

# INTELLECTUAL PROPERTY RIGHTS POLICY, COMPETITION AND INNOVATION

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**Daron Acemoglu**  
Massachusetts Institute of Technology

**Ufuk Akcigit**  
University of Pennsylvania

## Abstract

To what extent and in what form should the intellectual property rights (IPR) of innovators be protected? Should a company with a large technology lead over its rivals receive the same IPR protection as a company with a more limited advantage? In this paper, we develop a dynamic framework for the study of the interactions between IPR and competition, in particular to understand the impact of such policies on future incentives. The economy consists of many industries and firms engaged in cumulative (step-by-step) innovation. IPR policy regulates whether followers in an industry can copy the technology of the leader. We prove the existence of a steady-state equilibrium and characterize some of its properties. We then quantitatively investigate the implications of different types of IPR policy on the equilibrium growth rate and welfare. The most important result from this exercise is that full patent protection is not optimal; instead, optimal policy involves state-dependent IPR protection, providing greater protection to technology leaders that are further ahead than those that are close to their followers. This is because of a *trickle-down* effect: providing greater protection to firms that are further ahead of their followers than a certain threshold increases the R&D incentives also for all technology leaders that are less advanced than this threshold. (JEL: O31, O34, O41, L16)

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## 1. Introduction

What is the optimal extent and form of intellectual property rights (IPR) protection? Should a firm with a large technology lead receive the same IPR protection as a company with a more limited technological lead, or should IPR policy be coupled with antitrust and used to limit the monopoly power of technology leaders? Despite broad consensus that innovation is central to the long-run performance of an economy, there

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E-mail addresses: [daron@mit.edu](mailto:daron@mit.edu) (Acemoglu); [uakcigit@econ.upenn.edu](mailto:uakcigit@econ.upenn.edu) (Akcigit)

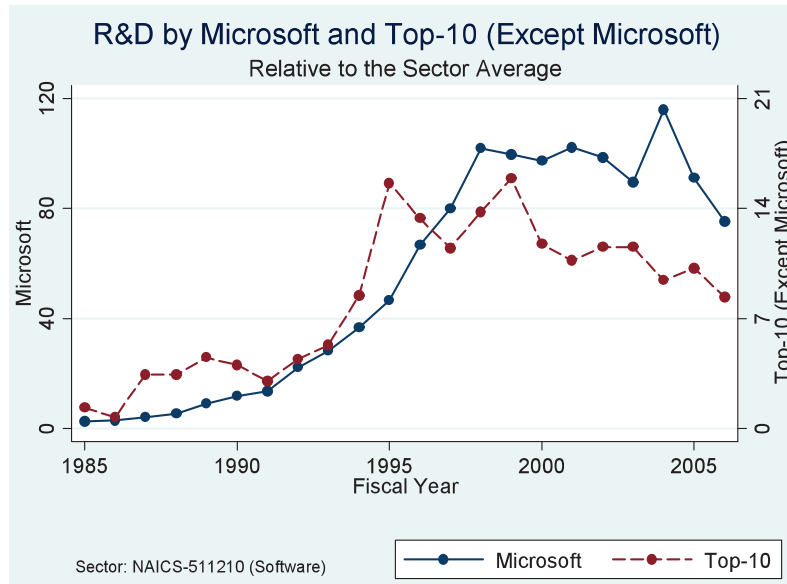


FIGURE 1. The evolution of R&D by Microsoft and by other top ten publicly traded R&D investors in the IT sector relative to the sector average before and after the start of the DOJ case. All data are from COMPUSTAT. Top ten firms is determined by the highest ten R&D investors (except Microsoft) in 1998: CA Inc, Continuum Inc, Intergraph Corp, Sterling Software Inc, Oracle Inc, Adobe Inc, Symantec Corp, Electronic Arts Inc, Sybase Inc, Intuit Inc. The patterns are similar for the top ten investors in 2000 or 1990, or if benchmarked it to the median of the industry rather than the mean.

is no consensus on the answers to such questions. A large literature on IPR (discussed in what follows) focuses on the *static* trade-offs between the positive incentive benefits of IPR protection and its costs in terms of reducing competition and increasing markups. In this paper, we argue that *dynamic* trade-offs between IPR protection and competition, which have so far been overlooked, may be equally or more important for developing answers to these questions.

These issues and the importance of these questions are highlighted by several recent high-profile cases. For example, motivated by antitrust concerns, a recent ruling of the European Commission ordered Microsoft to share secret information about its operating system and products with other software companies (*New York Times*, 22 December 2004).<sup>1</sup> Similar issues were also central to the US Department of Justice (DOJ) case against Microsoft, which started on 18 May 1998 and ultimately resulted in a ruling against Microsoft. Figure 1 shows the evolution of R&D by Microsoft and by other top ten publicly traded R&D investors in the IT sector relative to the sector average before and after the start of the DOJ case.

1. In addition to the Microsoft case, the issue of technological lead has been central in the Department of Justice investigations of Intel (*New York Times*, 11 May 2009) and the debates about Google's market share (*New York Times*, 21 February 2009).

The relative R&D spending by Microsoft and other industry leaders, which had been steadily—perhaps even exponentially—increasing since the mid-1980s, appear to decline after the DOJ action. While one might expect R&D by Microsoft to slow down for a variety of reasons, it is not obvious why there should be a relative decline in the R&D of other top companies, since they partly benefited from the weakening of, and the restrictions imposed on, Microsoft. This relative decline may have been caused by a combination of a slowdown in the R&D activities of these other companies or an increase in the R&D investments of smaller IT firms in response to the DOJ action, or by entirely different and unrelated factors. A systematic investigation of these issues requires a dynamic equilibrium framework where R&D activities of different types respond to changes in IPR and competition policy. In this paper, we take a step in this direction.

Our framework builds on and extends the step-by-step innovation models of Aghion, Harris, and Vickers (1997) and Aghion et al. (2001), where a number of (typically two) firms engage in price competition within an industry and undertake R&D in order to improve their production technology. The technology gap between the firms determines the extent of the monopoly power of the leader, and hence the price markups and profits. The purpose of R&D by the follower is to catch up and surpass the leader (as in standard Schumpeterian models of innovation, for example, Reinganum 1981, 1985; Aghion and Howitt 1992; Grossman and Helpman 1991), while the purpose of R&D by the leader is to escape the competition of the follower and increase its markup and profits. Despite the dynamic nature of these models, their policy implications are still mostly based on the same static trade-off already mentioned. For this reason, for example, Aghion et al. (2001, p. 481) conjecture that IPR protection should be limited and particularly so for firms with larger technological leads over their rivals (which face less competition and thus have greater monopoly power).

We extend these existing models in several directions. Most importantly, we explicitly introduce *state-dependent* patent/IPR protection policy, meaning a policy that makes the extent of patent or intellectual property rights protection conditional on the technology gap between different firms in the industry. As in racing-type models in general (for example, Harris and Vickers 1985, 1987; Budd, Harris, and Vickers 1993), a large gap between the leader and the follower discourages R&D by both. Consequently, overall R&D and technological progress are greater when the technology gap between the leader and the follower is relatively small.<sup>2</sup> One may then expect that full patent protection may be suboptimal in a world of step-by-step competition and permitting followers to copy or use the leaders' technologies would be particularly beneficial in industries where there is a large technology gap between leaders and followers.<sup>3</sup> However, crucially, this reasoning ignores the dynamic

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2. Aghion et al. (2005) provide empirical evidence from British industries consistent with the view R&D increases when there is a smaller technological gap between firms. See also Aghion and Griffith (2007).

3. This is indeed the basis of the conjecture of Aghion et al. already mentioned.

incentive effects, which are our main focus in this paper and emerge more clearly when IPR policy is explicitly state dependent.

Our analysis establishes that the *opposite* of the conjecture already mentioned is always true in such a dynamic equilibrium framework: optimal IPR policy should provide *greater* protection to technologically more advanced leaders. Underlying this result is what we refer to as the *trickle-down of incentives*: providing relatively low protection to firms with limited leads and greater protection to those that have greater leads not only improves the incentives of firms that are technologically advanced, but also encourages R&D by those that have limited leads because of the prospect of reaching levels of technology gaps associated with greater protection. A corollary of this result is that full IPR protection is not optimal, and there should be limited, but state-dependent, IPR protection for firms with only limited technology leads over their rivals.

More specifically, we show that in contrast to the standard disincentive effects of uniform relaxation of IPR policy, state-dependent relaxation that provides *greater* protection to technologically more advanced firms creates *a positive incentive effect*. This is because when a particular state for the technology leader (say being  $n^*$  steps ahead of the follower) becomes more profitable, this increases the incentives to perform R&D not only for leaders that are  $n^* - 1$  steps ahead, but for *all* leaders with a lead of size  $n \leq n^* - 1$ . It is this trickle-down effect that generates the positive incentive effect and makes state-dependent IPR, with greater protection for firms that are technologically more advanced than their rivals, preferable to uniform IPR.

We start with a partial equilibrium model and provide an explicit characterization of the trickle-down effect. We then develop a richer dynamic general equilibrium framework which allows a systematic analysis of how innovation depends on R&D by technology leaders and followers. Our baseline model focuses on *quick catch-up*, meaning that a follower can catch up with the technology leader with a single innovation regardless of the size of the gap between them. For this environment, we establish the existence of a stationary equilibrium and characterize some of its properties. We then study the form of optimal (welfare maximizing) IPR and competition policy quantitatively. The same effects as in the partial equilibrium analysis make state-dependent relaxation of IPR optimal. Quantitatively, we find that optimal state-dependent IPR policy can increase the growth rate of the economy from 1.86% to 2.04%, and does so with *fewer* workers employed in the R&D sector (because R&D workers are reallocated towards firms where their efforts directly lead to productivity growth). In contrast, uniform relaxation of IPR policy reduces both welfare and growth. These patterns are quite robust to different parameter values.

We next show how the framework can be extended to study these issues when there is *slow catch-up*, meaning that followers close the gap between themselves and technology leaders only gradually. The presence of slow catch-up also enables us to introduce different types of R&D efforts and different dimensions of IPR policy, in

particular, licensing and patent infringement fees.<sup>4</sup> We show that the trickle-down effect and the result that optimal IPR policy should be state dependent and provide *greater protection* to technologically more advanced firms are robust in these alternative environments. In most cases, optimal IPR policy also increases growth by a similar magnitude to our baseline model (though in some cases it increases welfare but not necessarily growth).<sup>5</sup>

Our paper is a contribution both to the IPR protection and the endogenous growth literatures. Previous work has focused on the static trade-off between ex-post monopoly rents and ex-ante R&D incentives (for example, Arrow 1962; Reinganum 1981; Tirole 1988; Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992; Green and Scotchmer 1995; Scotchmer 1999; Gallini and Scotchmer 2002; O'Donoghue and Zweimuller 2004).<sup>6</sup> Much of the literature discusses the trade-off between these two forces to determine the optimal length and breadth of patents. For example, Klemperer (1990) and Gilbert and Shapiro (1990) show that optimal patents should have a long duration in order to provide inducement to R&D, but a narrow breadth so as to limit monopoly distortions. A number of other papers, for example, Gallini (1992) and Gallini and Scotchmer (2002), reach opposite conclusions.

Another branch of the literature, including the seminal paper by Scotchmer (1999) and the recent interesting papers by Hopenhayn, Llobet, and Mitchell (2006) and Hopenhayn and Mitchell (2001, 2011), adopts a mechanism design approach to the determination of the optimal patent and intellectual property rights protection system. For example, Scotchmer (1999) derives the patent renewal system as an optimal mechanism in an environment where the cost and value of different projects are unobserved and the main problem is to decide which projects should go ahead. Hopenhayn, Llobet, and Mitchell (2006) consider optimal patent policy in the context of a model of sequential innovation with heterogeneous quality and private information. They show that allowing for a choice from a menu of patents will be optimal in this context. Hopenhayn and Mitchell (2011) build on an earlier version of our paper, Acemoglu and Akcigit (2006), and derive a form of trickle-down effect using a mechanism design approach in a model with recurring innovations.

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4. In particular, in this regime, we allow firms to undertake frontier as well as catch-up R&D. With frontier R&D, they can build on the technology leader's knowledge base, and if successful, they immediately overtake the leader, but might be liable for a *patent infringement fee*. We also allow followers to *license* the innovation of the technology leader by paying a prespecified license fee—that is, a *compulsory licensing* where the license fee is determined by IPR policy. We also show that voluntary licensing agreements would not achieve the same results, so our analysis establishes a potential need for compulsory licensing policy. Previous work emphasizing the importance of compulsory licensing includes Tandon (1982), Gilbert and Shapiro (1990), and Kremer (2002).

5. We also show that both licensing and the possibility of frontier R&D (subject to infringement fees) contributes to growth and welfare.

6. Boldrin and Levine (2008, 2009) or Quah (2004) argue that patent systems are not necessary for innovation.

Our paper builds on and extends Aghion, Harris, and Vickers (1997) and Aghion et al. (2001).<sup>7</sup> Although our model builds on these papers, it also differs from them in a number of significant ways. First, we introduce state-dependent IPR policy. This is crucial for most of the results in the paper, including the trickle-down of incentives and the form of optimal IPR. Second, we also introduce and analyze the slow catch-up regime, and in this context, we allow for compulsory licensing and for leapfrogging, which makes the followers directly contribute to the economic growth. We provide a full quantitative analysis of state-dependent IPR policy under these different scenarios. Third, our economy is a full general equilibrium model with competition between production and R&D for scarce labor.<sup>8</sup> Finally, we provide a general existence result and a number of analytical results for the general model (with or without IPR policy), while previous literature has focused on the special cases where innovations are either *drastic* (so that the leader never undertakes R&D) or very small, and has not provided existence or general characterization results for steady-state equilibria.

Lastly, our results are also related to the literature on tournaments and races (for example, Fudenberg et al. 1983; Harris and Vickers 1985, 1987; Choi 1991; Budd, Harris, and Vickers 1993; Taylor 1995; Fullerton and McAfee 1999; Baye and Hoppe 2003; Moscarini and Squintani 2010). This literature considers the impact of endogenous or exogenous prizes on effort in tournaments, races or R&D contests. In terms of this literature, state-dependent IPR policy can be thought of as *state-dependent handicapping* of different players (where the state variable is the gap between the two players in a dynamic tournament). To the best of our knowledge, these types of schemes have not been considered in the literature.

The rest of the paper is organized as follows. Section 2 introduces the partial equilibrium model and analytically demonstrates the trickle-down effect. Section 3 presents our baseline environment (where a successful innovation by followers closes the entire gap with technology leaders in one step, that is, there is quick catch-up). Section 4 proves the existence of a steady-state equilibrium and characterizes some of its key properties under both uniform and state-dependent IPR policy. Section 5 defines the social welfare objective and outlines our quantitative methods. Section 6 characterizes the structure of optimal IPR policy quantitatively. Section 7 extends the model to allow for slow catch-up, compulsory license fees and leapfrogging, and quantitatively characterizes the structure of optimal IPR policy under different

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7. Two other papers are also related: Segal and Whinston (2007) who analyze the impact of anti-trust policy on economic growth in a related model of step-by-step innovation, and Acemoglu, Gancia, and Zilibotti (2012) who analyze the role of IPR policy in a model with innovation and standardization based on imitation of new products.

8. This general equilibrium aspect is introduced to be able to close the model economy without unrealistic assumptions and makes our economy more comparable to other growth models (Aghion et al. 2001 assume a perfectly elastic supply of labor). We show that the presence of general equilibrium interactions does not significantly complicate the analysis and it is still possible to characterize the steady-state equilibrium. The endogenous allocation of labor between different firms and between production and R&D also enables us to show that optimal IPR policy can increase the growth rate of the economy while also reducing the fraction of the workforce employed in R&D (see Section 6 for details).

combinations of these policies. Section 8 concludes. The Web Appendix, which contains additional results and the proofs of all the results stated in the text, is available online.<sup>9</sup>

## 2. A Partial Equilibrium Illustration

We first illustrate the main economic force in this paper, the trickle-down effect, using a partial equilibrium model. Consider the following infinite horizon, step-by-step R&D race between two competing firms in continuous time. Each firm maximizes the expected net present discounted value of *net profits*, defined as operating profit minus R&D cost,

$$\mathbb{E}_t \int_t^\infty \exp(-r(s-t)) [\pi_i(s) - \Phi_i(s)] ds,$$

where  $\mathbb{E}_t$  denotes expectation at time  $t$ ,  $r > 0$  is the interest rate,  $\pi_i(t)$  is the instantaneous operating profit flow and  $\Phi_i(t)$  represents the R&D cost of firm  $i$  at time  $t$ . In this game, firm  $i \in \{1, 2\}$  invests in R&D to advance its position relative to its rival  $i' \neq i$ . Suppose that the positions of both firms in this race can be characterized by integer values on the real line, and denote the distance of firm  $i$  from its rival at time  $t$  by  $n_i(t)$ . In the partial equilibrium model, we simplify the analysis by following Aghion et al. (2001) and Aghion et al. (2005) in assuming that the maximum technology gap between a leader and a follower is 2; this assumption is relaxed in the full general equilibrium model analyzed in the rest of the paper. For now it simplifies the analysis by ensuring that the relative position of firm  $i$  can take five possible values,  $n_i(t) \in N_I \equiv \{-2, -1, 0, 1, 2\}$ . Let us denote the absolute gap between the two firms by  $n(t) \equiv \max\{n_i(t), n_{-i}(t)\}$ , and suppress the time subscripts to simplify notation.

The payoffs in this game are assumed to be stationary and only a function of the relative distance between the firms, thus represented by  $\pi : N_I \rightarrow \mathbb{R}_+$  (see equation (20) in Section 3). In particular,  $\pi_{n_i} \geq 0$  is the instantaneous payoff that firm  $i$  obtains when its distance from its competitor is  $n_i$  at time  $t$  and is assumed to be a strictly increasing function of  $n_i$ . To advance its relative position, firm  $i$  invests in R&D, which determines the Poisson rate of arrival of innovation,  $x_i \in \mathbb{R}_+$ . Let us also assume that the cost of R&D is linear in the arrival rate of innovation, that is,  $\Phi(x_i) = \phi x_i$ , with  $\Phi > 0$  (again see what follows for more general formulations). Each successful innovation is patented and advances firm  $i$ 's state (relative position) by one step, so that following a successful innovation by firm  $i$  at time  $t$  we have:  $n_i(t+\Delta t) = n_i(t) + 1$  (where  $n_i(t+\Delta t)$  stands for  $n_i$  immediately following time  $t$ ).

9. See <http://econ-www.mit.edu/faculty/acemoglu/paper> or <http://www.sas.upenn.edu/~uakcigit/web/Research.html>.

IPR policy governs the expected length of a patent. For simplicity, we model patent length by assuming that it terminates at a Poisson rate. Crucially for our focus, IPR policy is state dependent, and we represent it by the function:  $\eta : N_I \rightarrow \mathbb{R}_+$ . Here  $\eta(n) \equiv \eta_n < \infty$  is the flow rate at which the patent terminates (patent protection is removed) for a technology leader that is  $n$  steps ahead. When  $\eta_n = 0$ , there is full protection at technology gap  $n$ , in the sense that patent protection will never be removed. In contrast,  $\eta_n \rightarrow \infty$  implies that patent protection is removed immediately once technology gap  $n$  is reached. When the patent protection is removed, the firm that is behind copies the technology of its competitor and both firms end up *neck-and-neck*, that is  $n = 0$ .

Finally, we take the interest rate  $r$  as exogenous and assume that it satisfies  $r < (\pi_n - \pi_{n-1})/4\phi$  for each  $n \in N_I$ . This assumption ensures positive R&D by each firm when  $\eta_n = 0$ . Throughout we will focus on (stationary) Markov perfect equilibria (MPE), where strategies (R&D decisions) are only functions of the payoff-relevant state, which is  $n \in N_I$ . A more formal definition of the MPE in the general equilibrium environment is given in what follows.

The MPE can be characterized by writing the value functions of each firm as a function of the state  $n \in N_I$ . These value functions are given by the following recursions:

$$rv_2 = \pi_2 + x_{-2}[v_1 - v_2] + \eta_2[v_0 - v_2], \quad (1)$$

$$rv_1 = \max_{x_1 \geq 0} \{ \pi_1 - \phi x_1 + x_1[v_2 - v_1] + x_{-1}[v_0 - v_1] + \eta_1[v_0 - v_1] \}, \quad (2)$$

$$rv_0 = \max_{x_0 \geq 0} \{ \pi_0 - \phi x_0 + x_0[v_1 - v_0] + \tilde{x}_0[v_{-1} - v_0] \}, \quad (3)$$

$$rv_{-1} = \max_{x_{-1} \geq 0} \left\{ \begin{array}{l} \pi_{-1} - \phi x_{-1} + x_{-1}[v_0 - v_{-1}] + x_1[v_{-2} - v_{-1}] \\ + \eta_1[v_0 - v_{-1}] \end{array} \right\}, \quad (4)$$

$$rv_{-2} = \max_{x_{-2} \geq 0} \{ \pi_{-2} - \phi x_{-2} + x_{-2}[v_{-1} - v_{-2}] + \eta_2[v_0 - v_{-2}] \}. \quad (5)$$

In all equations, the first term represents current profits. In equations (2)–(5), the second term subtracts R&D costs from current profits, the third term represents the fact that the firm will successfully innovate at the flow rate  $x_n$  and increase its position by one step. The fourth term incorporates the change in value due to an innovation by the rival firm. In equations (1) and (2) the last term is the change in value for the leader due to patent expiration, which takes place at the rate  $\eta_n$ . In (4) and (5), the last term is the change in value for the follower. Finally, equation (3) has the same interpretation except that now  $n = 0$  and the two firms are neck-and-neck and thus there is no IPR policy (and the flow rate of innovation of the other firm is denoted by  $\tilde{x}_0$ , and naturally, in a symmetric equilibrium, we will have  $x_0 = \tilde{x}_0$ ). Note also that in equations (1) and (5), we used the fact that a two-step ahead firm does not undertake any R&D since it has already achieved the maximum feasible lead.

We now characterize the MPE under two different policy environments: uniform and state-dependent IPR policy.



*Uniform IPR Policy.* Uniform IPR policy corresponds to the case where  $\eta_n = \eta < \infty$ . Consequently, optimal R&D decisions in equations (2)–(5) can be solved out as (see the Web Appendix):

$$\begin{aligned} x_{-2}^* &= \max \{-4\eta + (\pi_2 - \pi_{-2}) / \phi - 4r, 0\}, \\ x_{-1}^* &= \max \{-3\eta + (\pi_1 - \pi_{-2}) / \phi - 3r, 0\}, \\ x_0^* &= \max \{-2\eta + (\pi_0 - \pi_{-2}) / \phi - 2r, 0\}, \\ x_1^* &= \max \{-\eta + (\pi_{-1} - \pi_{-2}) / \phi - r, 0\}. \end{aligned}$$

Inspection of these expressions immediately establishes the following result.

**PROPOSITION 1.** *Under uniform IPR policy regime, any relaxation of IPR policy (away from  $\eta = 0$ ) creates a disincentive effect and reduces all R&D levels.*

*State-Dependent IPR Policy.* We next consider state-dependent policy where the patent protection of a technology leader depends on the technology gap,  $n$ . Optimal R&D decisions can now be written out as (see the Web Appendix):

$$\begin{aligned} x_{-2}^* &= \max \{-4\eta_2 + (\pi_2 - \pi_{-2}) / \phi - 4r, 0\}, \\ x_{-1}^* &= \max \{-\eta_1 - 2\eta_2 + (\pi_1 - \pi_{-2}) / \phi - 3r, 0\}, \\ x_0^* &= \max \{-2\eta_2 + (\pi_0 - \pi_{-2}) / \phi - 2r, 0\}, \\ x_1^* &= \max \{\eta_1 - 2\eta_2 + (\pi_{-1} - \pi_{-2}) / \phi - r, 0\}. \end{aligned}$$

Inspection of these expressions shows that, in contrast to the uniform IPR case, relaxing patent protection, with a higher  $\eta_1$ , can increase the R&D effort of the one-step leader,  $x_1^*$ . In fact, these expressions capture the essence of the trickle-down of incentives: higher  $\eta_1$ , particularly when combined with low levels of  $\eta_2$ , creates greater incentives to undertake R&D for leaders that are one step ahead, in part to reach a greater lead and thus obtain better protection. The next proposition summarizes this result.

**PROPOSITION 2.** *Under a state-dependent IPR policy regime, relaxing IPR policy (away from  $\eta_n = 0$ ) by weakening current protection (that is, increasing  $\eta_1$ ) creates a positive incentive effect and increases  $x_1^*$ .*

Proposition 2 does not specify whether optimal IPR policy will involve  $\eta_1 > 0$  and/or  $\eta_2 > 0$ , which depends on the social returns from the  $x_n$ . For example, if  $x_1$  is socially more beneficial than  $x_{-1}$ ,  $\eta_1 > 0$  will always be preferred. We will next see that this is always the case in our general equilibrium model.

### 3. General Equilibrium Framework

We now describe our baseline dynamic general equilibrium model. Our baseline model assumes *quick catch-up*, meaning that one innovation by a follower is sufficient to

close the gap with the technology leader in the industry. The characterization of the equilibrium in this environment under the different policy regimes is presented in the next section. Alternative assumptions on the form of catch-up are investigated in Section 7.

### 3.1. Preferences and Technology

Consider the following continuous time economy with a unique final good. The economy is populated by a continuum of 1 individuals, each with 1 unit of labor endowment, which they supply inelastically. Preferences at time  $t$  are given by

$$\mathbb{E}_t \int_t^\infty \exp(-\rho(s-t)) \ln C(s) ds, \quad (6)$$

where  $\mathbb{E}_t$  denotes expectations at time  $t$ ,  $\rho > 0$  is the discount rate and  $C(t)$  is consumption at date  $t$ . The logarithmic preferences in (6) facilitate the analysis, since they imply a simple relationship between the interest rate, growth rate and the discount rate (see (7)).

Let  $Y(t)$  be the total production of the final good at time  $t$ . We assume that the economy is closed and the final good is used only for consumption (that is, there is no investment), so that  $C(t) = Y(t)$ . The standard Euler equation from (6) then implies that

$$g(t) \equiv \frac{\dot{C}(t)}{C(t)} = \frac{\dot{Y}(t)}{Y(t)} = r(t) - \rho, \quad (7)$$

where this equation defines  $g(t)$  as the growth rate of consumption and thus output, and  $r(t)$  is the interest rate at date  $t$ .

The final good  $Y$  is produced using a continuum 1 of intermediate goods according to the Cobb–Douglas production function

$$\ln Y(t) = \int_0^1 \ln y(j, t) dj, \quad (8)$$

where  $y(j, t)$  is the output of  $j$ th intermediate at time  $t$ . Throughout, we take the price of the final good as the numeraire and denote the price of intermediate  $j$  at time  $t$  by  $p(j, t)$ . We also assume that there is free entry into the final good production sector. These assumptions, together with the Cobb–Douglas production function (8), imply that the final good sector has the following demand for intermediates:

$$y(j, t) = \frac{Y(t)}{p(j, t)}, \quad \forall j \in [0, 1]. \quad (9)$$

Intermediate  $j \in [0, 1]$  comes in two different *varieties*, each produced by one of two infinitely-lived firms. We assume that these two varieties are perfect substitutes and these firms compete à la Bertrand.<sup>10</sup> Firm  $i = 1$  or 2 in industry  $j$  has the following

10. A more general case would involve these two varieties being imperfect substitutes, for example, with the output of intermediate  $j$  produced as

$$y(j, t) = \left[ \phi y_1(j, t)^{\frac{\sigma-1}{\sigma}} + (1-\phi) y_2(j, t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

technology:

$$y(j, t) = q_i(j, t)l_i(j, t), \quad (10)$$

where  $l_i(j, t)$  is the employment level of the firm and  $q_i(j, t)$  is its level of technology at time  $t$ . Each consumer in the economy holds a balanced portfolio of the shares of all firms. Consequently, the objective function of each firm is to maximize expected profits.

The production function for intermediate goods, (10), implies that the marginal cost of producing intermediate  $j$  for firm  $i$  at time  $t$  is

$$MC_i(j, t) = \frac{w(t)}{q_i(j, t)}, \quad (11)$$

where  $w(t)$  is the wage rate in the economy at time  $t$ .

When this causes no confusion, we denote the *technology leader* in each industry by  $i$  and the follower by  $-i$ , so that we have

$$q_i(j, t) \geq q_{-i}(j, t).$$

Bertrand competition between the two firms implies that all intermediates will be supplied by the leader at the *limit price*:<sup>11</sup>

$$p_i(j, t) = \frac{w(t)}{q_{-i}(j, t)}. \quad (12)$$

Equation (9) then implies the following demand for intermediates:

$$y(j, t) = \frac{q_{-i}(j, t)}{w(t)}Y(t). \quad (13)$$

### 3.2. Technology, R&D and IPR Policy under Quick Catch-up

R&D by the leader or the follower stochastically leads to innovation. We assume that when the leader innovates, its technology improves by a factor  $\lambda > 1$ .

The follower, on the other hand, can undertake R&D to catch up with the frontier technology. We will call this type of R&D as *catch-up R&D*.<sup>12</sup> Catch-up R&D can

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with  $\sigma > 1$ . The model analyzed in the text corresponds to the limiting case where  $\sigma \rightarrow \infty$ . Our results can be easily extended to this more general case with any  $\sigma > 1$ , but at the cost of additional notation. We therefore prefer to focus on the case where the two varieties are perfect substitutes. It is nonetheless useful to bear this formulation with imperfect substitutes in mind, since it facilitates the interpretation of *distinct* innovations by the two firms (when the follower engages in *catch-up* R&D).

11. If the leader were to charge a higher price, then the market would be captured by the follower earning positive profits. A lower price can always be increased while making sure that all final good producers still prefer the intermediate supplied by the leader  $i$  rather than that by the follower  $-i$ , even if the latter were supplied at marginal cost. Since the monopoly price with the unit elastic demand curve is infinite, the leader always gains by increasing its price, making the price given in (12) the unique equilibrium price.

12. This contrasts with *frontier R&D* introduced in Section 7, which will allow the follower to leapfrog the leader.

be thought of as R&D to discover an *alternative* way of performing the same task as the current leading-edge technology. Because this innovation applies to the follower's variant of the product (recall footnote 10) and results from its own R&D efforts, we assume in our baseline framework that it does not constitute infringement on the patent of the leader.

R&D by the leader and follower may have different costs and success probabilities. We simplify the analysis by assuming that both types of R&D have the same costs and the same probability of success. In particular, in all cases, we assume that innovations follow a controlled Poisson process, with the arrival rate determined by R&D investments. Each firm (in every industry) has access to the following R&D technology:

$$x_i(j, t) = F(h_i(j, t)), \quad (14)$$

where  $x_i(j, t)$  is the flow rate of innovation at time  $t$  and  $h_i(j, t)$  is the number of workers hired by firm  $i$  in industry  $j$  to work in the R&D process at  $t$ . This specification implies that within a time interval of  $\Delta t$ , the probability of innovation for this firm is  $x_i(j, t)\Delta t + o(\Delta t)$ .

We assume that  $F$  is twice continuously differentiable and satisfies  $F'(\cdot) > 0$ ,  $F''(\cdot) < 0$ ,  $F'(0) < \infty$  and that there exists  $\bar{h} \in (0, \infty)$  such that  $F'(h) = 0$  for all  $h \geq \bar{h}$ . The assumption that  $F'(0) < \infty$  implies that there is no Inada condition when  $h_i(j, t) = 0$ . The last assumption, on the other hand, ensures that there is an upper bound on the flow rate of innovation (which is not essential but simplifies the proofs). Recalling that the wage rate for labor is  $w(t)$ , the cost for R&D is therefore  $w(t)G(x_i(j, t))$  where

$$G(x_i(j, t)) \equiv F^{-1}(x_i(j, t)), \quad (15)$$

and the assumptions on  $F$  immediately imply that  $G$  is twice continuously differentiable and satisfies  $G'(\cdot) > 0$ ,  $G''(\cdot) > 0$ ,  $G'(0) > 0$  and  $\lim_{x \rightarrow \bar{x}} G'(x) = \infty$ , where

$$\bar{x} \equiv F(\bar{h}) \quad (16)$$

is the maximal flow rate of innovation (with  $\bar{h}$  already defined).

We next describe the evolution of technologies within each industry. Suppose that leader  $i$  in industry  $j$  at time  $t$  has a technology level of

$$q_i(j, t) = \lambda^{n_{ij}(t)}, \quad (17)$$

and that the follower  $-i$ 's technology at time  $t$  is

$$q_{-i}(j, t) = \lambda^{n_{-ij}(t)}, \quad (18)$$

where  $n_{ij}(t) \geq n_{-ij}(t)$  and  $n_{ij}(t), n_{-ij}(t) \in \mathbb{Z}_+$  denote the technology rungs of the leader and the follower in industry  $j$ . We refer to  $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$  as the *technology gap* in industry  $j$ . If the leader undertakes an innovation within a time interval of  $\Delta t$ , then its technology increases to  $q_i(j, t + \Delta t) = \lambda^{n_{ij}(t)+1}$  and the technology gap rises to

$n_j(t + \Delta t) = n_j(t) + 1$  (the probability of two or more innovations within the interval  $\Delta t$  will be  $o(\Delta t)$ , where  $o(\Delta t)$  represents terms that satisfy  $\lim_{\Delta t \rightarrow 0} o(\Delta t)/\Delta t = 0$ ).

In our baseline model, we assume that there is *quick catch-up* between followers and leaders. Namely, when the follower is successful in catch-up R&D within the interval  $\Delta t$ , then its technology improves to

$$q_{-i}(j, t + \Delta t) = \lambda^{n_{ijt}},$$

and thus it catches up with the leader immediately (regardless of how large the technology gap was). In this case, the technology gap variable becomes  $n_{jt+\Delta t} = 0$ .

In addition to catching up with the technology frontier with their own R&D, followers can also copy the technology frontier if and when patents expire. In particular, we assume that patents expire at some policy-determined Poisson rate  $\eta$ , and after expiration, followers can costlessly copy the frontier technology, jumping to  $q_{-i}(j, t + \Delta t) = \lambda^{n_{ijt}}$ .<sup>13</sup> As in the partial equilibrium model in Section 2, IPR policy governs the length of the patent and we allow it to be state dependent, so it is represented by the following function:

$$\eta : \mathbb{N} \rightarrow \mathbb{R}_+.$$

Here  $\eta(n) \equiv \eta_n < \infty$  is the flow rate at which the patent protection is removed from a technology leader that is  $n$  steps ahead of the follower. When  $\eta_n = 0$ , this implies that there is full protection at technology gap  $n$ , in the sense that patent protection will never be removed. In contrast,  $\eta_n \rightarrow \infty$  implies that patent protection is removed immediately once technology gap  $n$  is reached. Our formulation imposes that  $\eta \equiv \{\eta_1, \eta_2, \dots\}$  is time invariant. Given this specification, we can now write the law of motion of the technology gap in industry  $j$  as follows:

$$n_j(t + \Delta t) = \begin{cases} n_j(t) + 1 & \text{with prob. } x_i(j, t)\Delta t + o(\Delta t) \\ 0 & \text{with prob. } x_{-i}(j, t)\Delta t + \eta_{n_j(t)}\Delta t + o(\Delta t) \\ n_j(t) & \text{with prob. } \begin{bmatrix} 1 - x_i(j, t)\Delta t + x_{-i}(j, t)\Delta t \\ + \eta_{n_j(t)}\Delta t - o(\Delta t) \end{bmatrix}. \end{cases} \quad (19)$$

Here  $o(\Delta t)$  again represents second-order terms, in particular, the probabilities of more than one innovations within an interval of length  $\Delta t$ . The terms  $x_i(j, t)$  and  $x_{-i}(j, t)$  are the flow rates of innovation by the leader and the follower; and  $\eta_{n_j(t)}$  is the flow rate at which the follower is allowed to copy the technology of a leader that is  $n_j(t)$  steps ahead. Intuitively, the technology gap in industry  $j$  increases from  $n_j(t)$  to  $n_j(t) + 1$  if the leader is successful. The firms become *neck-and-neck* when the follower comes up with an alternative technology to that of the leader (flow rate  $x_{-i}(j, t)$ ) or the patent expires at the flow rate  $\eta_{n_j}$ .

13. Alternative modeling assumptions on IPR policy, such as a fixed patent length of  $T > 0$  from the time of innovation, are not tractable, since they lead to value functions that take the form of delayed differential equations.

### 3.3. Profits

We next write the instantaneous *operating* profits for the leader (that is, the profits exclusive of R&D expenditures). Profits of leader  $i$  in industry  $j$  at time  $t$  are

$$\begin{aligned}\Pi_i(j, t) &= [p_i(j, t) - MC_i(j, t)] y_i(j, t) \\ &= \left( \frac{w(t)}{q_{-i}(j, t)} - \frac{w(t)}{q_i(j, t)} \right) \frac{Y(t)}{p_i(j, t)} \\ &= (1 - \lambda^{-n_j(t)}) Y(t),\end{aligned}\tag{20}$$

where  $n_j(t) \equiv n_{ij}(t) - n_{-ij}(t)$  is the technology gap in industry  $j$  at time  $t$ . The first line simply uses the definition of operating profits as price minus marginal cost times quantity sold. The second line uses the fact that the equilibrium limit price of firm  $i$  is  $p_i(j, t) = w(t)/q_{-i}(j, t)$  as given by (12), and the final equality uses the definitions of  $q_i(j, t)$  and  $q_{-i}(j, t)$  from (17) and (18). The expression in (20) also implies that there will be zero profits in neck-and-neck industries, that is, in those with  $n_j(t) = 0$ . Also clearly, followers always make zero profits, since they have no sales.

The Cobb–Douglas aggregate production function in (8) is responsible for the form of the profits (20), since it implies that profits only depend on the technology gap of the industry and aggregate output. This will simplify the analysis that follows by making the technology gap in each industry the only industry-specific payoff-relevant state variable.

The objective function of each firm is to maximize the net present discounted value of *net profits* (operating profits minus R&D expenditures). In doing this, each firm will take the sequence of interest rates,  $[r(t)]_{t \geq 0}$ , the sequence of aggregate output levels,  $[Y(t)]_{t \geq 0}$ , the sequence of wages,  $[w(t)]_{t \geq 0}$ , and the R&D decisions of all other firms and policies as given.

### 3.4. Equilibrium

Let  $\mu(t) \equiv \{\mu_n(t)\}_{n=0}^{\infty}$  denote the distribution of industries over different technology gaps, with  $\sum_{n=0}^{\infty} \mu_n(t) = 1$ . For example,  $\mu_0(t)$  denotes the fraction of industries in which the firms are neck-and-neck at time  $t$ . Throughout, we focus on Markov perfect equilibria, where strategies are only functions of the payoff-relevant state variables.<sup>14</sup> This allows us to drop the dependence on industry  $j$ , thus we refer to R&D decisions by  $x_n$  for the technology leader that is  $n$  steps ahead and by  $x_{-n}$  for a follower that is  $n$  steps behind. Let us denote the list of decisions by the leader and the follower with technology gap  $n$  at time  $t$  by  $\xi_n(t) \equiv \langle x_n(t), p_i(j, t), y_i(j, t) \rangle$  and  $\xi_{-n}(t) \equiv \langle x_{-n}(t) \rangle$ .<sup>15</sup>

14. MPE is a natural equilibrium concept in this context, since it does not allow for implicit collusive agreements between the follower and the leader. While such collusive agreements may be likely when there are only two firms in the industry, in most industries there are many more firms and also many potential entrants, making collusion more difficult. Throughout, we assume that there are only two firms to keep the model tractable.

15. The price and output decisions,  $p_i(j, t)$  and  $y_i(j, t)$ , depend not only on the technology gap, aggregate output, and the wage rate, but also on the exact technology rung of the leader,  $n_j(t)$ . With a slight abuse of

Throughout,  $\xi$  will indicate the whole sequence of decisions at every state, so that  $\xi(t) \equiv \{\xi_n(t)\}_{n=-\infty}^{\infty}$ . We define an allocation as follows.

**DEFINITION 1 (Allocation).** Let  $\eta$  be the IPR policy sequence. Then an allocation is a sequence of decisions for a leader that is  $n = 0, 1, 2, \dots$  step ahead,  $[\xi_n(t)]_{t \geq 0}$ , a sequence of R&D decisions for a follower that is  $n = 1, 2, \dots$  step behind,  $[\xi_{-n}(t)]_{t \geq 0}$ , a sequence of wage rates  $[w(t)]_{t \geq 0}$ , and a sequence of industry distributions over technology gaps  $[\mu(t)]_{t \geq 0}$ .

For given IPR sequence  $\eta$ , MPE strategies, which are only functions of the payoff-relevant state variables, can be represented as follows:

$$\mathbf{x} : \mathbb{Z} \times \mathbb{R}_+^2 \times [0, 1]^\infty \rightarrow \mathbb{R}_+.$$

This mapping represents the R&D decision of a firm (both when it is the follower and when it is the leader in an industry) as a function of the technology gap,  $n \in \mathbb{Z}$ , the aggregate level of output and the wage,  $(Y, w) \in \mathbb{R}_+^2$ , and R&D decision of the other firm in the industry,  $\tilde{\mathbf{x}} \in [0, 1]^\infty$ . Consequently, we have the following definition of equilibrium.

**DEFINITION 2 (Equilibrium).** Given an IPR policy sequence  $\eta$ , a MPE is given by a sequence  $[\xi^*(t), w^*(t), Y^*(t)]_{t \geq 0}$  such that (i)  $[p_i^*(j, t)]_{t \geq 0}$  and  $[y_i^*(j, t)]_{t \geq 0}$  implied by  $[\xi^*(t)]_{t \geq 0}$  satisfy (12) and (13); (ii) R&D policy  $[\mathbf{x}^*(t)]_{t \geq 0}$  is a best response to itself, that is,  $[\mathbf{x}^*(t)]_{t \geq 0}$  maximizes the expected profits of firms taking aggregate output  $[Y^*(t)]_{t \geq 0}$ , wages  $[w^*(t)]_{t \geq 0}$ , government policy  $\eta$  and the R&D policies of other firms  $[\tilde{\mathbf{x}}^*(t)]_{t \geq 0}$  as given; (iii) aggregate output  $[Y^*(t)]_{t \geq 0}$  is given by (8); and (iv) the labor market clears at all times given the wage sequence  $[w^*(t)]_{t \geq 0}$ .

### 3.5. The Labor Market

Since only the technology leader produces, labor demand in industry  $j$  with technology gap  $n_j(t) = n$  can be expressed as

$$l_n(t) = \frac{\lambda^{-n} Y(t)}{w(t)} \quad \text{for } n \in \mathbb{Z}_+. \quad (21)$$

In addition, there is demand for labor coming for R&D from both followers and leaders in all industries. Using (14) and the definition of the  $G$  function, we can express industry demands for R&D labor as

$$h_n(t) = G(x_n(t)) + G(x_{-n}(t)) \quad \text{for } n \in \mathbb{Z}_+, \quad (22)$$

where  $G(x_n(t))$  and  $G(x_{-n}(t))$  refer to the demand of the leader and the follower in an industry with a technology gap of  $n$ . Note that in this expression,  $x_{-n}(t)$  refers to the R&D effort of a follower that is  $n$  steps behind.

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notation, throughout we suppress this dependence, since their product  $p_i(j, t)y_i(j, t)$  and the resulting profits for the firm, (20), are independent of  $n_{ij}(t)$ , and consequently, only the technology gap,  $n_j(t)$ , matters for profits, R&D, aggregate output, and economic growth.

The labor market clearing condition can then be expressed as

$$1 \geq \sum_{n=0}^{\infty} \mu_n(t) \left[ \frac{1}{\omega(t)\lambda^n} + G(x_n(t)) + G(x_{-n}(t)) \right], \quad (23)$$

and  $\omega(t) \geq 0$ , with complementary slackness, where

$$\omega(t) \equiv \frac{w(t)}{Y(t)} \quad (24)$$

is the labor share at time  $t$ . The labor market clearing condition, (23), uses the fact that total supply is equal to 1, and demand cannot exceed this amount. If demand falls short of 1, then the wage rate,  $w(t)$ , and thus the labor share,  $\omega(t)$ , have to be equal to zero (though this will never be the case in equilibrium). The right-hand side of (23) consists of the demand for production (the terms with  $\omega$  in the denominator), the demand for R&D workers from the neck-and-neck industries ( $2G(x_0(t))$  when  $n = 0$ ) and the demand for R&D workers coming from leaders and followers in other industries ( $G(x_n(t)) + G(x_{-n}(t))$  when  $n > 0$ ).

Defining the index of aggregate quality in this economy by the aggregate of the qualities of the leaders in the different industries, that is,

$$\ln Q(t) \equiv \int_0^1 \ln q_i(j, t) dj, \quad (25)$$

the equilibrium wage can be written as<sup>16</sup>

$$w(t) = Q(t)\lambda^{-\sum_{n=0}^{\infty} n\mu_n(t)}. \quad (26)$$

### 3.6. Steady State and the Value Functions under Quick Catch-Up

Let us now focus on steady-state (Markov perfect) equilibria, where the distribution of industries  $\boldsymbol{\mu}(t) \equiv \{\mu_n(t)\}_{n=0}^{\infty}$  is stationary,  $\omega(t)$  defined in (24) and  $g$ , the growth

16. Note that

$$\ln Y(t) = \int_0^1 \ln q_i(j, t) l(j, t) dj = \int_0^1 \left[ \ln q_i(j, t) + \ln \frac{Y(t)}{w(t)} \lambda^{-n_j} \right] dj,$$

where the second equality uses (21). Thus we have

$$\ln Y(t) = \int_0^1 [\ln q_i(j, t) + \ln Y(t) - \ln w(t) - n_j \ln \lambda] dj.$$

Rearranging and canceling terms, and writing

$$\exp \int n_j \ln \lambda dj = \lambda^{-\sum_{n=0}^{\infty} n\mu_n(t)},$$

we obtain (26).



rate of the economy, are constant over time. We will establish the existence of such an equilibrium and characterize a number of its properties. If the economy is in steady state at time  $t = 0$ , then by definition, we have  $Y^*(t) = Y_0 e^{g^* t}$  and  $w^*(t) = w_0 e^{g^* t}$ , where  $g^*$  is the steady-state growth rate. These two equations also imply that  $\omega(t) = \omega^*$  for all  $t \geq 0$ . Throughout, we assume that the parameters are such that the steady-state growth rate  $g^*$  is positive but not large enough to violate the transversality conditions. This implies that net present values of each firm at all points in time will be finite. This enables us to write the maximization problem of a leader that is  $n > 0$  steps ahead recursively.

First note that given an optimal policy  $\hat{x}$  for a firm, the net present discounted value of a leader that is  $n$  steps ahead at time  $t$  can be written as

$$V_n(t) = \mathbb{E}_t \int_t^\infty \exp(-r(s-t)) [\Pi(s) - w(s)G(\hat{x}(s))] ds,$$

where  $\Pi(s)$  is the operating profit at time  $s \geq t$  and  $w(s)G(\hat{x}(s))$  denotes the R&D expenditure at time  $s \geq t$ . All variables are stochastic and depend on the evolution of the technology gap within the industry.

Next taking as given the equilibrium R&D policy of other firms,  $x_{-n}^*(t)$ , the equilibrium interest and wage rates,  $r^*(t)$  and  $w^*(t)$ , and equilibrium profits  $\{\Pi_n^*(t)\}_{n=1}^\infty$  (as a function of equilibrium aggregate output), this value can be written as (see the Web Appendix for the derivation of this equation)<sup>17</sup>

$$\begin{aligned} & r^*(t)V_n(t) - \dot{V}_n(t) \\ &= \max_{x_n(t) \geq 0} \left\{ \begin{aligned} & [\Pi_n^*(t) - w^*(t)G(x_n(t))] + x_n(t)[V_{n+1}(t) - V_n(t)] \\ & + (x_{-n}^*(t) + \eta_n)[V_0(t) - V_n(t)] \end{aligned} \right\}, \end{aligned} \quad (27)$$

where  $\dot{V}_n(t)$  denotes the derivative of  $V_n(t)$  with respect to time. The first term is current profits minus R&D costs, while the second term captures the fact that the firm will undertake an innovation at the flow rate  $x_n(t)$  and increase its technology lead by one step. The remaining terms incorporate changes in value due to quick catch-up by the follower (flow rate  $x_{-n}^*(t) + \eta_n$  in the second line).

In steady state, the net present value of a firm that is  $n$  steps ahead,  $V_n(t)$ , will also grow at a constant rate  $g^*$  for all  $n \in \mathbb{Z}_+$ . Let us then define the normalized values as

$$v_n(t) \equiv \frac{V_n(t)}{Y(t)} \quad (28)$$

for all  $n \in \mathbb{Z}$ , which will be independent of time in steady state, that is,  $v_n(t) = v_n$ .

Using (28) and the fact that from (7),  $r(t) = g(t) + \rho$ , the recursive form of the steady-state value function (27) can be written as

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{aligned} & (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] \\ & + [x_{-n}^* + \eta_n][v_0 - v_n] \end{aligned} \right\} \text{ for } n \in \mathbb{N}, \quad (29)$$

17. Clearly, this value function could be written for any arbitrary sequence of R&D policies of other firms. We set the R&D policies of other firms to their equilibrium values,  $x_{-n}^*(t)$ , to reduce notation in the main body of the paper.

where  $x_{-n}^*$  is the equilibrium value of R&D by a follower that is  $n$  steps behind, and  $\omega^*$  is the steady-state labor share (while  $x_n$  is now explicitly chosen to maximize  $v_n$ ).

Similarly the value for neck-and-neck firms is

$$\rho v_0 = \max_{x_0 \geq 0} \{-\omega^* G(x_0) + x_0[v_1 - v_0] + x_0^*[v_{-1} - v_0]\}, \quad (30)$$

while the values for followers are given by

$$\rho v_{-n} = \max_{x_{-n} \geq 0} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + [x_{-n} + \eta_n][v_0 - v_{-n}] \\ + x_n^*[v_{-n-1} - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N}. \quad (31)$$

For neck-and-neck firms and followers, there are no instantaneous profits, which is reflected in equations (30) and (31). In the former case this is because neck-and-neck firms sell at marginal cost, and in the latter case, this is because followers have no sales. These normalized value functions emphasize that, because of growth, the effective discount rate is  $r(t) - g(t) = \rho$  rather than  $r(t)$ .

The maximization problems in equations (29)–(31) immediately imply that any steady-state equilibrium R&D policies,  $\mathbf{x}^*$ , must satisfy

$$x_n^* = \max \left\{ G'^{-1} \left( \frac{[v_{n+1} - v_n]}{\omega^*} \right), 0 \right\}, \quad (32)$$

$$x_{-n}^* = \max \left\{ G'^{-1} \left( \frac{[v_0 - v_{-n}]}{\omega^*} \right), 0 \right\}, \quad (33)$$

$$x_0^* = \max \left\{ G'^{-1} \left( \frac{[v_1 - v_0]}{\omega^*} \right), 0 \right\}, \quad (34)$$

where the normalized value functions, the  $v$ , are evaluated at the equilibrium, and  $G'^{-1}(\cdot)$  is the inverse of the derivative of the  $G$  function. Since  $G$  is twice continuously differentiable and strictly concave,  $G'^{-1}$  is continuously differentiable and strictly increasing. These equations therefore imply that innovation rates, the  $x_n^*$ , will increase whenever the incremental value of moving to the next step is greater and when the cost of R&D, as measured by the normalized wage rate,  $\omega^*$ , is less. Note also that since  $G'(0) > 0$ , these R&D levels can be equal to zero, which is taken care of by the max operator.

The response of innovation rates,  $x_n^*$ , to the increments in values,  $v_{n+1} - v_n$ , is the key economic force in this model. A policy that reduces the patent protection of leaders that are  $n + 1$  steps ahead (by increasing  $\eta_{n+1}$ ) will make being  $n + 1$  steps ahead less profitable, thus reduce  $v_{n+1} - v_n$  and  $x_n^*$ . This corresponds to the standard *disincentive effect* of relaxing IPR policy. This result corresponds to fact (1) in the toy model. In contrast to existing models, however, here relaxing IPR policy can also create a *positive incentive effect*. Somewhat paradoxically, lower protection for technology leaders that are  $n + 1$  steps ahead will tend to reduce  $v_{n+1}$ , thus increasing  $v_{n+2} - v_{n+1}$  and  $x_{n+1}^*$ . This result is very similar to fact (2) in the toy model. We will see this positive incentive

effect plays an important role in the form of optimal state-dependent IPR policy. In addition to the incentive effects, relaxing IPR protection may also create a beneficial *composition effect*; this is because, typically,  $\{v_{n+1} - v_n\}_{n=0}^{\infty}$  is a decreasing sequence, which implies that  $x_{n-1}^*$  is higher than  $x_n^*$  for  $n \geq 1$  (see, for example, Proposition 4). Weaker patent protection (in the form of shorter patent lengths) will shift more industries into the neck-and-neck state and potentially increase the equilibrium level of R&D in the economy. Finally, weaker patent protection also creates a beneficial *level effect* by influencing equilibrium markups and prices (as shown in equation (12)) and by reallocating some of the workers engaged in *duplicative* R&D to production. This level effect will also feature in our welfare computations. The optimal level and structure of IPR policy in this economy will be determined by the interplay of these various forces.

Given the equilibrium R&D decisions  $\mathbf{x}^*$ , the steady-state distribution of industries across states  $\boldsymbol{\mu}^*$  has to satisfy the following accounting identities:

$$(x_{n+1}^* + x_{-n-1}^* + \eta_{n+1})\mu_{n+1}^* = x_n^*\mu_n^* \quad \text{for } n \in \mathbb{N}, \quad (35)$$

$$(x_1^* + x_{-1}^* + \eta_1)\mu_1^* = 2x_0^*\mu_0^*, \quad (36)$$

$$2x_0^*\mu_0^* = \sum_{n=1}^{\infty} (x_{-n}^* + \eta_n)\mu_n^*. \quad (37)$$

The first expression equates exit from state  $n + 1$  (which takes the form of the leader going one more step ahead or the follower catching up the leader) to entry into the state (which takes the form of a leader from state  $n$  making one more innovation). The second equation, (36), performs the same accounting for state 1, taking into account that entry into this state comes from innovation by either of the two firms that are competing neck-and-neck. Finally, equation (37) equates exit from state 0 with entry into this state, which comes from innovation by a follower in any industry with  $n \geq 1$ .

The labor market clearing condition in steady state can then be written as

$$1 \geq \sum_{n=0}^{\infty} \mu_n^* \left[ \frac{1}{\omega^* \lambda^n} + G(x_n^*) + G(x_{-n}^*) \right] \quad \text{and} \quad \omega^* \geq 0, \quad (38)$$

with complementary slackness.

The next proposition characterizes the steady-state growth rate. As with all the other results in the paper, the proof of this proposition is provided in Web Appendix E.

**PROPOSITION 3.** *Let the steady-state distribution of industries and R&D decisions be given by  $\langle \boldsymbol{\mu}^*, \mathbf{x}^* \rangle$ . Then the steady-state growth rate is*

$$g^* = \ln \lambda \left[ 2\mu_0^* x_0^* + \sum_{n=1}^{\infty} \mu_n^* x_n^* \right]. \quad (39)$$

This proposition clarifies that the steady-state growth rate of the economy is determined by two factors: (1) R&D decisions of industries at different levels of technology gap,  $\mathbf{x}^* \equiv \{x_n^*\}_{n=-\infty}^{\infty}$ ; (2) the distribution of industries across different technology gaps,  $\boldsymbol{\mu}^* \equiv \{\mu_n^*\}_{n=0}^{\infty}$ . IPR policy affects these two margins in different directions as illustrated by the previous discussion.

#### 4. Existence and Characterization of Steady-State Equilibria

We now define a steady-state equilibrium in a more convenient form, which will be used to establish existence and derive some of the properties of the equilibrium.

**DEFINITION 3** (*Steady-State Equilibrium*). Given an IPR policy  $\eta$ , a steady-state equilibrium is a tuple  $\langle \boldsymbol{\mu}^*, \mathbf{v}, \mathbf{x}^*, \omega^*, g^* \rangle$  such that the distribution of industries  $\boldsymbol{\mu}^*$  satisfy (35), (36), and (37), the values  $\mathbf{v} \equiv \{v_n\}_{n=-\infty}^{\infty}$  satisfy (29), (30), and (31), the R&D decision  $\mathbf{x}^*$  is given by (32), (33), and (34), the steady-state labor share  $\omega^*$  satisfies (38) and the steady-state growth rate  $g^*$  is given by (39).

We next provide a characterization of the steady-state equilibrium, starting first with the case in which there is uniform IPR policy.

##### 4.1. Uniform IPR Policy

Let us first focus on the case where IPR policy is uniform, that is  $\eta_n = \eta < \infty$  for all  $n \in \mathbb{N}$  and we denote this by  $\eta^{uni}$ . In this case, (31) implies that the problem is identical for all followers, so that  $v_{-n} = v_{-1}$  for  $n \in \mathbb{N}$ . Consequently, (31) can be replaced with the following simpler equation:

$$\rho v_{-1} = \max_{x_{-1} \geq 0} \{-\omega^* G(x_{-1}) + [x_{-1} + \eta][v_0 - v_{-1}]\}, \quad (40)$$

implying optimal R&D decisions for all followers of the form

$$x_{-1}^* = \max \left\{ G'^{-1} \left( \frac{[v_0 - v_{-1}]}{\omega^*} \right), 0 \right\}. \quad (41)$$

Let us denote the sequence of value functions under uniform IPR as  $\{v_n\}_{n=-1}^{\infty}$ . We next establish the existence of a steady-state equilibrium under uniform IPR and characterize some of its most important properties. Establishing the existence of a steady-state equilibrium in this economy is made complicated by the fact that the equilibrium allocation cannot be represented as a solution to a maximization problem. Instead, as emphasized by Definition 3, each firm maximizes its value taking the R&D decisions of other firms as given; thus an equilibrium corresponds to a set of R&D decisions that are best responses to themselves and a labor share (wage rate)  $\omega^*$  that clears the labor market. Nevertheless, there is sufficient structure in the model

to guarantee the existence of a steady-state equilibrium and monotonic behavior of values and R&D decisions.

**PROPOSITION 4.** *Consider a uniform IPR policy  $\eta^{umi}$  and suppose that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$ . Then a steady-state equilibrium  $(\mu^*, \mathbf{v}, \mathbf{x}^*, \omega^*, \mathbf{g}^*)$  exists. Moreover, in any steady-state equilibrium  $\omega^* < 1$ . In addition, if either  $\eta > 0$  or  $x_{-1}^* > 0$ , then  $\mathbf{g}^* > 0$ . For any steady-state R&D decisions  $\mathbf{x}^*$ , the steady-state distribution of industries  $\mu^*$  is uniquely determined.*

*In addition, we have the following results.*

- $v_{-1} \leq v_0$  and  $\{v_n\}_{n=0}^\infty$  forms a bounded and strictly increasing sequence converging to some  $v_\infty \in (0, \infty)$ .
- $x_0^* > x_1^*$ ,  $x_0^* \geq x_{-1}^*$ , and  $x_{n+1}^* \leq x_n^*$  for all  $n \in \mathbb{N}$  with  $x_{n+1}^* < x_n^*$  if  $x_n^* > 0$ . Moreover, provided that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$  and  $x_0^* > x_{-1}^*$ .

*Proof.* See Web Appendix E. □

**REMARK 1.** The condition that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$  ensures that there will be positive R&D in equilibrium. If this condition does not hold, then there exists a trivial steady-state equilibrium in which  $x_n^* = 0$  for all  $n \in \mathbb{Z}_+$ , that is, an equilibrium in which there is no innovation and thus no growth (see the Web Appendix E for more details). Moreover, provided that  $\eta > 0$ , this equilibrium would also involve  $\mu_0^* = 1$ , so that in every industry two firms with equal costs compete à la Bertrand and charge price equal to marginal cost, leading to zero aggregate profits and a labor share of output equal to 1. The assumption that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta)) > 0$ , on the other hand, is sufficient to rule out  $\mu_0^* = 1$  and thus  $\omega^* = 1$ . If, in addition, the steady-state equilibrium involves some probability of catch-up or innovation by the followers, that is, either  $\eta > 0$  or  $x_{-1}^* > 0$ , then the growth rate is also strictly positive.

In addition to the existence of a steady-state equilibrium with positive growth, Proposition 4 shows that the sequence of values  $\{v_n\}_{n=0}^\infty$  is strictly increasing and converges to some  $v_\infty$ , and more importantly that  $\mathbf{x}^* \equiv \{x_n^*\}_{n=1}^\infty$  is a decreasing sequence, which implies that technology leaders that are further ahead undertake less R&D. Intuitively, the benefits of further R&D are decreasing in the technology gap, since greater values of the technology gap translate into smaller increases in the equilibrium markup (recall (20)). Moreover, the R&D level of neck-and-neck firms,  $x_0^*$ , is greater than both the R&D level of technology leaders that are one step ahead and followers that are one step behind (that is,  $x_0^* > x_1^*$  and  $x_0^* \geq x_{-1}^*$ ). This implies that with uniform policy neck-and-neck industries are *most R&D intensive*, while industries with the largest technology gaps are *least R&D intensive*. This is closely related to the *escape competition* mechanism discussed in Aghion et al. (2001), whereby incumbents undertake more R&D when their lead over the followers is more limited. It is also the basis of the conjecture mentioned in the Introduction that reducing protection given to technologically advanced leaders might be useful for increasing R&D by bringing them into the neck-and-neck state.

#### 4.2. State-Dependent IPR Policy

We now extend the results from the previous section to the environment with state-dependent IPR policy, though results on monotonicity of values and R&D efforts no longer hold.<sup>18</sup>

**PROPOSITION 5.** *Consider the state-dependent IPR policy  $\eta$  and suppose that  $G'^{-1}((1 - \lambda^{-1})/(\rho + \eta_1)) > 0$ . Then a steady-state equilibrium  $(\mu^*, v, x^*, \omega^*, g^*)$  exists. Moreover, in any steady-state equilibrium  $\omega^* < 1$ . In addition, if either  $\eta_1 > 0$  or  $x_{-1}^* > 0$ , then  $g^* > 0$ .*

*Proof.* See Web Appendix E. □

Unfortunately, it is not possible to determine the optimal (welfare- or growth-maximizing) state-dependent IPR policy analytically. For this reason, in Section 5, we undertake a quantitative investigation of the form and structure of optimal state-dependent IPR policy using plausible parameter values.

### 5. Optimal IPR Policy: Towards A Quantitative Investigation

In the remainder of the paper, we investigate the implications of various different types of IPR policies on R&D, growth, and welfare using numerical computations of the steady-state equilibrium. Our purpose is not to provide a detailed calibration of the model economy but to highlight its qualitative implications for optimal IPR policy under plausible parameter values. We focus on optimal policy, defined as steady-state welfare-maximizing choice of policy (growth-maximizing policies give very similar results and are omitted to save space). In this section, we introduce the measure of steady-state welfare and describe our quantitative methodology. Results are reported in the subsequent sections.

#### 5.1. Welfare

Our focus so far has been on steady-state equilibria (mainly because of the very challenging nature of transitional dynamics in this class of models). In our quantitative analysis, we continue to focus on steady states and thus look at steady-state welfare. In a steady-state equilibrium, welfare at time  $t = 0$  can be written as

$$\text{Welfare}(0) = \int_0^{\infty} e^{-\rho t} \ln(Y(0)e^{g^*t}) dt = \frac{\ln Y(0)}{\rho} + \frac{g^*}{\rho^2}, \quad (42)$$

18. This is because IPR policies could be very sharply increasing at some technology gap, making a particular state very unattractive for the leader. For example, we could have  $\eta_n = 0$  and  $\eta_{n+1} \rightarrow \infty$ , which would imply that  $v_{n+1} - v_n$  is negative.

where the first equality uses the facts that all output is consumed, utility is logarithmic (recall (6)), output and consumption at date  $t = 0$  are given by  $Y(0)$ , and in the steady-state equilibrium output grows at the rate  $g^*$ . The second equality simply evaluates the integral. Next, note that

$$\begin{aligned}
 \ln Y(t) &= \int_0^1 \ln y(j, t) dj \\
 &= \int_0^1 \ln \left( \frac{q_{-i}(j, t) Y(t)}{\omega(t)} \right) dj \\
 &= \int_0^1 \ln q_{-i}(j, t) dj - \ln \omega(t) \\
 &= \ln Q(t) - \ln \lambda \left( \sum_{n=0}^{\infty} n \mu_n(t) \right) - \ln \omega(t), \tag{43}
 \end{aligned}$$

where the first line simply uses the definition in (8), the second line substitutes for  $y(j, t)$  from (13), the third line uses the definition of the labor share  $\omega(t)$ , and the final line uses the definition of  $Q(t)$  from (25) together with the fact that in the steady state  $q_i(j, t) = \lambda^n q_{-i}(j, t)$  in a fraction  $\mu_n(t)$  of industries. The expression in (43) implies that output simply depends on the quality index,  $Q(t)$ , the distribution of technology gaps,  $\mu(t)$  (because this determines markups), and also on the labor share,  $\omega(t)$ . In steady-state equilibrium, the distribution of technology gaps and labor share are constant, while output and the quality index grow at the steady-state rate  $g^*$ . Therefore, for steady-state comparisons of welfare across economies with different policies, it is sufficient to compare two economies with the same level of  $Q(0)$ , but with different policies. We can then evaluate steady-state welfare with the distribution of industries given by their steady-state values in the two economies, and output and the quality index growing at the corresponding steady-state growth rates. Expression (43) also makes it clear that only the aggregate quality index  $Q(0)$  needs to be taken to be the same in the different economies. Given  $Q(0)$ , the dispersion of industries in terms of the quality levels has no effect on output or welfare (though, clearly, the distribution of industries in terms of technology gaps between leaders and followers,  $\mu$ , influences the level of markups and output, and thus welfare).

However, note one difficulty with welfare comparisons highlighted by equations (42) and (43); proportional changes in steady-state welfare due to policy changes will depend on the initial level of  $Q(0)$ , which is an arbitrary number. Therefore, proportional changes in welfare are not informative, though this has no effect on ordinal rankings and thus welfare-maximizing policy is well defined and independent of the level of  $Q(0)$ . Equations (42) and (43) also make it clear that changes in steady-state welfare will be the sum of two components: the first is the *growth effect*, given by  $g^*/\rho^2$ , whereas the second is due to changes in  $\ln \lambda(\sum_{n=0}^{\infty} n \mu_n)/\rho - \ln \omega(0)$ . Since changes in the labor share  $\omega(0)$  are largely driven by the distribution of industries, we refer to this as the *distribution effect*. Policies will typically affect both of these quantities. In what follows, we give the welfare rankings of different policies and then

report the relative magnitudes of the growth and the distribution effects. This will show that the growth effects will be one or two orders of magnitude greater than the distribution effects and dominate welfare comparisons. So if the reader wishes, he or she may think of the magnitudes of the changes in welfare as given by the proportional changes in growth rates.

It is useful to note that in general, welfare is not maximized in the equilibrium we have characterized so far for three reasons. First, there are the usual monopoly distortions as prices deviate systematically from marginal costs. Second, only part of the returns from their innovation is appropriated by leaders. For example, if both leaders and followers improve their technologies by one step, consumers are better off, but profits remain unchanged. Third, the social benefit of innovation by followers differs from private benefits; socially such innovation is beneficial because it reduces markups and tends to increase future R&D investments (as it brings the industry to a neck-and-neck state where R&D is greater), while privately it enables followers to become leaders in the future. IPR policy can mostly close the gap between the equilibrium and the social optimum through the second channel, though it indirectly also manipulates the third.

## 5.2. Quantitative Methods and Parameter Choices

For our quantitative exercise, we take the annual discount rate as 5%, that is,  $\rho_{year} = 0.05$ . In all our computations, we work with the monthly equivalent of this discount rate in order to increase precision, but throughout the tables, we convert all numbers to their annual counterparts to facilitate interpretation.

The theoretical analysis considered a general production function for R&D given by (14). The empirical literature typically assumes a Cobb–Douglas production function. For example, Kortum (1993) considers a function of the form

$$\text{Innovation}(t) = B_0 \exp(\kappa t)(\text{R\&D inputs})^\gamma, \quad (44)$$

where  $B_0$  is a constant and  $\exp(\kappa t)$  is a trend term, which may depend on general technological trends, a drift in technological opportunities, or changes in general equilibrium prices (such as wages of researchers, etc.). The advantage of this form is not only its simplicity, but also the fact that most empirical work estimates a single elasticity for the response of innovation rates to R&D inputs. Consequently, they essentially only give information about the parameter  $\gamma$  in terms of equation (44). A low value of  $\gamma$  implies that the R&D production function is more concave. For example, Kortum (1993) reports that estimates of  $\gamma$  vary between 0.1 and 0.6 (see also Pakes and Griliches 1980; Hall, Hausman, and Griliches 1988). For these reasons, throughout, we adopt a R&D production function similar to (44):

$$x = Bh^\gamma, \quad (45)$$

where  $B, \gamma > 0$ . In terms of our previous notation, equation (45) implies that  $G(x) = [x/B]^{1/\gamma} w$ , where  $w$  is the wage rate in the economy (thus in terms of the function



already mentioned, it is captured by the  $\exp(\kappa t)$  term).<sup>19</sup> Equation (45) does not satisfy the boundary conditions we imposed so far and can be easily modified to do so without affecting any of the results, since in all numerical exercises only a finite number of states are reached.<sup>20</sup> Following the estimates reported in Kortum (1993), we start with a benchmark value of  $\gamma = 0.35$ , and then report sensitivity checks for  $\gamma = 0.1$  and  $\gamma = 0.6$ . The other parameter in (45),  $B$ , is chosen so as to ensure an annual growth rate of approximately 1.9%, that is,  $g^* \simeq 0.019$ , in the benchmark economy which features indefinitely-enforced patents. This growth rate together with  $\rho_{year} = 0.05$  also pins down the annual interest rate as  $r_{year} = 0.069$  from equation (7).

We choose the value of  $\lambda$  using a reasoning similar to Stokey (1995). Equation (39) implies that if the expected duration of time between any two consecutive innovations is about 3 years in an industry, then a growth rate of about 1.9% would require  $\lambda = 1.05$ .<sup>21</sup> This value is also consistent with the empirical findings of Bloom, Schankerman and Van Reenen (2005).<sup>22</sup> We take  $\lambda = 1.05$  as the benchmark value. We then check the robustness of the results to  $\lambda = 1.01$  and  $\lambda = 1.2$  (expected duration of 8 months and 13 years, respectively). Finally, without loss of generality, we normalize labor supply to 1. This completes the determination of all the parameters in the model except the IPR policy.

As already noted, we begin with the full patent protection regime, that is,  $\eta = \{0, 0, \dots\}$ . We then move to a comparison of the optimal (welfare-maximizing) uniform IPR policy  $\eta^{uni}$  to the optimal state-dependent IPR policy. Since it is computationally impossible to calculate the optimal value of each  $\eta_n$ , we limit our investigation to a particular form of state-dependent IPR policy, whereby the same  $\eta$  applies to all industries that have a technology gap of  $n = 5$  or more. In other words, the IPR policy can be represented as

IPR policy $\rightarrow$	<i>none</i>	$\eta_1$	$\eta_2$	$\eta_3$	$\eta_4$	$\eta_5$								
Technology gap: $n \rightarrow$	0	1	2	3	4	5	6	7	8	9	10	11	. . .	$\infty$

We checked and verified that allowing for further flexibility (for example, allowing  $\eta_5$  and  $\eta_6$  to differ) has little effect on our results.

19. More specifically, (45) can be alternatively written as  $\text{Innovation}(t) = B w(t)^{-\nu} (\text{R\&D expenditure})^\nu$ , thus would be equivalent to (44) as long as the growth of  $w(t)$  can be approximated by constant rate.

20. For example, we could add a small linear term to the production function for R&D, (45), and also make it flat after some level  $\bar{h}$ . For example, the following generalization of (45),  $x = \min\{Bh^\nu + \varepsilon h; B\bar{h}^\nu + \varepsilon\bar{h}\}$  for  $\varepsilon$  small and  $\bar{h}$  large, makes no difference to our simulation results.

21. In particular, in our benchmark parameterization with full protection without licensing, 24% of industries are in the neck-and-neck state. This implies that improvements in the technological capability of the economy is driven by the R&D efforts of the leaders in 76% of the industries and the R&D efforts of both the leaders and the followers in 24% of the industries. Therefore, the growth equation, (39), implies that  $g \simeq \ln \lambda \times 1.24 \times x$ , where  $x$  denotes the average frequency of innovation in a given industry. A major innovation on average every three years implies a value of  $\lambda \simeq 1.05$ .

22. The production function for the intermediate good, (10), can be written as  $\ln(y(j, t)) = n(j, t) \ln(\lambda) + \ln(l(j, t))$ , where  $n(j, t)$  is the number of innovations to date in sector  $j$  and represents the *knowledge stock* of this industry. Bloom, Schankerman, and Van Reenen (2005) proxy the knowledge stock in an industry by the stock of R&D in that industry and estimate the elasticity of sales with respect to the stock of R&D to be approximately 0.06. In terms of the exercise here, this implies that  $\ln(\lambda) = 0.06$ , or that  $\lambda \approx 1.06$ .

The numerical methodology we pursue relies on *uniformization* and value function iteration. The details of the uniformization technique are described in the proof of Lemma 1 in Web Appendix E (for details of value function iteration, see Judd 1999). In particular, we first take the IPR policy  $\eta$  as given and make an initial guess for the equilibrium labor share  $\omega^*$ . Then for a given  $\omega^*$ , we generate a sequence of values  $\{v_n\}_{n=-\infty}^{\infty}$ , and we derive the optimal R&D policies,  $\{x_n^*\}_{n=-\infty}^{\infty}$  and the steady-state distribution of industries,  $\{\mu_n^*\}_{n=0}^{\infty}$ . After convergence, we compute the growth rate  $g^*$  and welfare, and then check for market clearing in the labor market from equation (23). Depending on whether there is excess demand for or supply of labor,  $\omega^*$  is varied and the numerical procedure is repeated until the entire steady-state equilibrium for a given IPR policy is computed. The process is then repeated for different IPR policies.

In the state-dependent IPR case, the optimal (welfare-maximizing) IPR policy sequences,  $\eta^*$ , are computed one element at a time, until we find the welfare-maximizing value for that component, for example,  $\eta_1$ . We then move the next component, for example,  $\eta_2$ . Once the welfare-maximizing value of  $\eta_2$  is determined, we go back to optimize over  $\eta_1$  again, and this procedure is repeated recursively until convergence.<sup>23</sup>

## 6. Optimal IPR Policy

In this section, we present a quantitative analysis of our baseline model.

### 6.1. Full IPR Protection

We start with the benchmark with full protection, which is the case that the existing literature has considered so far (for example, Aghion et al. 2001). In terms of our model, this corresponds to  $\eta_n = 0$  for all  $n$ . We choose the parameter  $B$  in terms of (45), so that the benchmark economy has an annual growth rate of 1.86%.

The value function for this benchmark case is shown in Figure 2 (solid line). The value function has decreasing differences for  $n \geq 0$ , which is consistent with the results in Proposition 4, and features a constant level for all followers (since there is no state dependence in the IPR policy). Figure 3 shows the level of R&D efforts for leaders and followers in this benchmark (again solid line). Again consistent with Proposition 4, this figure also shows that the R&D level of a leader declines as the technology gap increases and that the highest level of R&D is for firms that are neck-and-neck (that is, at the technology gap of  $n = 0$ ). Since there is no state-dependent IPR policy, all followers undertake the same level of R&D effort, which is also shown in the figure.

Figure 4 shows the distribution of industries according to technology gaps (again the solid line refers to the benchmark case). The mode of the distribution is at the technology gap of  $n = 1$ , but there is also a significant concentration of industries at

23. After we find a maximizer ( $\eta^*$ ), we also evaluate several random policy combinations around the maximizer to verify the solution.

TABLE 1. Optimal patent length in quick catch-up regime.

$\lambda = 1.05,$ $\gamma = 0.35,$ $B = 0.1$	Full IPR	Optimal uniform IPR	Optimal state – dependent IPR
$\eta_1$	0	0	0.71
$\eta_2$	0	0	0.08
$\eta_3$	0	0	0
$\eta_4$	0	0	0
$\eta_5$	0	0	0
$x_{-1}^*$	0.22	0.22	0.12
$x_0^*$	0.35	0.35	0.25
$x_1^*$	0.29	0.29	0.41
$\mu_0^*$	0.24	0.24	0.46
$\mu_1^*$	0.33	0.33	0.19
$\mu_2^*$	0.20	0.20	0.13
$\omega^*$	0.95	0.95	0.96
Researcher ratio	0.032	0.032	0.028
$\ln C(0)$	33.78	33.78	34.20
$g^*$	0.0186	0.0186	0.0204
Welfare	683.0	683.0	692.1

\*Notes: Tables 1–4 give the results of the numerical computations with  $\rho = 0.05$ , under three different IPR policy regimes. They consider a different environment (quick catch-up, slow catch-up, licensing and leapfrogging) at a time. Depending on the applicability and necessity, the tables report the steady-state equilibrium values of the difference in the values  $v_1 - v_{-3}$  and  $v_0 - v_{-3}$ ; the (annual) catch-up and frontier R&D rates of a follower that is one step behind,  $(x_{-1}^{C*}, x_{-1}^{F*})$ ; the (annual) R&D rate of neck-and-neck competitors,  $x_0^*$ ; the (annual) R&D rate of one-step leader,  $x_1^*$ ; fraction of industries in neck-and-neck competition,  $\mu_0^*$ ; fraction of industries at a technology gap of  $n = 1, 2$ ; the value of “labor share,”  $\omega^*$ ; the ratio of the labor force working in research; log of initial (annual) consumption,  $\ln C(0)$ ; the annual growth rate,  $g^*$ ; and the welfare level according to equation (42). They also report the welfare-maximizing uniform and state-dependent IPR policies.

Web Appendix Tables A.1–A.4 contain robustness checks for the benchmark results of Table 1 with alternative step sizes and R&D elasticity parameters. Web Appendix Table C.1 combines the three environments (slow catch-up, licensing and leapfrogging). Web Appendix Table D.1 reports the robustness checks of the state-dependent results of Table C.1 with alternative step sizes and R&D elasticity parameters.

technology gap  $n = 0$ , because innovations by the followers take them to the *neck-and-neck* state.

The first column of Table 1 also reports the results for this benchmark simulation. As already noted, in each case  $B$  is chosen such that the annual growth rate is equal to 0.0186, which is recorded at the bottom of Table 1 together with the initial consumption and welfare levels according to equations (42) and (43). The table also shows the R&D levels  $x_0^*$ ,  $x_{-1}^*$ , and  $x_1^*$  (0.35, 0.22, and 0.29), the frequencies of industries with technology gaps of 0, 1, and 2. The steady-state value of  $\omega^*$  is 0.95. Since labor is the only factor of production in the economy,  $\omega^*$  should not be thought of as the labor share in GDP. Instead,  $1 - \omega^*$  measures the share of pure monopoly profits in value added. In the benchmark parameterization, this corresponds to 5% of GDP, which is reasonable.<sup>24</sup> Finally, the table also shows that in this benchmark parameterization

24. Bureau of Economic Analysis (2004) reports that the ratio of before-tax profits to GDP in the US economy in 2001 was 7% and the after-tax ratio was 5%.

3.2% of the workforce is working as researchers, which is also consistent with US data.<sup>25</sup> These results are encouraging for our simple quantitative exercise, since with very few parameter choices, the model generates reasonable numbers, especially for the share of the workforce allocated to research.<sup>26</sup>

### 6.2. Optimal Uniform IPR Protection

For reference, we now characterize optimal uniform IPR policy, that is, we impose that  $\eta_n = \eta$  for all  $n$ , and look for values of  $\eta^*$  that maximizes the welfare in the economy. Column 2 of Table 1 shows that the welfare-maximizing value of  $\eta^*$  is not different from zero at the three-digit level. Therefore the results of the full protection case carries over to uniform policy as well. The main reason for this result is the quick catch-up assumption. Recall that the uniform IPR policy discourages innovation, but generates a potential benefit because of the composition effect (bringing more firms into neck-and-neck position). In the quick catch-up regime, firms come into neck-and-neck position at a Poisson rate of 0.22, which results in 35% of sectors being in state 0 and 77% at two-step gap or below. This implies that there are only limited composition gains. In this light, it is not surprising that relaxing the IPR protection uniformly is not beneficial; it generates a significant disincentive effect and little benefit. Therefore, optimal IPR policy is to set full protection,  $\eta^* = 0$ , and thus the value functions, innovation rates and industry distributions under optimal uniform IPR policy are given by the solid lines in Figures 2–4.

### 6.3. Optimal State-Dependent IPR

We next turn to our major question; whether state-dependent IPR makes a significant difference relative to the uniform IPR. In particular, we look for the combination of  $\{\eta_1, \dots, \eta_5\}$  that maximizes the welfare. The new value function, innovation rates and industry distribution are plotted in Figures 2–4 and the numerical results are shown in column 3 of Table 1.

Two features are worth noting. First, the growth rate increases noticeably relative to column 1; it is now 2.04% instead of 1.86%. Second and more important, we see the key pattern that will be present in all of our quantitative results: optimal state-dependent policy  $\{\eta_1^*, \dots, \eta_5^*\}$  provides *greater* protection to technology leaders that are further ahead. In particular, we find that the optimal policy involves  $\eta_1^* = 0.71$ ,  $\eta_2^* = 0.08$ , and  $\eta_3^* = \eta_4^* = \eta_5^* = 0$ . This corresponds to very little patent protection for firms that are

25. According to National Science Foundation (2006), the percentage of scientists and engineers in the US workforce in 2001 is about 4%.

26. Most endogenous growth models imply that a significantly greater fraction of the labor force should be employed in the research sector and one needs to introduce various additional factors to reduce the profitability of research or to make entry into research more difficult. In the current model, the step-by-step nature of innovation and competition plays this role and generates a plausible allocation of workers between research and production.

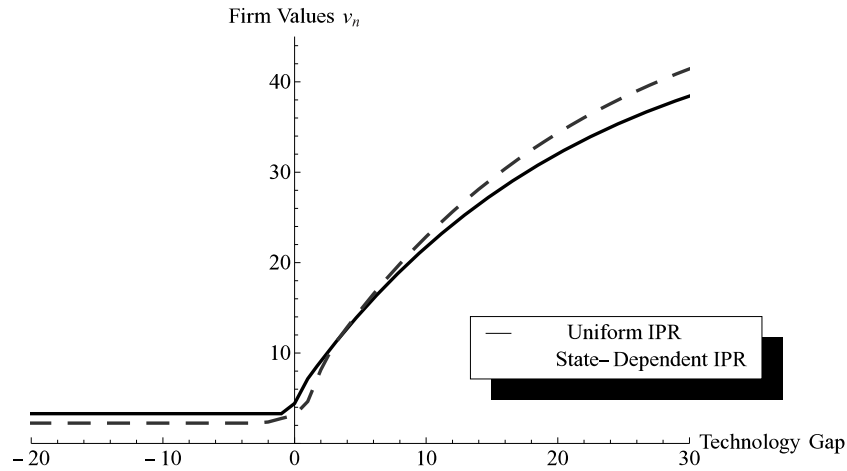


FIGURE 2. Value functions.

one step ahead of the followers. In particular, since  $\eta_1^* = 0.71$  and  $x_{-1}^* = 0.12$ , in this equilibrium firms that are one step behind followers are more than *six times* as likely to catch up with the technology leader because of the expiration of the patent of the leader as they are likely to catch up because of their own successful R&D. Then, there is a steep increase in the protection provided to technology leaders that are two steps ahead, and  $\eta_2^*$  is 1/12th of  $\eta_1^*$ . Perhaps even more remarkably, after a technology gap of three or more steps, optimal IPR involves full protection, and patents never expire.

This pattern of greater protection for technology leaders that are further ahead may go against a naïve intuition that state-dependent IPR policy should try to boost the growth rate of the economy by bringing the industries with largest technology gaps (where leaders engage in little R&D) into neck-and-neck competition. This composition effect is present, but dominated by another, more powerful force, the *trickle-down* effect. The intuition for the trickle-down effect is as follows: by providing secure patent protection to firms that are three or more steps ahead of their rivals, optimal state-dependent IPR increases the R&D effort of leaders that are one and two steps ahead as well. This is because technology leaders that are only one or two steps ahead now face greater returns to R&D, which will not only increase their profits but also the security of their intellectual property. Mechanically, high levels of  $\eta_1$  and  $\eta_2$  reduce  $v_1$  and  $v_2$ , while high IPR protection for more advanced firms increases  $v_n$  for  $n \geq 3$ , and this increases the R&D incentives of leaders at  $n = 1$  or at  $n = 2$ .<sup>27</sup>

27. An alternative intuition, suggested by an anonymous referee, is that when the technology gap is greater, leaders will lose more from a relaxation of IPR. However, this intuition can only be partial, since, as shown in Section 2, state-dependent relaxation of IPR in this form creates a positive incentive effect, which is central to our results (and this is independent of how much technology leaders lose as a result of the relaxation of IPR). As a result, we believe that the trickle-down of incentives is the more correct intuition for our results.

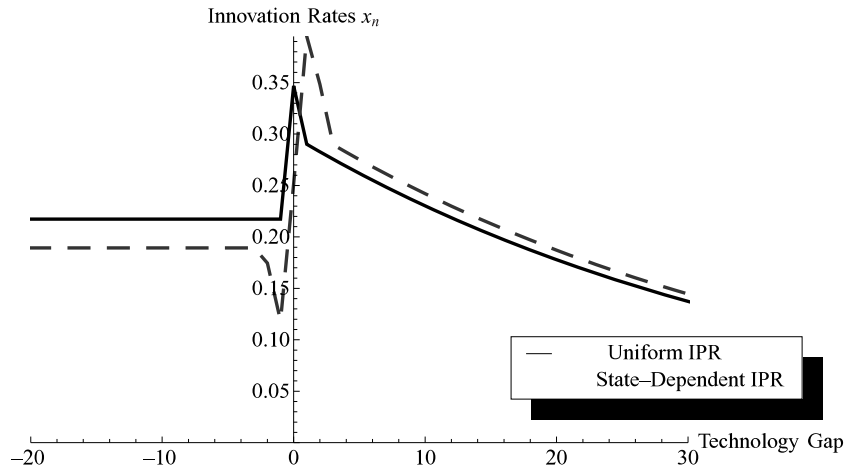


FIGURE 3. R&D efforts.

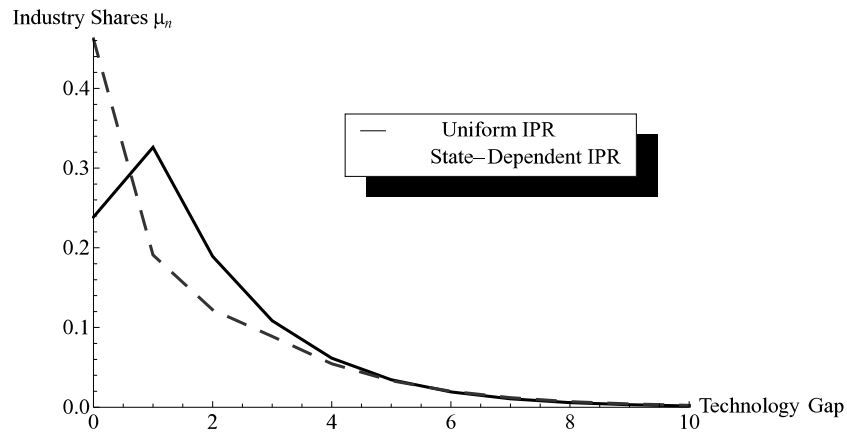


FIGURE 4. Industry shares.

Providing more secure patent protection through less frequent catch-up benefits an  $n$ -step leader more than  $(n + 1)$ -step leader since the preserved profit is higher for a more advanced firm. This results in a steeper value function, as illustrated in Figure 2. The slope of the value function is the key determining factor for R&D decisions and this increase in slope reflects itself in overall higher R&D effort by the leaders in Figure 3. It is also notable that state-dependent IPR introduces positive incentive effect while gaining also from the composition. Figure 4 shows that the mode of the new distribution is at  $n = 0$ . The average innovation rate is higher (as reflected on a higher growth rate,  $g^* = 2.04\%$ ) and the average mark-up is lower ( $C(0)$  increases by 52%). This pattern of greater R&D investments under state-dependent IPR contrasts with uniform IPR, which always reduces R&D of all firms. The possibility that imperfect

state-dependent IPR protection can *increase* (rather than reduce) R&D incentives is a novel feature of our approach and has also been shown explicitly in the partial equilibrium model of Section 2.

#### 6.4. Robustness

The patterns shown in Figures 2–4 and Table 1 are quite robust. In Tables 1–4, we report results from the same exercise for various different combinations of values of  $\gamma$  and  $\lambda$  (in particular, varying  $\gamma$  to  $\gamma = 0.1$  and  $\gamma = 0.6$ , and  $\lambda$  to  $\lambda = 1.01$  and  $\lambda = 1.2$ ). In each case, we change the parameter  $B$  in equation (45) so that the growth rate of the benchmark economy with full IPR protection without licensing is the same as in Table 1, that is,  $g^* = 1.86\%$ . The overall pattern and in fact the quantitative magnitudes are remarkably similar to the baseline results reported here. We therefore conclude that the trickle-down of incentives and the form of the optimal state-dependent IPR policy apply for a range of plausible parameter values.

### 7. Optimal IPR Policy in the Slow Catch-up Regime

In this section, we extend our analysis to an environment where followers close the gap with technology leaders also step by step. We then also introduce different types of R&D efforts by followers and study several different dimensions of IPR policy.

#### 7.1. Value Functions

The environment is the same as in Section 3, except that we now assume that successful R&D by followers close is the gap between themselves and the technology leader by one step. We will allow for different types of R&D in what follows. The equivalent expressions for the value functions (29)–(31) in this case are

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{l} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n [v_{n+1} - v_n] \\ + x_{-n}^* [v_{n-1} - v_n] + \eta_n [v_0 - v_n] \end{array} \right\} \text{ for } n \in \mathbb{N}, \quad (46)$$

$$\rho v_0 = \max_{x_0 \geq 0} \{-\omega^* G(x_0) + x_0 [v_1 - v_0] + x_0^* [v_{-1} - v_0]\}, \quad (47)$$

$$\rho v_{-n} = \max_{x_{-n} \geq 0} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + x_{-n} [v_{-n+1} - v_{-n}] \\ + x_n^* [v_{-n-1} - v_{-n}] + \eta_n [v_0 - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N}. \quad (48)$$

These expressions are intuitive in light of those presented in Section 3, in particular, (29)–(31). The only difference from equations (29)–(31) is that, when a follower innovates, an  $n$ -step leader's value changes from  $v_n$  to  $v_{n-1}$  instead of dropping all the way to  $v_0$ , since this innovation closes the technology gap only by one step. Similarly, in this event, the follower's value changes from  $v_{-n}$  to  $v_{-n+1}$  instead of increasing all the way to  $v_0$ . The rest of the analysis mirrors that in Section 4. In particular, existence

TABLE 2. Optimal patent length in slow catch-up regime.

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1,$ $\zeta_n = \infty, \vartheta_n = \infty$	Full IPR	Optimal uniform IPR	Optimal state – dependent IPR
$\eta_1$	0.02	0.11	0.69
$\eta_2$	0.02	0.11	0.20
$\eta_3$	0.02	0.11	0.14
$\eta_4$	0.02	0.11	0.12
$\eta_5$	0.02	0.11	0.08
$x_{-1}^*$	0.75	0.27	0.17
$x_0^*$	0.99	0.14	0.32
$x_1^*$	1.10	0.15	0.51
$\mu_0^*$	0.02	0.16	0.30
$\mu_1^*$	0.03	0.19	0.15
$\mu_2^*$	0.03	0.14	0.10
$\omega^*$	0.56	0.90	0.90
Researcher ratio	0.150	0.055	0.059
$\ln C(0)$	31.31	34.31	34.57
$g^*$	0.025	0.023	0.025
Welfare	636.3	695.3	701.2

Note: See Table 1.

of stationary equilibria can be proved using an analogous argument to that provided in the Web Appendix E, but we are not able to prove the analogue of the second part of Proposition 4.

## 7.2. Quantitative Results

We next investigate the form of optimal IPR policy in the baseline slow catch-up regime.

*Full IPR Protection.* Under the slow catch-up regime, setting  $\eta_n = 0$ , that is, providing full protection via infinite patent length generates too little catch-up by the followers. Consequently, the steady-state distribution has little mass at or around the *neck-and-neck* state ( $n = 0$ ). To generate a more plausible distribution with a nonzero share of industries in the neck-and-neck state, we instead impose  $\eta_n = 0.02$ , which implies an expected length of patent protection of 50 years (as the full protection benchmark) under slow catch-up regime.

The first column of Table 2 reports the results under this scenario. Even with 50 years of protection, the share of industries that are neck-and-neck is only 2%, and the total share of industries that have a gap of less than two steps is only 8%. One implication of this pattern is that a relaxation of IPR policy may now be more powerful because it can affect the composition of industries, reduce the average mark-up in the economy, and perhaps have a large effect on average R&D. Therefore, this is a particularly relevant environment for investigating whether the trickle-down of



incentives identified in the previous section is present and robust in different and perhaps more realistic environments.

*Optimal Uniform IPR Protection.* The second column of Table 2 shows optimal uniform IPR policy in this case. Consistent with Proposition 1 in Section 2, relaxing IPR protection creates a powerful disincentive effect. However, it also generates a beneficial composition effect by bringing more and more firms into neck-and-neck competition. For this reason, optimal uniform IPR policy is no longer full protection.

The results in the table show that the optimal policy reduces patent length from  $\eta = 0.02$  (average protection of 50 years) to  $\eta^* = 0.11$  (average protection of nine years). This involves a lower innovation rate for technology leaders that are one-step ahead (from 1.1 to 0.15). Similarly average R&D is also reduced and the aggregate growth rate declines from 2.5% to 2.3%. However, because of the increase in the share of neck-and-neck industries (from 2% to 16%) and the increase in the total share of industries that are in the first three states (from 8% to 49%), the average mark-up in the economy decreases. This enables a large (19-fold) increase in initial consumption  $C(0)$  (which is the reason why this policy is optimal even though it reduces growth).

*Optimal State-Dependent IPR.* Once again, the most interesting case is when IPR policy is state dependent. In this case, the optimal policy not only benefits from the composition effect, but can do so without sacrificing growth (by exploiting the positive incentive and the trickle-down effects highlighted in Proposition 2 in Section 2).

The optimal state-dependent policies shown in column 3 of Table 2. Under this optimal policy, the share of the first three states increases by an additional six percentage points (55%) and the initial consumption further increases relative to the uniform IPR policy by 30%. More interestingly, the innovation rate of a one-step leader now increases from 0.15 to 0.51 (relative to the uniform policy case) and the growth rate increases back to 2.5%. It is noteworthy that these gains are achieved by providing stronger protections to more advanced firms, and thus exploiting the trickle-down effect. For example, under the optimal policy one-step leader is caught up seven times more frequently than a five-step leader due to patent expiration.

### 7.3. Compulsory Licensing

In this section, we introduce (compulsory) licensing. Several recent empirical papers suggest that licensing has a significant positive impact on firm innovation (for example, Moser and Voena 2011; Almeida and Fernandes 2008). Consistent with these findings, we model licensing as a way of generating knowledge spillovers to the licensee. In particular, we assume that in addition to independent R&D to proceed one step in the quality ladder, followers can also close all intervening steps by reverse-engineering the current leading-edge technology. But this is only possible by making use of the knowledge generated by the leading-edge technology, and the follower will have to pay a prespecified license fee  $\hat{\zeta}_n(t) \geq 0$  to the leader. The licensing decision of the

follower  $-i$  is denoted by  $a_{-i}(j, t) = 1$  ( $a_{-i}(j, t) = 0$  corresponds to independent R&D). Throughout, we allow  $a_{-i}(j, t) \in [0, 1]$  for mathematical convenience. The fees in question are compulsory license fees imposed by policy and are state dependent, and thus we represent them as

$$\hat{\zeta}(t) : \mathbb{N} \rightarrow \mathbb{R}_+ \cup \{+\infty\}.$$

Note that  $\hat{\zeta}(t) \equiv \{\hat{\zeta}_1(t), \hat{\zeta}_2(t), \dots\}$  is a function of time. This is natural, since in a growing economy, license fees should not remain constant. As in (28), in what follows we assume that license fees are also scaled up by GDP, so that  $\zeta_n \equiv \hat{\zeta}_n(t)/Y(t)$ , to keep the equilibrium stationary.<sup>28</sup>

*Value Functions with Compulsory Licensing.* With a similar reasoning to before, relevant value functions in this case can be written as

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{l} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] \\ + a_{-n}^* x_{-n}^* [v_0 - v_n + \zeta_n] + \\ (1 - a_{-n}^*) x_{-n}^* [v_{n-1} - v_n] + \eta_n [v_0 - v_n] \end{array} \right\} \text{ for } n \in \mathbb{N},$$

where  $a_{-n}^*$  is the equilibrium value of licensing decision by a follower that is  $n$  steps behind, and  $\zeta_n$  is the license fee that it has to pay. The value for neck-and-neck firms remain unchanged while the values for followers becomes

$$\rho v_{-n} = \max_{\substack{x_{-n} \geq 0, \\ a_{-n} \in [0, 1]}} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + a_{-n} x_{-n} [v_0 - v_{-n} - \zeta_n] \\ + (1 - a_{-n}) x_{-n} [v_{-n+1} - v_{-n}] + \\ x_{-n}^* [v_{-n-1} - v_{-n}] + \eta_n [v_0 - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N}.$$

Note that licensing  $a_{-n} \in [0, 1]$  is the new additional decision variable of the follower.

Full IPR protection in this case corresponds to prohibitively high licensing fees, that is,  $\zeta_n = \infty$  for all  $n$ , and as in the previous subsection, patent protection has expected duration of 50 years ( $\eta = 0.02$ ). Therefore, the results in this case will be identical to those reported for full protection in the previous section (column 1 of Table 2). This is indeed the case; these results are repeated in column 1 of Table 3 for ease of comparison with the remaining results in this table.

*Optimal Uniform IPR Protection.* Uniform compulsory licensing policy now corresponds to  $\zeta_n = \zeta^* \geq 0$ . The results under the optimal choice of such uniform compulsory licensing policy are reported in the second column of Table 3. This optimal policy involves  $\zeta^* = 1.61$ , which is more than half of the surplus that a three-step follower generates from licensing,  $v_0 - v_{-3} = 2.9$ .

28. Voluntary licensing is briefly discussed in the Web Appendix, where we show that it cannot in general achieve the same results as compulsory licensing.

TABLE 3. Licensing in slow catch-up regime.

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1,$ $\eta_n = 0.02, \vartheta_n = \infty$	Full IPR	Optimal uniform IPR	Optimal state – dependent IPR
$\zeta_1$	$\infty$	1.61	0
$\zeta_2$	$\infty$	1.61	1.54
$\zeta_3$	$\infty$	1.61	2.45
$\zeta_4$	$\infty$	1.61	2.92
$\zeta_5$	$\infty$	1.61	3.32
$v_0 - v_{-3}$	10.1	2.9	3.2
$x_{-1}^*$	0.75	0.27	0.31
$x_0^*$	0.99	0.39	0.45
$x_1^*$	1.10	0.43	0.46
$\mu_0^*$	0.02	0.21	0.18
$\mu_1^*$	0.03	0.25	0.20
$\mu_2^*$	0.03	0.20	0.12
$\omega^*$	0.56	0.94	0.91
Researcher ratio	0.150	0.043	0.071
$\ln C(0)$	31.31	34.13	34.47
$g^*$	0.025	0.021	0.025
Welfare	636.3	690.9	699.3

Note: See Table 1.

Since this type of licensing allows for more frequent catch-up by followers, a greater share of industries are now in tight competition: the total share of industries with one or two step gaps goes up to 66% (this number was 8% under full protection). This again corresponds to a powerful composition effect and generates a significant reduction in the average mark-up and a corresponding increase in initial consumption. However, consistent with our previous results, this type of uniform licensing again generates a significant disincentive effect on technology leaders. In particular, more frequent catch-up implies a shorter durations of positive profits. As a result, innovation incentives are reduced; the innovation rate of a one-step leader is now 0.43 instead of 1.1 and the average growth rate declines from 2.5% to 2.1%.

*Optimal State-Dependent IPR.* As in our previous exercises, the negative incentive effects of uniform relaxations of IPR protection are rectified when policy is state dependent. Optimal state-dependent policy has in fact qualitatively very similar pattern to those already reported. Most importantly, column 3 of Table 3 shows that optimal state-dependent policy provides greater protection to technology leaders that are more advanced. For example, while a two-step leader receives a license fee of  $\zeta_2^* = 1.5$ , a five-step leader receives more than its double,  $\zeta_5^* = 3.3$ . Given this pattern, the *trickle-down effect* is again at work and generates positive innovation incentives: the innovation rate of a one-step leader increases to  $x_1^* = 0.46$  and the aggregate growth rate goes back to 2.5% from 2.1%. This positive gain is generated without sacrificing the composition effect. Under this policy, 50% of total industries operate with a technology gap less

than two and the initial consumption  $C(0)$  is now even higher than under uniform policy (by 40%).

#### 7.4. Leapfrogging and Infringement under Slow Catch-up

Finally, we allow the follower to engage in frontier R&D and *leapfrog* the technology leader. This exercise is useful for two reasons. First, the models analyzed so far do not allow R&D by followers to directly contribute to aggregate growth. One might conjecture that this feature strengthens the *trickle-down* effect. Second, frontier R&D and leapfrogging by followers will allow us to introduce another relevant and important dimension of IPR policy, patent infringement fees.

Suppose, now, that followers can undertake two types of R&D. The first, which is what we have focused on so far, is *catch-up R&D*, corresponding to R&D directed at discovering an *alternative* way of performing the same task as the current leading-edge technology. Catch-up R&D improves the technology of the follower by one step as before. The alternative, *frontier R&D*, involves followers improving the current leading-edge technology. If this type of R&D succeeds, the follower will have improved the leading-edge technology. However, following such an event, the follower will be judged (for example, by courts) to have infringed the patent of technology leader with probability  $\tau \in (0, 1)$  and will be required to pay a prespecified infringement penalty (fee)  $\hat{\vartheta}_n \geq 0$  to the leader. The infringement fees are also state dependent and represented by

$$\hat{\vartheta}(t) : \mathbb{N} \rightarrow \mathbb{R}_+ \cup \{+\infty\},$$

and we again adopt the normalization  $\vartheta_n \equiv \hat{\vartheta}_n(t)/Y(t)$ , and denote the Poisson arrival rate of innovation by catch-up R&D and frontier R&D by  $x_n^c$  and  $x_n^f$ , respectively. Then the new value of an  $n$ -step leader takes the following form:

$$\rho v_n = \max_{x_n \geq 0} \left\{ \begin{array}{l} (1 - \lambda^{-n}) - \omega^* G(x_n) + x_n[v_{n+1} - v_n] \\ + x_{-n}^{c*}[v_{n-1} - v_n] + x_{-n}^{f*}[v_{-1} - v_n + \tau \vartheta_n] \\ + \eta_n[v_0 - v_n] \end{array} \right\} \text{ for } n \in \mathbb{N}.$$

The main difference in this equation is that the follower has two different arrival rates of innovation. If the follower is successful with frontier R&D, the current leader falls one step behind the follower. However, in this event, with probability  $\tau$ , it receives an infringement fee of  $\vartheta_n$ . With a similar reasoning, the value of an  $n$ -step follower now becomes

$$\rho v_{-n} = \max_{x_{-n}^c \geq 0, x_{-n}^f \geq 0} \left\{ \begin{array}{l} -\omega^* G(x_{-n}) + x_{-n}^c[v_{-n+1} - v_{-n}] \\ + x_{-n}^f[v_1 - v_{-n} - \tau \vartheta_n] \\ + x_n^*[v_{-n-1} - v_{-n}] + \eta_n[v_0 - v_{-n}] \end{array} \right\} \text{ for } n \in \mathbb{N}.$$

The value of a neck-and-neck firm is unchanged.

TABLE 4. Leapfrogging in slow catch-up regime.

$\lambda = 1.05, \gamma = 0.35$ $B = 0.1,$ $\eta_n = 0.02, \zeta_n = \infty$	Full IPR	Optimal uniform IPR	Optimal state – dependent IPR
$\vartheta_1$	$\infty$	14	0
$\vartheta_2$	$\infty$	14	18.1
$\vartheta_3$	$\infty$	14	31.3
$\vartheta_4$	$\infty$	14	36.6
$\vartheta_5$	$\infty$	14	43.7
$v_1 - v_{-3}$	21.4	2.7	4.1
$x_{-1}^{c*}$	0.75	0.15	0.14
$x_{-1}^{f*}$	0	0.23	0.33
$x_0^*$	0.99	0.30	0.29
$x_1^*$	1.10	0.30	0.39
$\mu_0^*$	0.02	0.14	0.12
$\mu_1^*$	0.03	0.42	0.35
$\mu_2^*$	0.03	0.22	0.17
$\omega^*$	0.56	0.95	0.94
Researcher ratio	0.150	0.028	0.058
$\ln C(0)$	31.31	35.48	36.17
$g^*$	0.025	0.026	0.031
Welfare	636.3	720.0	735.9

Note: See Table 1.

The quantitative analysis requires an empirical estimate for  $\tau$ . Lanjouw and Schankerman (2001) report that around 10% of the US utility patents are filed for infringement. We therefore set  $\tau = 0.1$ .

Note also that since the followers now improve the technology frontier through frontier R&D, the aggregate growth rate becomes

$$g^* = \ln \lambda \left[ 2\mu_0^* x_0^* + \sum_{n=1}^{\infty} \mu_n^* (x_n^* + x_{-n}^{f*}) \right]. \quad (49)$$

Full protection in this case corresponds to infinite patent infringement fees, that is,  $\vartheta_n = \infty$ , and given the same parameter choices as before, will be identical to column 1 of Table 2. We repeat these results in column 1 of Table 4 for ease of comparison with the rest of the table.

*Optimal Uniform IPR Protection.* In the uniform policy case, we set  $\vartheta_n = \vartheta \geq 0$ . Column 2 of Table 4 shows that the optimal uniform policy in this case is  $\vartheta^* = 14$ . Recall that when a follower undertakes frontier innovation, the probability that it will have to make this payment is  $\tau = 0.1$ . Therefore the expected infringement payment is  $\tau \times \vartheta^* = 1.4$  which is more than half of the surplus that a three-step follower generates out of leapfrogging,  $v_1 - v_{-3} = 2.7$ .

Column 2 also shows that under this policy, followers undertake more frontier R&D ( $x_{-1}^{*f} = 0.23$ ) than catch-up R&D ( $x_{-1}^{*c} = 0.15$ ). Parallel to the previous uniform policies, the shorter duration of monopoly position resulting from innovation reduces

innovation incentives. For example, one-step leaders now innovate at the rate 0.3 instead of 1.1. However, despite this disincentive effect, the growth rate increases slightly because leapfrogging allows followers to directly contribute to aggregate growth, as shown by equation (49).

Column 2 also shows that the share of industries in one-step gap is now much larger,  $\mu_1 = 0.42$ . This is because leapfrogging puts the follower one-step ahead of the previous leader. Thanks to this effect, optimal uniform IPR protection achieves lower average mark-up and higher initial consumption as well as higher growth.

*Optimal State-Dependent IPR.* State-dependent IPR policy once again exploits the trickle-down effect and creates positive incentive effects on innovation. The form of state-dependent policy is the same as before: technologically more advanced leaders receive more protection in the form of higher fees when followers infringe their patents. While a two-step leader receives  $\vartheta_1^* = 18.1$  in case of infringement, a five-step leader receives more than double of this fee,  $\vartheta_5^* = 43.7$ . In expectation, a three-step follower pays almost three-quarters of the surplus that it generates from leapfrogging ( $\tau \times \vartheta_3^* = 3.1$  versus  $v_1 - v_{-3} = 4.1$ ). As a result of this pattern, state-dependent policy not only generates a greater welfare gain in terms of the initial consumption ( $C(0)$  is now approximately twice the level under the optimal uniform policy), but it also exploits the trickle-down effect and increases the equilibrium growth rate by an additional 0.5 percentage point relative to the uniform policy.

*Additional Results and Robustness.* In the Web Appendix, we show that the results are similar when all three IPR policies are simultaneously present. In particular, the optimal pattern of R&D involves infinitely long patents with prohibitively high compulsory license fees. The only dimension in which IPR protection is not full is because of moderate infringement fees, which permit followers to undertake frontier R&D and leapfrog technology leaders. Crucially, this aspect of IPR is state-dependent and exploits the trickle-down effect. We also report robustness checks for different values of the parameters  $\lambda$  and  $\gamma$  (again increasing or reducing  $\lambda$  to 1.2 or 1.01, and increasing or reducing  $\gamma$  to 0.6 or 0.1). In all cases, the pattern of optimal IPR is similar: infringement fees are state dependent and provide greater protection to technologically more advanced leaders.

## 8. Conclusions

In this paper, we have emphasized the importance of dynamic interactions between IPR protection and competition for understanding the structure of optimal IPR policy. Our main result highlights the importance of a new and powerful effect, *the trickle-down effect*, which implies that protection given to companies with significant technological leads over their rivals also dynamically incentivizes companies with more limited technological leads—as further innovation will not only increase their productivity but also grant them additional IPR protection. This new effect implies that optimal IPR

policy should be *state dependent* and provide greater protection to companies with significant technological leads and only limited IPR protection for those without.

To systematically investigate these issues, we developed a dynamic general equilibrium framework with cumulative (step-by-step) innovations. In each industry, technology leaders innovate in order to widen the gap between themselves and the followers, which enables them to charge higher markups. Followers innovate to catch up with or surpass the technology leaders in their industry (by undertaking *frontier R&D*), and can also license the technology of leaders. IPR policy regulates the length of patents, whether licensing is possible and the size of patent infringement fees.

We provided existence and characterization results, and a quantitative analysis of the form of *optimal* (welfare-maximizing) IPR policy. In several different environments and under different parameter values, we consistently found that the trickle-down effect is present and powerful. It implies that optimal IPR should be state-dependent and should provide greater protection to firms with greater technological lead over their rivals. In our benchmark parameterization, for example, optimal IPR policy increases the growth rate of the economy from 1.86% to 2.04%, and does so while also significantly increasing initial consumption (and in fact reducing the overall amount of resources allocated to the R&D sector). We also showed that similar qualitative and quantitative results are obtained when followers catch up with technology leaders only slowly. In this extended environment, we also investigated the form of optimal compulsory licensing fees and patent infringement fees, and found them to be similarly state dependent (in a way that provides greater protection to firms that are technologically more advanced relative to their rivals). These extensions further showed that compulsory licensing, which allows followers to build on the leading-edge technology in return of a license fee, also has a major impact on the equilibrium growth rate.

Our main results go against a naïve intuition that providing less protection to technologically more advanced firms is socially beneficial because it would exploit a composition effect (bringing firms that are furthest apart into *neck-and-neck* competition to both reduce markups and increase R&D which results from tight competition). This naïve intuition is not correct precisely because of the trickle-down effect we have already emphasized. The trickle-down effect implies that providing greater protection to sufficiently advanced technology leaders not only increases their R&D efforts but also raises the R&D efforts of all technology leaders that are less advanced than this level. This is because the reward to innovation now includes the greater protection that they will receive once they reach this higher level of technology. Our analysis and results suggest that in addition to the reasoning based on the static trade-off between IPR protection and competition, the trickle-down effect should also be factored into policy analysis, and naturally calls for future empirical work to estimate its empirical magnitude.

In this context, it should be emphasized that our objective in this paper has not been to derive practical policy prescriptions. There is little doubt that our model is simplified, excludes a whole host of important factors, and ignores potential limitations on the form

and complexity of IPR policies. Nevertheless, we believe that our results demonstrate a range of robust and new effects that should be further investigated in future work.

More generally, the analysis in this paper suggests that a move to a richer menu of IPR policies, in particular, a move towards optimal state-dependent policies, may significantly increase innovation, economic growth, and welfare. The results also show that the form of optimal IPR policy may depend on the industry structure (and the technology of catch-up within the industry). The next step in this line of research should be to investigate the robustness of these effects in different models of industry dynamics. It would also be useful to study whether the relationship between the form of optimal IPR policy and industry structure suggested by our analysis also applies when variation in industry structure has other sources (for example, differences in the extent of fixed costs or demand structure causing differential gaps between technology leaders and followers across industries). The most important area for future work is a detailed empirical investigation of the form of optimal IPR policy, using both better estimates of the effects of IPR policy on innovation rates and also structural models that would enable the evaluation of the effects of different policies on equilibrium growth and welfare.

### Supporting Information

Additional Supporting Information may be found in the online version of this article:

**Appendix:** additional results and proofs (pdf file).

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