(p), c^j_{NL}(p)\}_{p\in\Psi}$ to maximise expected profits.[6] Following Rothschild and Stiglitz,[2] I define a Competitive Nash Equilibrium as an equilibrium of a two-stage game. In the first stage, insurance companies offer contract menus, A_j. In the second stage, agents observe the total set of consumption bundles offered in the market, $A^U=\cup_{j\in J}A_j$, and choose the bundle which maximises their utility. The outcome of this game can be described as an allocation which satisfies the following constraints.

---

[4] The model is adapted from Hendren (2013), which derives the no-trade condition analogue of Akerlof in the binary loss environment but does not provide any discussion of competitive equilibriums.

[5] To my knowledge, Riley (1979) was the first paper to discuss this environment with a continuum of types.

[6] In contrast to Rothschild and Stiglitz (1976), I allow the insurance companies to offer menus of consumption bundles, consistent with the real-world observation that insurance companies offer applicants menus of premiums and deductibles.

Definition 1: An allocation $A=\{c_L(p),c_{NL}(p)\}_{p\in\Psi}$ is a *Competitive Nash Equilibrium* if

1. A makes non-negative profits

$$\int_{p\in\Psi} [p(w-l-c_L(p))+(1-p)(w-c_{NL}(p))]\mathrm{d}F(p)\geqslant 0.$$

2. A is incentive compatible

$$pu(c_L(p))+(1-p)u(c_{NL}(p))\geqslant pu(c_L(\tilde{p})) + (1-p)u(c_{NL}(\tilde{p})) \quad \forall p,\tilde{p}\in\Psi.$$

3. A is individually rational

$$pu(c_L(p))+(1-p)u(c_{NL}(p))\geqslant pu(w-l)+(1-p)u(w) \quad \forall p\in\Psi.$$

4. A has no profitable deviations: For any $\hat{A}=\{\hat{c}_L(p),\hat{c}_{NL}(p)\}_{p\in\Psi}$, it must be that

$$\int_{p\in D(\hat{A})} [p(w-l-c_L(p))+(1-p)(w-c_{NL}(p))]\mathrm{d}F(p)\leqslant 0,$$

where

$$D(\hat{A})=\left\{p\in\Psi \mid \max_{\hat{p}}\{pu(\hat{c}_L(\hat{p}))+(1-p)u(\hat{c}_{NL}(\hat{p}))\}>pu(c_L(p)). + (1-p)u(c_{NL}(p))\right\}.$$

The first three constraints require that a Competitive Nash Equilibrium must yield non-negative profits, must be incentive compatible, and must be individually rational. The last constraint rules out the existence of profitable deviations by insurance companies. For A to be a competitive equilibrium, there cannot exist another allocation that an insurance company could offer and make positive profits on the (sub)set of people who would select the new allocation (given by $D(\hat{A})$).

Mutually exclusive occurrences

I first show that, in this model, the insurance market has the potential to unravel in the sense of Akerlof.[1]

Theorem 1: The endowment, $\{(w,w-l)\}_{p\in\Psi}$, *is the unique Competitive Nash Equilibrium if and only if*

$$\frac{p}{1-p}\frac{u'(w-l)}{u'(w)}\leqslant\frac{E[P \mid P\geqslant p]}{1-E[P \mid P\geqslant p]} \quad \forall p\in\Psi\setminus\{1\}. \tag{1}$$

Proof: The no-trade theorem of Hendren[7] shows that Condition (1) characterises when the endowment is the only allocation satisfying incentive compatibility, individual rationality and non-negative profits. Now, suppose $A=\{(w-l, l)\}$ and consider any allocation, $\hat{A}=\{(w-l, l)\}_{p\in\Psi}$. Suppose $\hat{A}$ delivers positive profits. Because A is the endowment, I can WLOG assume all agents choose $\hat{A}$ (since $\hat{A}$ can provide the endowment to types p at no cost). But then $\hat{A}$ would be an allocation other than A satisfying incentive compatibility, individual rationality and non-negative profits, contradicting the no-trade theorem of Hendren.[7] □

The market unravels *à la* Akerlof[1] if and only if no one is willing to pay the pooled cost of worse risks in order to obtain some insurance. This is precisely the logic of Akerlof[1] but provided in an environment with an endogenous contract space. When Condition (1) holds, no contract or menu of contracts can be traded because they would not deliver positive profits given the set of risks that would be attracted to the contract. This is precisely the unravelling intuition provided in Akerlof[1] in which the demand curve lies everywhere below the average cost curve. Notice that when this no-trade condition holds, the endowment is indeed a Nash equilibrium. Since no one is willing to pay the pooled cost of worse risks to obtain insurance, there exist no profitable deviations for insurance companies to break the endowment as an equilibrium.

Theorem 1 also shows that whenever the no-trade condition holds, there must exist a Competitive Nash Equilibrium. Thus, whenever the market unravels *à la* Akerlof,[1] the competitive equilibrium cannot unravel *à la* Rothschild and Stiglitz.[2] Unravelling in the sense of Akerlof[1] is a mutually exclusive occurrence from unravelling in the sense of Rothschild and Stiglitz.[2]

Two-type case

To relate to previous literature, it is helpful to illustrate how Theorem 1 works in the canonical two-type model of Rothschild and Stiglitz.[2] So, let $\Psi=\{p^L, p^H\}$ with $p^H > p^L$ denote the type space and let λ denote the fraction of types p^H. When $p^H<1$, Corollary 1 of Hendren[7] shows that the market cannot unravel *à la* Akerlof.[8] Hence, the mutual exclusivity of Akerlof and Rothschild and Stiglitz holds trivially. But, when $p^H=1$, the situation is perhaps more interesting. To see this, Figure 1 replicates the canonical Rothschild and Stiglitz[2] graphs in the case when $p^H=1$.

The vertical axis is consumption in the event of a loss, c_L; the horizontal axis is consumption in the event of no loss, c_{NL}. Point 1 is the endowment $\{w-l, w\}$. Because $p^H=1$, the horizontal line running through the endowment represents both the indifference curve of type p^H and the actuarially fair line for type p^H. Notice that

[7] Hendren (2013).
[8] If $p^H<1$, then Eq. (1) would be violated at $p^H=1$ by the assumption of strict concavity of u.

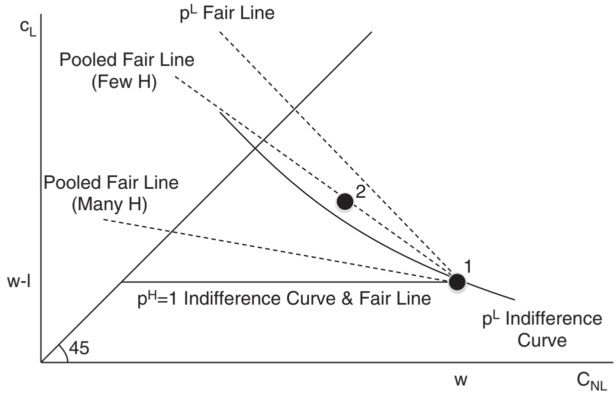

Figure 1. Two-type model with $p^H = 1$.

type p^H prefers any allocation bundle that lies above this line (intuitively, she cares only about consumption in the event of a loss).

The low type indifference curve runs through the endowment (point 1) and intersects the 45-degree line parallel to her actuarially fair line. As noted by Rothschild and Stiglitz,[2] the outcomes in this environment depend crucially on the fraction of low vs high types. Figure 1 illustrates the two cases. If there are few p^H types (λ is small), then point 2 is a feasible pooling deviation from the endowment. When such a deviation is feasible, unravelling *à la* Akerlof does not occur: the low type is willing to pay the pooled cost of the worse risks. But the existence of such a deviation is precisely what breaks the existence of a competitive equilibrium in Rothschild and Stiglitz.[2] Point 2 involves pooling across types and cannot be a competitive equilibrium. Hence, if the market unravels *à la* Akerlof, the endowment is the unique competitive equilibrium. If the market unravels *à la* Rothschild and Stiglitz, there exists implementable allocations other than the endowment and Akerlof's notion of unravelling does not occur.

As one might gather from Figure 1, when there is a type arbitrarily close to 1, the only feasible competitive equilibrium is the endowment, i.e., there is no possibility of a pair of separating contracts with the p^L-type receiving partial coverage in equilibrium. I now make this point in the more general setting that does not require any mass of types at $p^H = 1$.

Exhaustive occurrences

I now show that not only are these two notions of unravelling mutually exclusive, but they are also exhaustive of the possibilities that can occur in model environments when the type distribution satisfies the following regularity condition.

Assumption 1: Either (a) there exists $a<b$ such that $[a, b]\subset\Psi$ or (b) $1\in\Psi$ (i.e. $F(p)<1$ for all $p<1$).

Assumption 1 assumes that the support of the type distribution includes either (a) a continuous interval or (b) the point $p=1$. Note any distribution can be approximated arbitrarily closely by distributions satisfying this regularity condition.

I now show that competitive equilibriums cannot sustain cross-subsidisation, an insight initially provided in Rothschild and Stiglitz.[2]

Lemma 1: (Rothschild and Stiglitz[2]) Suppose A is a Competitive Nash Equilibrium. Then

$$pc_L(p)+(1-p)c_{NL}(p) = w-pl \quad \forall p \in \Psi.$$

Proof: See Rothschild and Stiglitz[2] for a full discussion. Clearly, competition requires zero profits on any consumption bundle. Hence, it suffices to show that no allocation can pool types into the same consumption bundle other than the endowment. Suppose multiple types are allocated to the same consumption bundle (distinct from the endowment). Then, an insurance company could offer a new allocation, $\hat{A}$, arbitrarily close to the current allocation but that is only preferred by the lowest p in the pool. Hence, this allocation will provide strictly positive profits and will render the original consumption bundle unprofitable, thereby breaking the Nash equilibrium with pooling. Therefore, $pc_L(p)+(1-p)c_{NL}(p)=w-pl$ for all p. □

Now, consider the two cases in Assumption 1. If $p=1$ is in the support of the type distribution, it is straightforward to see that there cannot exist any Competitive Nash Equilibrium other than the endowment, since trade requires cross-subsidisation towards types near $p=1$. Now, suppose that $p=1$ is not in the support of the type distribution but that the type distribution contains an interval. Here, the non-existence of a Nash equilibrium is perhaps less straightforward, but a proof is actually contained in Riley.[9,10] Given the interval $[a, b]\subset\Psi$, Riley's derivations show that there exists a profitable deviation which pools types near $p=b$; hence there can be no Nash equilibrium other than the endowment. Theorem 2 follows.

Theorem 2: Suppose Assumption 1 *holds. Then, there exists a Competitive Nash Equilibrium if and only if Condition* (1) *holds.*

When Assumption1 holds, trade requires risk types to be willing to enter risk pools which pool *ex ante* heterogeneous types. Such *ex ante* pooling is not possible in a

[9] Riley (1979).
[10] See Theorem 3 in Section 4, pages 341–343.

Competitive Nash Equilibrium. Therefore, when the no-trade condition (1) does not hold, there does not exist any Competitive Nash Equilibrium: the equilibrium unravels *à la* Rothschild and Stiglitz.[2]

Conclusion

This memo uses a generalised binary model of insurance to highlight the distinction between Akerlof's notion of unravelling, in which an equilibrium exists in which no trade can occur, and Rothschild and Stiglitz' notion of unravelling, in which a standard notion of competitive equilibrium (pure strategy Nash) cannot exist. In the latter case, there are (Pareto) gains to trade; but in a generic sense described in Assumption 1, the realisation of these gains to trade require cross-subsidisation of types. Such cross-subsidisation cannot be sustained under the canonical notion of competition.[11] Hence, Akerlof unravelling shows when private information can lead to the absence of trade in insurance markets. Rothschild and Stiglitz unravelling shows that the canonical model of competition (Nash equilibrium) is inadequate to describe the behaviour of insurance companies in settings where there are potential gains to trade.

References

Akerlof, G. (1970) The market for lemons: Qualitative uncertainty and the market mechanism, *Quarterly Journal of Economics* 84(3): 488–500.

Hendren, N. (2013) Private information and insurance rejections, *Econometrica* 81(5): 1713–1762.

Miyazaki, H. (1977) The rat race and internal labor markets, *The Bell Journal of Economics* 8(2): 394–418.

Riley, J. (1979) Informational equilibrium, *Econometrica* 47(2): 331–359.

Rothschild, M. and Stiglitz, J. (1976) Equilibrium in competitive insurance markets: An essay on the economics of imperfect information, *The Quarterly Journal of Economics* 90(4): 629–649.

Spence, A. (1978) Product differentiation and performance in insurance markets, *Journal of Public Economics* 10(3): 427–447.

Wilson, C. (1977) A model of insurance markets with incomplete information, *Journal of Economic Theory* 16(2): 167–207.

About the Author

Nathaniel Hendren is an assistant professor of economics at Harvard.

[11] In the modified models of competition, proposed by Miyazaki (1977), Wilson (1977) or Spence (1978), such gains to trade will be realised in equilibrium.