Networks, Expectations and Convention

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EXPECTATIONS, NETWORKS, AND CONVENTIONS

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(i) information (signals) (ii) interpretation (priors) (ii) coordination concerns (interaction)

beliefs and higher-order beliefs networks

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Contribution: Analyze effects of (i), (ii), (iii) together via reduction of all three to a network. Yields **unification** and **new purely informational results**.

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Agents $i \in N$

External state $\theta \in \Theta$

i's fundamental $y^i : \Theta \longrightarrow [-M, M]$

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ex post payoff

$$u^i = -\beta \sum_j \gamma^{ij} (a^i - a^j)^2 - (1-\beta)\,(a^i - y^i(\theta))^2$$

$\beta \in (0,1)$

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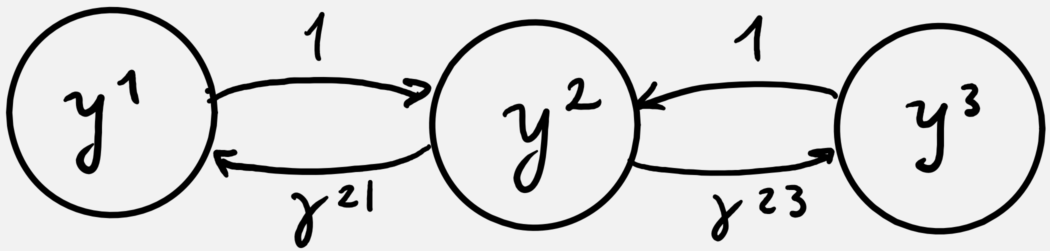
Ex. Net Game, Complete Info.

$$u^i = -\beta \sum_j \gamma^{ij} (a^i - a^j) - (1-\beta)(a^i - y^i)^2$$



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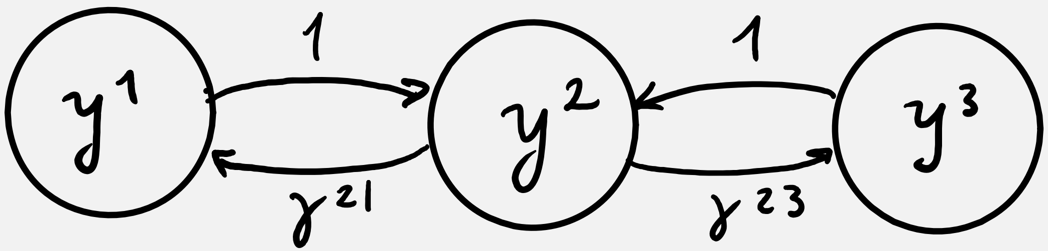

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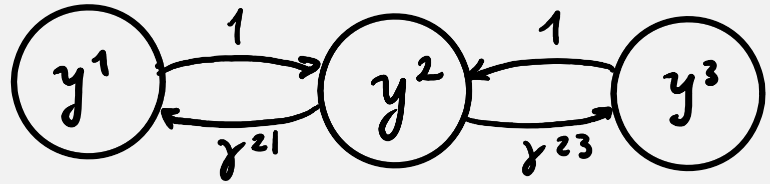

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QUESTION: How does play depend on (i) information; (ii) priors (iii) network ?

FOCUS: Conventions: play as $\beta \uparrow 1$.

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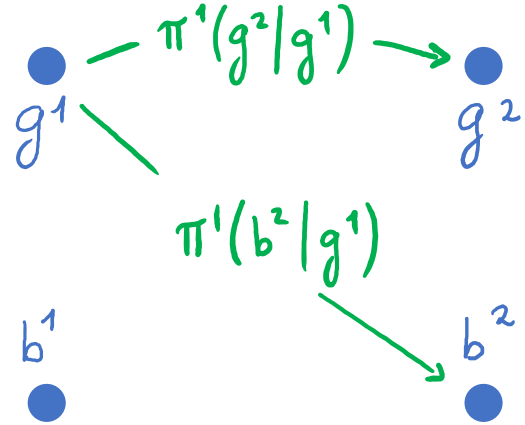

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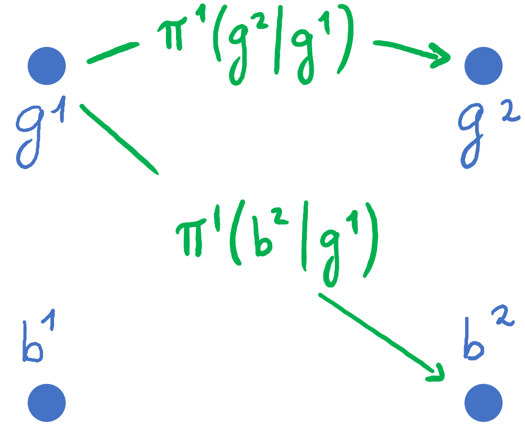
$$a_{eqm} = (1-\beta)(I - \beta B)^{-1} f$$

where $f(t^i) = (E^i y^i)(t^i)$



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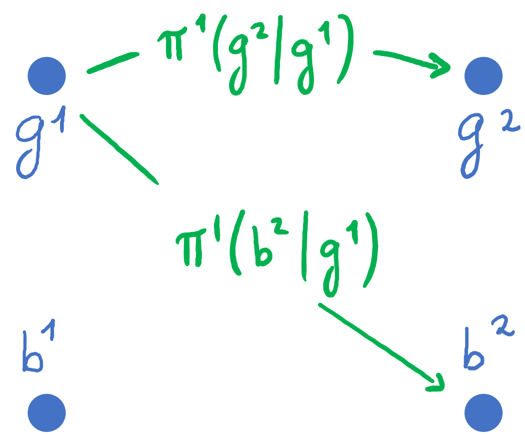

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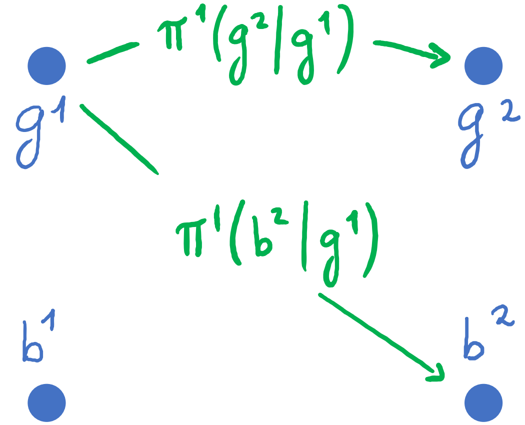

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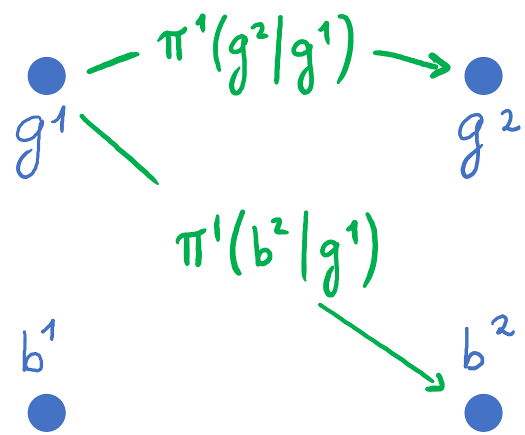

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$$a = BR(a) \iff a = \beta B a + (1-\beta) f$$

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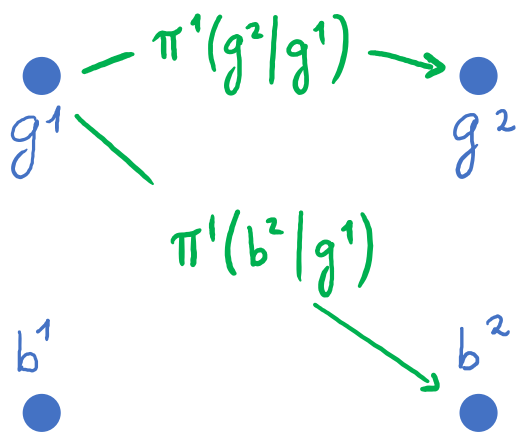

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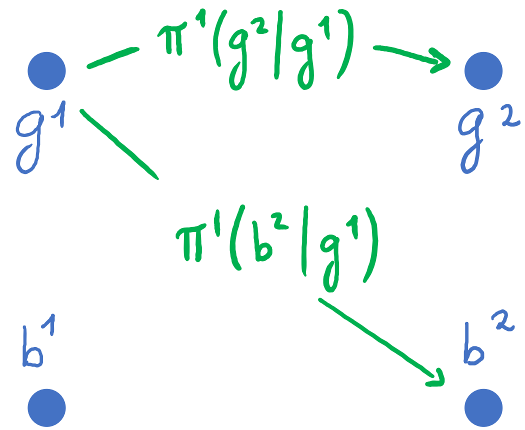

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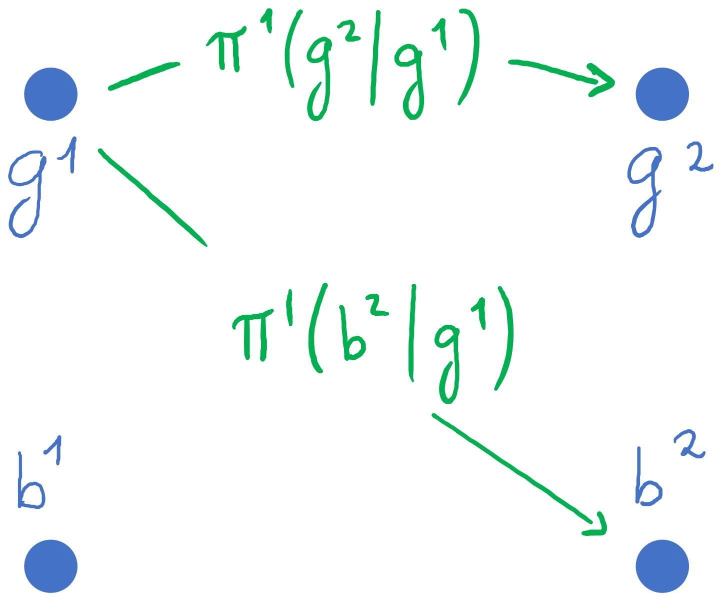

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where p is unique $p \in \Delta(S)$ s.t. $pB = p$

I.e. p is the stationary distribution of B, viewed as a Markov chain.

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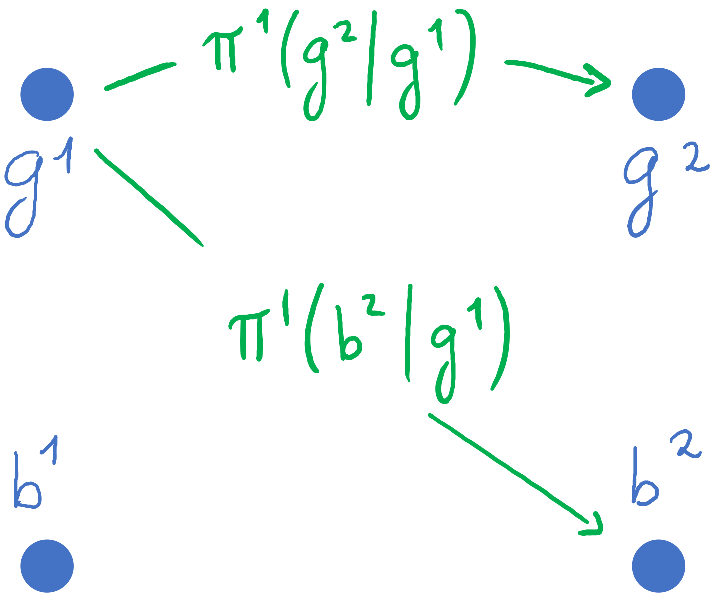
Shin and Williamson (*GEB* 96) "How Much Common Belief is Necessary for a Convention?"

Morris (1997) "Interaction Games"
Morris (*REStud* 2000) "Contagion"



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CONTAGION OF OPTIMISM

Suppose each i is certain each counterparty has $E^j y \geq E^i y + \delta$, unless $E^i y \geq \bar{f}$ — then $E^j y \geq E^i y$.

Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f}$

Reason: for t^i s.t. $f(t^i) < \bar{f}$, the B process can only move upward.

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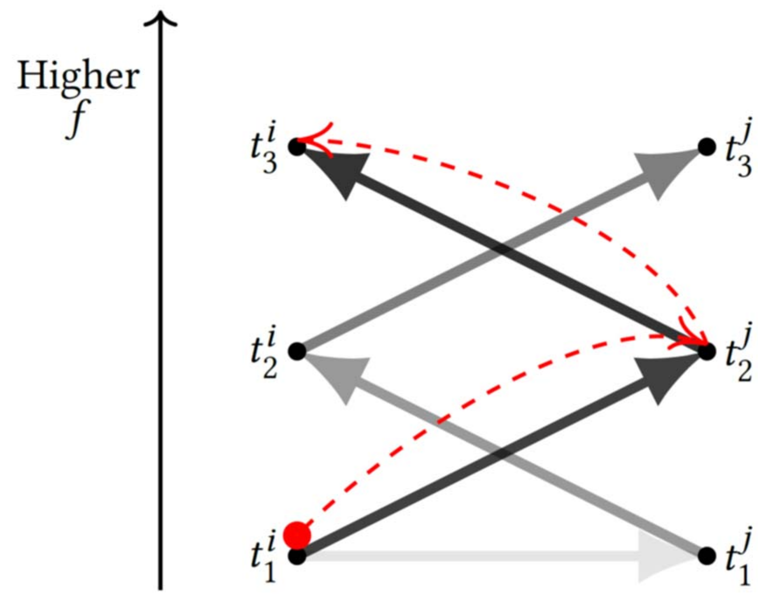

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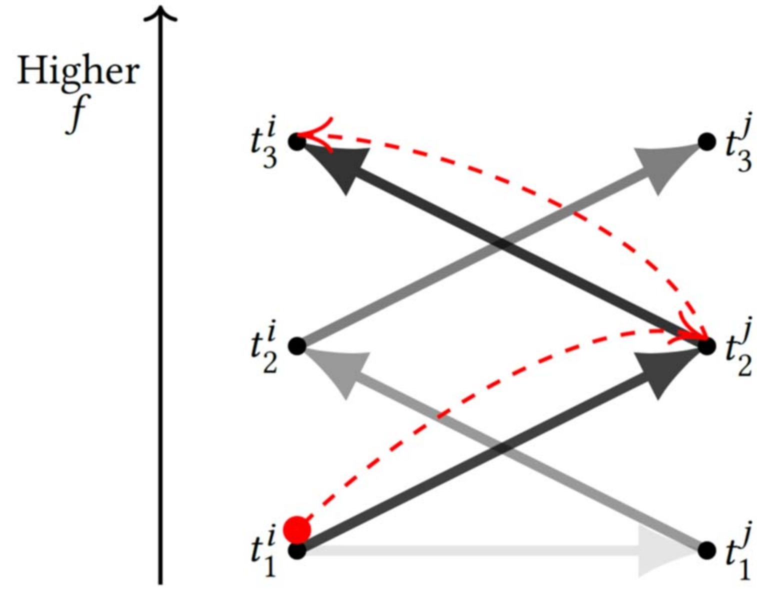
Contagion of Optimism

Suppose each i is second-order optimistic (on avg)

$\sum_j \gamma^{ij} E^i E^j y \geq E^i y + \delta$, unless $E^i y \geq \bar{f}$ — then " " $\geq E^i y$.

Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f} / (1 + \varepsilon/\delta)$

Reason: for t^i s.t. $f(t^i) < \bar{f}$, B process moves upward on average



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Suppose each i is second-order optimistic

$$\sum_j \gamma^{ij} E^i E^j y \geq E^i y + \delta,$$

unless $E^i y \geq \bar{f} -$

then " " $\geq E^i y$.

Then $c(y; \vec{\pi}, \Gamma) \geq \bar{f} / (1 + \varepsilon/\delta)$

Reason: for t^i s.t. $f(t^i) < \bar{f}$, B process moves upward on average

PROP 1 $c(\vec{y}; \vec{\pi}, \vec{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i)$

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PROOF: Take MC $W_1, W_2, \ldots,$ with ergodic dist p. Suppose $\exists\ \delta, \varepsilon$ s.t.

$$f(s) < \bar{f} \Rightarrow \mathbb{E}_{W_0 = s}[f(W_1)] \geq f(s) + \delta$$

$$f(s) \geq \bar{f} \Rightarrow \mathbb{E}_{W_0 = s}[f(W_1)] \geq f(s) - \varepsilon$$

Then $p(s : f(s) \geq \bar{f}) \geq \dfrac{1}{1 + \varepsilon/\delta}$

Follows from

$$\mathbb{E}_{W_0 \sim p}[W_1 - W_0] = 0$$

Harrison and Kreps (*QJE* 1978) "Speculative Investor Behavior…"

Izmalkov and Yildiz (*AEJ:Micro* 2010) "Investor Sentiments"

Han and Kyle (*MS* 2017) "Speculative Equilibrium with Differences in Higher-Order Beliefs"

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KEY IDEA: Incomplete-info. aspect can be reduced to network aspect
$\to$ analyze how info. struct. matters.

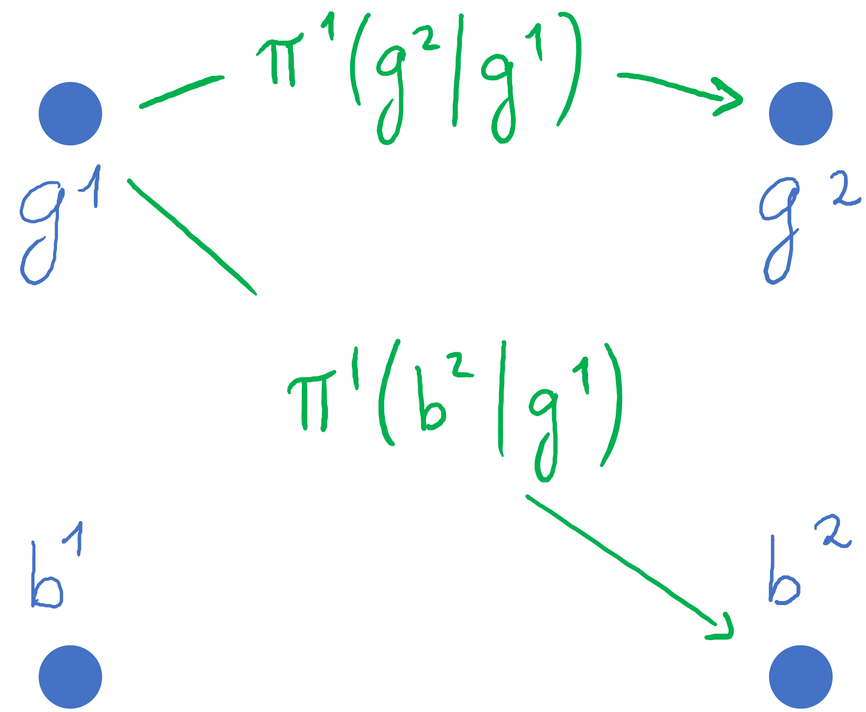
KEY DEVICE: "interaction structure"

nodes:

$$S = \bigcup_i T^i$$

edges:

$$B(t^i, t^j) = \gamma^{ij} \pi^i (t^j | t^i)$$



APPLICATIONS

1 Contagion of Optimism

2 (Pseudo) Common Prior
Influence $\propto$ net centrality

3 Tyranny of least-informed

Common Priors & Influence

Def Common priors over signals (CPS)

π^i all compatible w/ a $\hat{\mu} \in \Delta(T)$

$\Downarrow$

$\exists$ priors $(\mu^i)_{i \in N}$ s.t.

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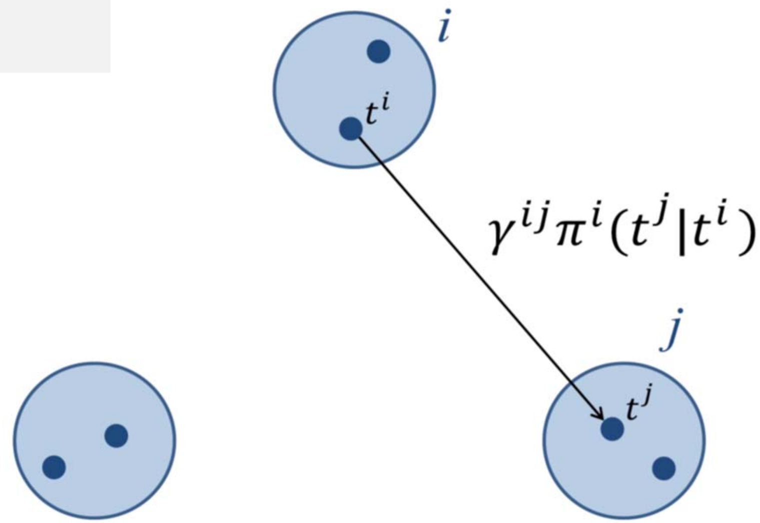
$$\sum_i p(t^i) = e^i$$



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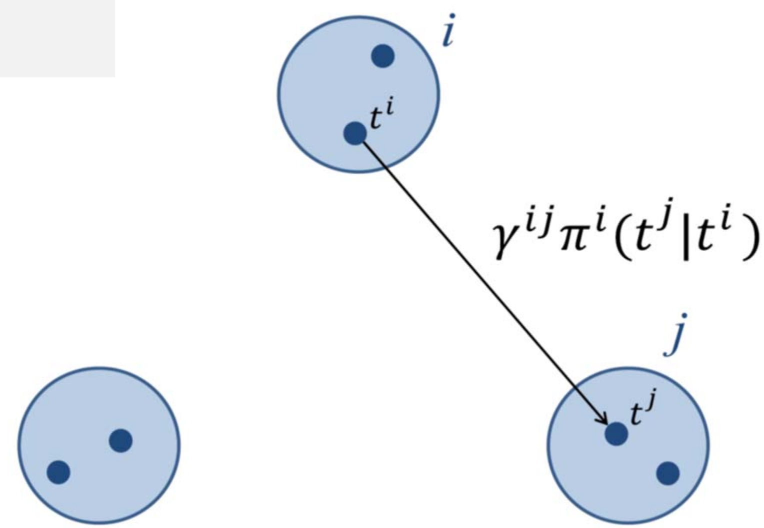

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Ballester, Calvó-Armengol, and Zenou (*Econometrica* 2006) "Who's Who in Networks"

Calvó-Armengol, de Marti, and Prat (*TE* 2015) "Communication and Influence"

Bergemann, Heumann, Morris (2017) "Information and Interaction"

Myatt and Wallace (2017), "Information Acquisition and Use by Networked Players"

Higher-Order Average Expectations

$$x^i_{t^i}(1) = E^i[y^i \mid t^i]$$

1st-order expectation of y^i given i's info

$$x^i_{t^i}(2) = \sum_j \gamma^{ij} E^i[x^j(1) \mid t^i]$$

2nd-order avg. expectation
an average of 1st-order exp. given i's info

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$$a_{eqm} = (1-\beta)(I - \beta B)^{-1} f = (1-\beta) \sum_{n=0}^{\infty} \beta^n B^n f$$

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Samet (*JET* 98) “Iterated Expectation and Common Priors”

Our companion paper: “Higher-Order Expectations”

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for all $i \neq 1$ $q^i \geq 1-\varepsilon$ at most ε-noisy

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Ex 2 agents, incomplete info

$$-u^i = \beta(a^i - a^j)^2 + (1-\beta)(a^i - y^i(\theta))^2$$

$\theta \in \{\theta_1, \dots, \theta_K\}$

$\rho^i \in \Delta(\Theta)$ i's prior

$t^i \in \{t^i_1, \dots, t^i_K\}$ matches θ w.p. q^i

Otherwise full support noise.

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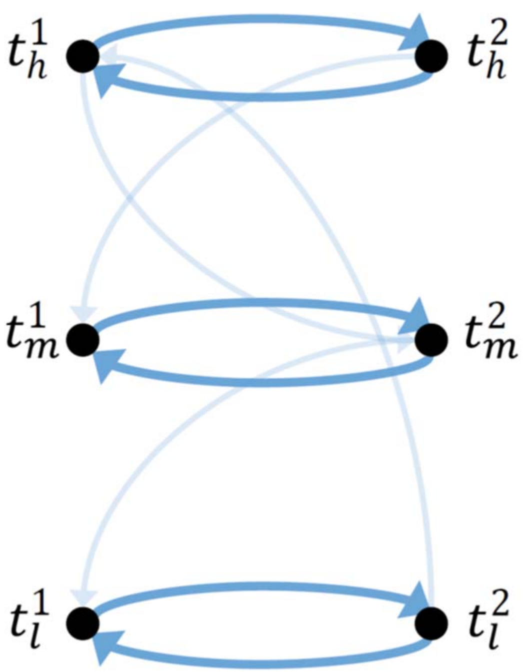

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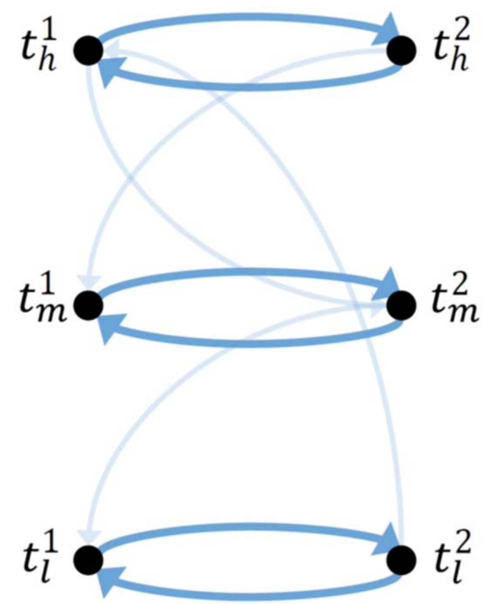

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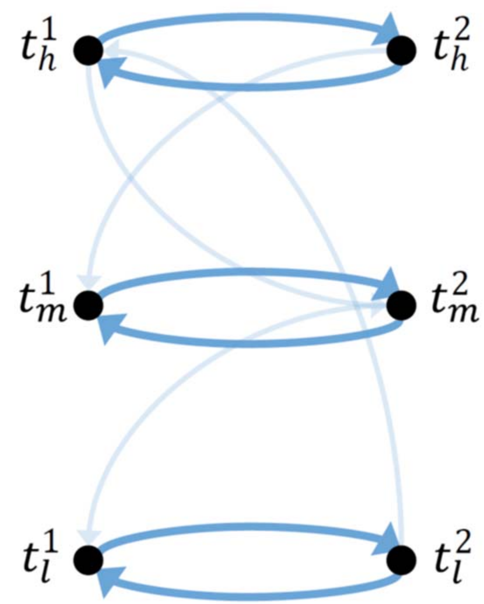
- each $i \neq 1$ knows θ
- 1's info. unchanged



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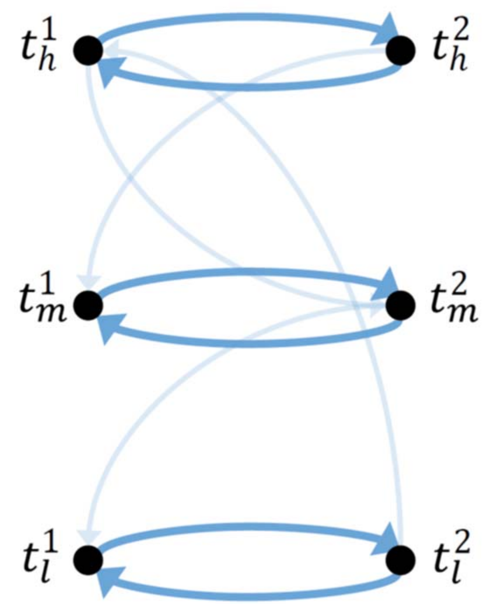

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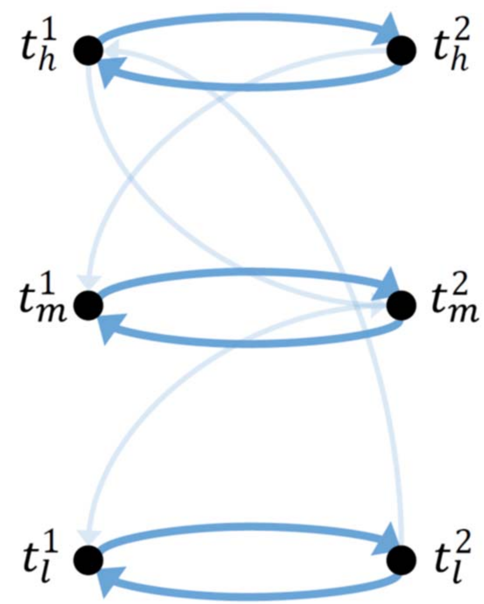

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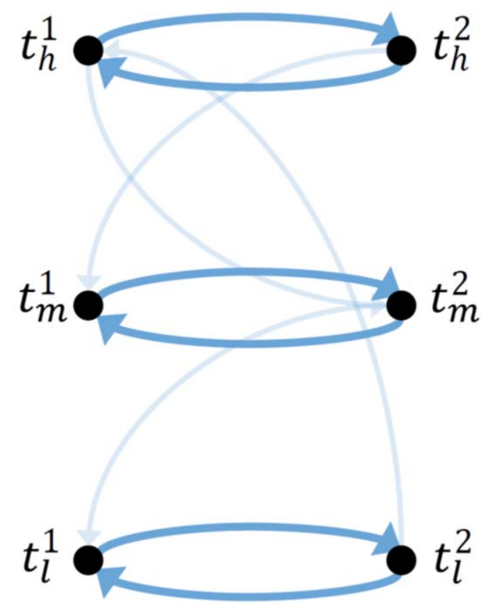
Cho and Meyer (00) “Markov chain sensitivity measured by mean first passage times”



CONCLUSION

Interaction structure captures (interim) beliefs and network simultaneously: a method for studying how behavior depends on

(i) information (signals) | (ii) interpretation (priors) | (ii) coordination concerns (interaction)

General characterization of conventions in terms of eigenvector **centrality** in **interaction structure**. Reduction to a complete-information network game.

Illustrate with three applications.

Contagion of optimism – small local bias (in common direction) leads to extreme conventions.

Under **common prior over signals**, agents' prior expectations matter in proportion to their **centrality in the network** Γ only.

Under **common interpretation of signals** and precise private information, get **tyranny of the least-informed**.