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JOINT LIFE ANNUITIES AND ANNUITY DEMAND BY MARRIED COUPLES

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ABSTRACT

This article summarizes the range of joint-life annuity products that are currently available to married couples, and it explores the potential utility gain that such couples receive from access to actuarially fair annuity markets. It is more difficult to estimate this utility gain for couples than for individuals because a couple's value of annuitization will depend in part on the survivor benefits that are available after one spouse has died but while the other is still alive. Valuing joint and survivor annuities also requires recognition of the potentially important interactions between the members of a married couple, such as joint consumption, interdependent utilities, and correlated mortality rates. This article considers each of these issues. It develops an annuity valuation model for married couples and it estimates their "annuity equivalent wealth," the amount of wealth that couples would need in the absence of actuarially fair annuity markets in order to achieve the same utility level that they receive when such markets are available. The utility gain from annuitization is smaller for couples than for single individuals. Since most potential annuity buyers are married, this finding may help to explain the limited size of the market for single premium annuities in the United States.

INTRODUCTION

Annuities play an important role in the standard theory of consumer choice when life length is uncertain. Yaari (1965) showed that an individual with a fixed stock of resources and an uncertain lifetime should purchase an annuity contract to insure against the risk of outliving his resources. More recent work, such as Mitchell, Poterba, Warshawsky, and Brown (1999) (hereafter MPWB), finds substantial gains to

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annuitization for individual life-cycle consumers. Typical results suggest that a 65-year-old man who does not have access to an actuarially fair annuity market would be willing to forgo roughly one-third of his wealth if by doing so he could purchase an actuarially fair annuity with his remaining wealth.

In spite of Yaari's (1965) insight, a number of studies have observed that the market for individual annuity contracts in the United States is very small. Standard explanations for the limited flow of new annuity purchases are adverse selection in the annuity market, individual bequest motives, and the presence of other annuitized resources. Several empirical studies, including Friedman and Warshawsky (1988, 1990) and MPWB (1999), have explored the extent of adverse selection. Numerous other studies surveyed in Laitner (1997) and (more briefly) in Altonji, Hayashi, and Kotlikoff (1997) and Laitner and Juster (1996) have focused on intergenerational altruism as a potential explanation for limited annuity demand. Simple altruistic models do not appear to provide a satisfactory explanation for observed patterns of intergenerational transfers. Auerbach, Kotlikoff, and Weil (1992) show that when Social Security benefits, private defined benefit pension plan payments, and Medicare benefits are added together, more than half of the resources of the current elderly in the United States take the form of life-contingent payouts. Thus many households are already "annuitized." This may also explain the limited demand for additional private annuities.

The functioning of annuity markets has recently attracted attention as part of the global policy debate on individual accounts Social Security systems. A central design issue in such systems concerns the way a retiree would spread accumulated resources over his or her remaining lifetime, or his or her lifetime and that of his or her spouse. Private annuities offer one way of spreading such resources; mandatory government-provided annuities are another.

Although the treatment of married couples is an important issue in retirement income security, virtually all of the previous research on annuities has focused on individuals rather than couples as decision-making units. This article tries to address this research gap. It describes the structure of joint-life annuity products that insurers currently offer, and it presents new evidence on the value of annuity contracts from the standpoint of married couples.

Two previous studies have recognized the importance of couples rather than individuals in post-retirement consumption behavior. The first, by Kotlikoff and Spivak (1981), focused on the demand for *individual* annuities by married rather than single individuals. This article showed that the benefits of purchasing an individual annuity contract were smaller for married individuals, or more generally those in an extended family, than for autarchic individuals. Risk sharing within families can potentially offer a substitute for risk sharing in an organized annuity market. Kotlikoff and Spivak (1981) did not consider the demand for joint life annuities by married couples. The second study, Hurd (1999), investigates optimal consumption by married couples when both members of the couple face uncertainty about length of life. This study shows that consumption patterns will depend on the couple's level of annuity income, but it does not explore the demand for annuities by married couples.

Married couples play a central role in determining annuity demand in private markets, so it is somewhat surprising that their annuity demand has not received more

attention. In the over-65 age category in the United States in 1995, 77 percent of men were married, as were 43 percent of women. Most individuals are married at the beginning of their retirement years, when many annuity purchases take place. LIMRA (1997) reports that married persons buy 77 percent of all annuity contracts and 85 percent of single-premium immediate annuity (SPIA) contracts. Single-life annuities without any provisions for spouses or survivors are not as common as period certain options and joint-and-survivor annuities, both of which provide some spousal protection.

In spite of the importance of married individuals in the annuity-buying age range, joint life annuities currently represent a relatively small share of the SPIA market, although they account for a very large fraction of the annuities written in conjunction with defined benefit pension plans. LIMRA (1997) reports that in 1996, 51 percent of SPIA premiums were spent on period certain annuities—annuities that did not involve *any* life contingency. Joint annuities accounted for 11 percent of SPIA premium payments, with 7 percent of such payments for joint life annuities with a “period certain” payout, while roughly one-third of SPIA premiums were devoted to single life annuities. More than half of the single-life products included period certain provisions.

In this article, the authors describe the structure of joint life annuity products that are currently available and show that these products provide married couples with asset allocation options that are simply not achievable with single-life products. The authors then extend previous work on the gains from annuitization to consider the utility gains for married couples. They specify a household utility function and compute the increase in wealth that would be needed to compensate couples who initially had access to an actuarially fair annuity market for the closure of this market. The authors’ calculations recognize the presence of pre-existing annuity benefits such as Social Security, and they allow for possible dependencies in the mortality rates of husbands and wives. The latter is the so-called “broken heart” effect, the tendency for mortality rates of surviving spouses to be somewhat higher for several years after their spouse’s death than the mortality rates for similar individuals who have not lost a spouse.

The article is divided into five sections. The first describes the structure of joint annuity products and develops expressions for actuarially fair joint-life annuity products. The second section presents the authors’ algorithm for evaluating the utility gains that flow from participating in an actuarially fair joint annuity market. The third section discusses a range of difficult calibration issues that arise in modeling household rather than individual annuity valuation. The fourth section reports the authors’ basic results on “annuity equivalent wealth,” which measures the household utility gain from access to an actuarially fair joint annuity market. It also reports evidence on how the authors’ findings depend on assumptions about the correlation between mortality rates for husbands and wives and the degree of pre-existing annuitization. The fifth section concludes and suggests directions for further research.

JOINT AND SURVIVOR ANNUITY CONTRACTS

In studying the demand for joint life annuity contracts, it is important to recognize that a married couple is concerned with four distinct states of the world. The first

state is that in which both members of the couple are alive and income is used to support the consumption of both individuals. The three other states are those in which the wife is a widow, in which the husband is a widower, and in which both spouses are deceased. A couple making rational financial decisions will seek to allocate wealth optimally across these four states. A joint annuity contract allows a couple to make its income stream contingent on its survival state.

To explain further how joint life annuities allow a couple to provide for such state-contingent consumption, the authors now consider the two primary types of joint annuity contracts. The first is a joint life annuity with a *last survivor* payout rule. This rule specifies a periodic payment, typically monthly or quarterly, that the annuitants will receive provided both of them are still alive. In addition, it specifies a fraction of this payment, ϕ , that will be paid to the survivor after the death of one member of the couple. The fraction ϕ is usually set at one, two-thirds, or one-half, although LIMRA (1997) reports that insurance companies will provide virtually any fractional survivor benefit at the request of the annuity buyer. In the special case of $\phi = 1$, the annuity provides a level payout stream from the time it is purchased until the death of the second-to-die spouse; this is sometimes referred to simply as a "joint and survivor annuity."

To define an actuarially fair joint and survivor annuity with a last survivor provision, let A denote the fixed nominal benefit that is paid as long as both members of the couple are alive. Define q_t^m as the probability that the husband dies in period t , conditional on surviving to period $t - 1$. The authors use years as the period of analysis, define period 0 as the date of annuity purchase, and let T equal the number of years between the time of annuity purchase and the maximum length of life (assumed to be 115 years). The annuity payout A is the *annual* payout in this setting. Let S_t^m denote the probability that the husband survives for at least t years after purchasing the annuity. Equation (1) relates this cumulative survival probability S_t^m to the yearly mortality rate q_t^m :

$$S_t^m = \prod_{j=1}^t (1 - q_j^m). \tag{1}$$

Similarly, define S_t^f as the wife's t -year survival probability. If the authors assume that the mortality rates of the husband and wife are independent, an assumption that they discuss further and relax in what follows, then an actuarially fair premium (P) for a joint and survivor annuity contract satisfies:

$$P = \sum_{t=1}^T \left\{ \frac{A \cdot S_t^m \cdot S_t^f + \phi \cdot A \cdot (S_t^m (1 - S_t^f) + S_t^f (1 - S_t^m))}{(1 + i)^t} \right\}. \tag{2}$$

The authors use i to denote the nominal interest rate at which the insurance company discounts future payouts. In the case of a real annuity, the payout flows in the numerator of Equation (2) would be measured in constant dollars rather than nominal dollars, and the discount rate would need to be the real interest rate.

The second type of joint life annuity policy is known as a "joint life policy with a contingent survivor benefit." The key distinction between this type of policy and a last survivor policy is that a contingent benefit policy specifies one member of the couple as a primary annuitant. Provided the primary annuitant is alive, the annuity payout is A per period. If the primary annuitant predeceases the secondary annuitant, however, the payout declines to a fraction θ of the primary annuitant's payment. If $\theta = 1$, then the contingent survivor annuity is equivalent to a last survivor annuity with $\phi = 1$, but when $\theta < 1$, the policy differs from a last survivor policy with $\phi < 1$. The key distinction is that with a contingent survivor annuity, the *order* in which the two annuitants die matters for the time profile of benefits. The spouses are treated asymmetrically with regard to the survivor benefit.

The pricing equation for an actuarially fair annuity with a joint and contingent survivor benefit differs slightly from Equation (2) above. If the authors assume that the husband is the primary annuitant, and again maintain that the husband and wife have independent mortality rates, then an actuarially fair premium must satisfy:

$$P = \sum_{t=1}^T \left\{ \frac{A \cdot S_t^m + \theta \cdot A \cdot S_t^f (1 - S_t^m)}{(1+i)^t} \right\}. \quad (3)$$

Contingent payout annuities are likely to be most attractive to couples with clear ideas about their relative consumption needs. Because the authors do not have a solid basis for specifying such consumption needs, most of their analysis focuses on last survivor joint annuities.

Joint life annuities play a potentially important role in completing the market for life-contingent claims. Although a joint life annuity can be structured to perfectly replicate any combination of single life annuities by adjusting the survivorship ratios, the reverse is not true. For example, consider a couple for whom the optimal allocation of income across states is to have either surviving spouse receive half of the income that the couple has while both were alive. This is easily achieved through the use of single life annuities by dividing the wealth in such a way that the annuity income generated for each spouse is equal. Purchasing a "joint and 50 percent survivor" annuity can also achieve this.

Now consider a couple for whom the optimal policy is to have no drop in income associated with the death of the first-to-die spouse. A joint and full survivor annuity can generate this state-contingent income stream, but a portfolio of individual annuities cannot replicate this income flow. Any income that is contingent upon one life will, by definition, cease upon that individual's death. A couple desiring this income flow could buy two single life annuities and use some of the proceeds from each annuity to purchase life insurance. The survivor could then use the life insurance payout associated with the first-to-die spouse to purchase a larger annuity contract after the death of the first spouse. In the presence of transaction costs and actuarially unfair insurance markets, however, this would be an expensive way to replicate a joint life annuity.

Although joint annuities represent a small fraction of the single premium individual annuity market, they represent a substantial fraction of the annuities written in con-

junction with the group annuity policies that are associated with private defined benefit pension plans. This is partly a result of legislation. ERISA, enacted in 1974, includes "Joint-and-Survivor Annuity Requirements" that specify that pension plans must offer a default annuitization option that provides at least a "joint and 50 percent survivor" annuity. The expected present discounted value of this option must be equal to that of a single life, individual worker annuity. Holden (1997) reports that while 48.1 percent of married men with pensions initiated before ERISA chose a survivor benefit, 63.9 percent of married men initiating pensions after 1974 did so. King (1996) also reports an increase in the fraction of two-life annuities chosen by TIAA-CREF annuitants in the years following the passage of the 1984 Retirement Equity Act, which required a spouse's notarized signature when the survivor option is not selected. Before this amendment, the worker could select a single life annuity without the spouse's consent or notification.

The importance of pension annuities to the retired population can be illustrated with tabulations from the September 1994 Health and Pension Benefit Supplement to the Current Population Survey. In that survey, 17.4 million individuals over the age of 55 reported that they were retired from private-sector jobs, and 7.2 million (41.3 percent) reported that they were receiving annuity income from a private pension plan. The mean annuity payment for this group was \$9,714, and the total amount of annuity income was \$69.9 billion. A very substantial component of this income flow is probably paid as joint annuities.

Although previous studies have explored the value of annuitization from the standpoint of individual consumers, there are two reasons for expecting differences in annuity valuation by individuals and married couples. First, the joint and survivor mortality curve facing a married couple differs from that facing an individual. Because the mortality rates of two members of a married couple are not perfectly correlated, the probability that at least one member of the couple will be alive any number of years into the future is always higher than the probability that a single individual will be alive. A couple's joint life expectancy is longer than that of either individual in the couple.

Second, individuals and couples may have differences in the time paths of their respective consumption needs. The consumption needs of a couple may change in a predictable way when one member of the couple dies. Under well-known but restrictive conditions (Yagi and Nishigaki, 1993), an individual may find a level real consumption stream to be optimal. There is no guarantee that such a consumption path will be optimal for a married couple, so an annuity that offers a constant payout may be less attractive to a couple than to an individual. The authors now consider the utility gains associated with joint and survivor annuity products.

THE "ANNUITY EQUIVALENT WEALTH" FROM ACCESS TO ACTUARIALLY FAIR ANNUITIES

To estimate the value of being able to annuitize retirement resources, the authors consider the following thought experiment. Consider a couple with an initial stock of wealth that can be fully annuitized in an actuarially fair market for joint life annuities. Then take away access to these annuities and calculate the incremental amount

of wealth that the couple would need to achieve the same level of utility that it had before the annuity market was closed. The authors label the required wealth in the no-annuity setting the couple's "annuity equivalent wealth."

The Couple's Utility Maximization Problem Without Annuities

To calculate the amount of additional wealth that couples with access to annuity markets would need to stay on the same indifference curve if the annuity market were to close, the authors develop a model of optimal consumption by couples. The authors consider a setting in which a couple maximizes an intertemporally separable, expected lifetime utility function (L) of the form:

$$L = \sum_{t=1}^T \left(\frac{U_c(C_t^m, C_t^f) \cdot S_t^m \cdot S_t^f + U_m(C_t^m, 0) \cdot S_t^m \cdot (1 - S_t^f) + U_f(0, C_t^f) \cdot S_t^f \cdot (1 - S_t^m)}{(1 + \rho)^t} \right). \quad (4)$$

The assumption that the authors can express the household's joint optimization problem as the maximization of a "household" utility function is a convenient fiction that enables the authors to avoid many difficult issues about preference aggregation that are outlined in Samuelson (1956). The couple's lifetime budget constraint requires that the present value of all future consumption cannot exceed W_0 , the initial resource endowment:

$$W_0 \geq \sum_{t=1}^T \frac{C_t^m + C_t^f}{(1 + r)^t}. \quad (5)$$

In Equation (4), U_c represents the one-period utility function of the married couple when both are alive, and U_m and U_f represent the one-period utility functions of the surviving male or female spouses, respectively. C_t^m and C_t^f represent the consumption of each spouse during period t . The rate of time preference, or utility discount rate, is denoted by ρ . In Equation (5), r is the household's rate of return. Note that in the absence of an annuity market, a couple facing uncertain mortality and maximizing Equation (4) subject to Equation (5) would be likely to leave an unplanned bequest since, except in the final period, it would never be optimal to consume all remaining resources.

The authors assume that the utility of the married couple when both spouses are alive, U_c , depends on the consumption of the husband (C_t^m) and the wife (C_t^f). In particular, the authors assume that the household utility function is a weighted sum of the husband's and wife's subutility functions, U_m and U_f :

$$U_c(C_t^m, C_t^f) = U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m). \quad (6)$$

The parameter ϕ determines the relative weights of the husband's and wife's utility in the household utility aggregate. Kotlikoff and Spivak (1981) used a similar specification in their analysis of the gains from annuitization for married individuals. One could imagine using other utility functions to model household behavior, or allowing for within-household bargaining by husbands and wives. The authors will focus on the case of $\phi = 1$ and will assume that the functional forms for the person-specific

subutility functions are the same for husbands and wives. These conditions imply that $C_i^m = C_i^f$ for any period in which both members of the couple are alive. Considering other cases does not present any analytical difficulties, but the authors are not aware of data that would guide the calibration of such models.

The authors extend the specification used by Kotlikoff and Spivak (1981) to allow for complementarities in consumption, or "consumption externalities," between the two members of a couple. In particular, the authors allow the utility of the husband to depend on $C^m + \lambda C^f$ and make a symmetric assumption for the utility of the wife. This allows for the possibility that some goods (for example, newspapers) are "household public goods," so that a purchase by one member of the couple also permits the other member to consume this good. When $\lambda = 0$, there is no jointness in consumption and only the husband's own consumption enters his subutility function. When $\lambda = 1$, all consumption is joint, and the consumption needs of a surviving spouse are the same as those of the couple. Pollak (1976) offers a more elaborate discussion of interdependent preferences within married couples.

Joint consumption can also be modeled by modifying the household's budget constraint, rather than by writing each individual's utility as a function of consumption outlays by both members of the couple. In this alternative case, the authors would set $\lambda = 0$ in Equation (6) and define the couple's total consumption spending as $C = (C^m + C^f)/(1 + \sigma)$, where σ controls the degree of joint consumption. When the husband and wife have identical utility functions, as in the authors' analysis, and $\varphi = 1$, these two approaches are equivalent. Moreover, it is possible to demonstrate that $\sigma = \lambda$.

The degree of jointness (λ) affects the couple's utility level for a given stock of lifetime resources and therefore the value of access to actuarially fair annuity markets. In the special case of logarithmic utility with equal division of consumption within the couple, however, the value of λ does not matter. Since $C_i^m = C_i^f$ in this case, the utility of each member of the couple is determined by $(1 + \lambda) \cdot C_i^m$, and the term $(1 + \lambda)$ can be factored out of the objective function as a constant. The authors develop their analytical framework in the next section assuming that $\lambda = 0$, although in their numerical results they consider varying degrees of joint consumption as well.

To obtain numerical estimates of the value of annuitization, the authors must make assumptions about the functional form of the individual utility functions in Equations (4) and (6). The authors assume that the husband and wife have constant relative risk aversion (CRRA) subutility functions:

$$U_m(C_i^m, C_i^f) = \frac{(C_i^m + \lambda C_i^f)^{1-\beta}}{1-\beta} \quad (7a)$$

and

$$U_f(C_i^f, C_i^m) = \frac{(C_i^f + \lambda C_i^m)^{1-\beta}}{1-\beta} \quad (7b)$$

The parameter β is the coefficient of risk aversion. The authors assume that the risk aversion parameter β is the same for the husband and the wife, which implies that the wife's share of total household consumption will be $1/(1 + \varphi^{-1/\beta})$. The assumption that the husband and wife have the same subutility functions could be relaxed, although once again there is limited empirical work that can be used to calibrate the functions separately.

The authors assume that married couples try to maximize expected lifetime utility from Equation (4) subject to their lifetime budget constraint from Equation (5). When it is not possible to purchase annuities, couples will set their consumption by balancing the marginal utility associated with current period consumption against the expected utility of future period wealth. In some simple cases, it is possible to solve analytically for the couple's optimal consumption path. In some of the more complex cases the authors consider, such as those in which individuals cannot borrow against their pre-existing annuity wealth, there are no closed-form solutions for the couple's consumption path. The authors therefore resort to numerical solutions to the optimal consumption problem.

Stochastic dynamic programming is the standard method for solving dynamic consumption problems. Blanchard and Fisher (1989) explain that dynamic programming reduces multi-period problems to a sequence of two-period decision problems through the use of a "value function." In the authors' setting, a couple's value function at time t , $V(W_t)$, depends on its current wealth. This function describes the present discounted value of expected utility evaluated along the couple's optimal consumption path, conditional on the couple's financial wealth at the beginning of period t , W_t , the discount rate ρ , and the joint distribution of future mortality rates. The authors continue to use q_t^m and q_t^f to denote the one-period mortality rates for the husband and wife, respectively.

With probability $(1 - q_{t+1}^m)(1 - q_{t+1}^f)$ both members of the couple survive to consume in the next period. In this case, the value function at t equals the discounted value of the same value function, evaluated using wealth at $t + 1$. If one member of the couple does not survive until the next period, however, then the appropriate value function for the next period is the one-person value function for the surviving husband or the wife. The authors denote the value function at time t for the surviving husband as $M(W_t)$, which represents his present discounted value of expected utility, again evaluated along the optimal consumption path. $F(W_t)$ is defined similarly for the surviving wife. Hurd (1999) uses a similar approach in his analysis of optimal consumption behavior by married couples.

The value functions of the survivors satisfy a recursive relationship, Bellman's equation in dynamic programming, that equates the value function at time t to the utility of consumption at time t plus the expected value of the discounted value function at time $t + 1$. For a surviving husband and a surviving wife, these value functions satisfy:

$$M(W_t) = \text{Max}_{\{C_t^m\}} U(C_t^m) + \frac{(1 - q_{t+1}^m) \cdot M(W_{t+1})}{(1 + \rho)} \quad (8a)$$

and

$$F(W_t) = \text{Max}_{\{C_t^f\}} \phi U(C_t^f) + \frac{(1 - q_{t+1}^f) \cdot F(W_{t+1})}{(1 + \rho)}. \quad (8b)$$

In defining the value function in period t , the authors also need to consider the possibility, which develops with probability $(q_{t+1}^m)(q_{t+1}^f)$, that neither member of the household survives to the end of the next period. Since the authors assume that individuals do not have bequest motives, the couple derives no utility from any wealth in this state.

The Bellman equation that defines the couple's value function in this setting is:

$$V(W_t) = \text{Max}_{\{C_t^m, C_t^f\}} U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) + \frac{(1 - q_{t+1}^m)(1 - q_{t+1}^f)V(W_{t+1})}{1 + \rho} \\ + \frac{(1 - q_{t+1}^m)q_{t+1}^f M(W_{t+1})}{1 + \rho} + \frac{q_{t+1}^m(1 - q_{t+1}^f)F(W_{t+1})}{1 + \rho}. \quad (9)$$

When the couple does not have any pre-existing annuity income, its optimal consumption planning problem consists of maximizing Equation (9) subject to a non-negativity constraint on wealth in each period and a dynamic budget constraint. These constraints are:

$$W_t \geq 0 \text{ for all } t > 0 \quad (10a)$$

and

$$W_{t+1} = (W_t - C_t^m - C_t^f)(1 + r). \quad (10b)$$

Note that Equation (10b) is the dynamic version of the more familiar present discounted value budget constraint in Equation (5).

Equations (9), (10a), and (10b) define an optimization problem that can be solved for the optimal consumption path using the standard discretization methods that are described in Judd (1998). These techniques involve starting in the final period T and solving for the optimal consumption choice conditional on the amount of financial wealth brought to period T . Since the authors have assumed that individuals do not live beyond a fixed maximum age, and they do not allow bequest motives, they know that in the last period of life all wealth will be consumed. This determines $V(W_T)$, $M(W_T)$, and $F(W_T)$ for each value of W_T .

With this information, the authors can solve for the optimal consumption choices of a two-person couple, a surviving husband, and a surviving spouse in period $T-1$, given a stock of wealth at the beginning of that period. The authors solve for the optimal consumption rule at $T-1$ using the Bellman equation and the dynamic budget constraint. With the resulting solutions for $V(W_{T-1})$, $M(W_{T-1})$, and $F(W_{T-1})$, the authors can continue the recursion and solve for the value functions in period $T-2$. The authors

continue these recursions, numerically, until they find the value function at all ages, including the age at which they assume that an annuity would be purchased. This yields $V(W_0)$, the couple's present discounted value of utility from time 0 forward, assuming that it pursues an optimal consumption strategy. In the authors' base case analysis, they consider a couple consisting of a 65-year-old husband and a 62-year-old wife at the time of annuity purchase.

The Household Problem With Pre-Existing Annuities

When the household receives some income from a pre-existing annuity, such as Social Security or a pension plan payout, the budget constraint differs from the one specified above. Annuities provide an income component each period that offsets the wealth reduction associated with consumption outlays. This makes it important to specify three features of the annuity flow: the amount that the couple receives when both spouses are alive (A^c), the amount that a surviving husband would receive (A^m), and the amount that a surviving wife would receive (A^f).

With a pre-existing annuity, the modified dynamic budget constraint that is comparable to Equation (10b) above is:

$$W_{t+1} = (W_t + A_t - C_t^m - C_t^f)(1 + r). \tag{11}$$

In this expression, A_t denotes the survivor-contingent annuity flow that the household receives in period t , and it could equal A^c , A^m , or A^f . The presence of annuity income does not change the other aspect of the budget constraint, namely the non-negativity constraint on wealth holdings.

Pre-existing annuity income requires the authors to modify all of the value functions defined above to allow for the various annuity flows that the household may receive. These inflows affect the lifetime utility level of the household. The Bellman equation corresponding to Equation (9) now becomes:

$$\begin{aligned} V(W_t, A_t^c, A_t^m, A_t^f) = & \underset{\{C_t^m, C_t^f\}}{\text{Max}} U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) \\ & + \frac{(1 - q_{t+1}^m)(1 - q_{t+1}^f)V(W_{t+1}, A_{t+1}^c, A_{t+1}^m, A_{t+1}^f)}{1 + \rho} \\ & + \frac{(1 - q_{t+1}^m)q_{t+1}^f M(W_{t+1}, A_{t+1}^m)}{1 + \rho} + \frac{q_{t+1}^m(1 - q_{t+1}^f)F(W_{t+1}, A_{t+1}^f)}{1 + \rho}. \end{aligned} \tag{12}$$

Although this problem is more complex than the one without any pre-existing annuity wealth, it can be solved using the same stochastic dynamic programming techniques.

To parameterize the pre-existing annuities that a household may have access to, the authors focus on the ratio of the expected present discounted value (EPDV) of annuity income, relative to household wealth. This EPDV is given by:

$$EPDV = \sum_{t=1}^T \frac{A_t^c \cdot S_t^m \cdot S_t^f + A_t^f \cdot S_t^f \cdot (1 - S_t^m) + A_t^m \cdot S_t^m \cdot (1 - S_t^f)}{(1 + r)^t}. \tag{13}$$

The authors set *EPDV* equal to half of the household's wealth in their calculations below. They also attempt to allow for realistic profiles of the relative payouts to the couple and to either surviving spouse, corresponding roughly to current Social Security and private pension payouts.

The Household Problem With Access to Annuity Markets

The focus of the authors' analysis is the change in the expected present value of utility that occurs when a household with access to an actuarially fair annuity market loses that access. To calculate this change, the authors first find the value function for a couple that uses its wealth to purchase an annuity. In this case the household will potentially receive two annuity streams for as long as either member of the household is alive: one from its pre-existing annuity and the other from its privately purchased annuity. The authors denote the private market annuity flows with \bar{A} and apply survivor superscripts as appropriate for each period. The authors assume that when a couple purchases a private annuity, it uses all of its wealth to buy this product, so $W_0 = 0$. The couple may, nevertheless, choose to save some of its annuity income early in the retirement period, thereby accumulating positive wealth values in the years after annuitization.

The problem that the couple must now solve is given by:

$$\begin{aligned}
 V(W_t, A_t^c, A_t^m, A_t^f, \bar{A}_t^c, \bar{A}_t^m, \bar{A}_t^f) = & \text{Max}_{\{C_t^m, C_t^f\}} \left\{ U_m(C_t^m + \lambda C_t^f) + \phi U_f(C_t^f + \lambda C_t^m) \right\} \\
 & + \frac{(1 - q_{t+1}^m)(1 - q_{t+1}^f) \cdot V(W_{t+1}, A_{t+1}^c, A_{t+1}^m, A_{t+1}^f, \bar{A}_{t+1}^c, \bar{A}_{t+1}^m, \bar{A}_{t+1}^f)}{1 + \rho} \\
 & + \frac{(1 - q_{t+1}^m)q_{t+1}^f \cdot M(W_{t+1}, A_{t+1}^m, \bar{A}_t^m)}{1 + \rho} + \frac{q_{t+1}^m(1 - q_{t+1}^f) \cdot F(W_{t+1}, A_{t+1}^f, \bar{A}_t^f)}{1 + \rho}. \quad (14)
 \end{aligned}$$

The couple maximizes this value function subject to:

$$W_{t+1} = (W_t + A_t + \bar{A}_t - C_t^m - C_t^f)(1 + r). \quad (15)$$

Once again, the non-negativity constraints on wealth at all dates, Equation (10a), apply.

The authors assume that a couple reaches the date of annuitization with a wealth stock W_0^* . The feasible vector of private sector annuity payouts that the couple can purchase, $\{\bar{A}_t^c, \bar{A}_t^m, \bar{A}_t^f\}$, is determined by a zero-profit condition for insurers:

$$W_0^* = \sum_{t=1}^T \frac{\bar{A}_t^c \cdot S_t^m \cdot S_t^f + \bar{A}_t^f \cdot S_t^f \cdot (1 - S_t^m) + \bar{A}_t^m \cdot S_t^m \cdot (1 - S_t^f)}{(1 + i)^t}. \quad (16)$$

The authors consider annuities that pay a level nominal payout stream while any member of the couple is still alive, a level nominal payout stream with survivor payouts that are less than the payouts while both members of the couple are alive, and a level real payout stream.

Utility Comparisons and the "Annuity Equivalent Wealth"

To describe how much a couple values access to an actuarially fair annuity market like the hypothetical one that satisfies Equation (16), the authors first find the value of the couple's expected discounted lifetime utility when the husband is 65, the wife is 62, and the couple has fully annuitized its wealth. Allowing for the possibility of pre-existing annuities, this yields the value function $V(0, A_t^c, A_t^m, A_t^f, \bar{A}_t^c, \bar{A}_t^m, \bar{A}_t^f)$. The authors then compute the corresponding value function for the case in which the couple cannot purchase private annuities. This is $V(W_0^*, A_t^c, A_t^m, A_t^f)$. Since annuitization in an actuarially fair market provides access to insurance that was not otherwise available, it will typically raise the couple's utility level, so

$$V(0, A_t^c, A_t^m, A_t^f, \bar{A}_t^c, \bar{A}_t^m, \bar{A}_t^f) > V(W_0^*, A_t^c, A_t^m, A_t^f). \quad (17)$$

If the structure of payouts within the private annuity is very unattractive relative to the household's prospective consumption needs, this inequality might be reversed, but in most plausible cases it should hold.

The authors measure the couple's benefit from access to the annuity market by asking how much more wealth it would need to have in the no-access case to obtain the same expected utility that it could obtain with its original wealth and annuity market access. This is just the multiple α defined implicitly by:

$$V(0, A_t^c, A_t^m, A_t^f, \bar{A}_t^c, \bar{A}_t^m, \bar{A}_t^f) = V(\alpha W_0^*, A_t^c, A_t^m, A_t^f). \quad (18)$$

The value α represents the ratio of the amount of wealth that is required to make a couple as well-off without annuities as it was with annuities to its initial wealth in the case when it could purchase annuities. The authors refer to α as the couple's "annuity equivalent wealth." It is the fractional amount of nonannuitized wealth that yields the same lifetime expected utility for the couple as initial annuitized wealth W_0^* .

This annuity equivalent wealth measure is identical to that used in Brown, Mitchell, and Poterba's (2001) study of how individuals value real and nominal annuities. It differs slightly, however, from the "wealth equivalence" measure that MPWB (1999) use to describe annuity valuation. The tables in MPWB (1999) report the amount of wealth that individuals would be prepared to give up in order to invest their remaining wealth in actuarially fair annuities. In MPWB (1999), the central focus is on the divergence between the expected present discounted value of annuity payouts and the purchase price of annuity contracts. Because this present value is less than the annuity's purchase price, the natural question to ask is what wealth fraction an individual or a couple would rationally forgo in order to obtain an annuity.

In the absence of any pre-existing annuity wealth, the "annuity equivalent wealth" measure used here is the reciprocal of the "wealth equivalence" measure. A wealth equivalence value of 0.67 corresponds to an annuity equivalent wealth of 1.5. When individuals and couples have pre-existing annuity income, however, the reciprocal relationship no longer holds exactly, although it still provides some guidance for comparing results from these two approaches. The authors focus on the annuity equivalent wealth in the current article primarily because of computational concerns.

Although the foregoing discussion focuses on the case of actuarially fair annuity products, which the authors define as products with no load for administrative or other expenses, the analysis generalizes easily to the case of annuity markets in which insurers charge a load. The annuity equivalent wealth measure and the wealth equivalence measure provide important insights on the extent to which load factors on actual annuity products discourage households from purchasing these products. The wealth equivalence measure, the reciprocal of the annuity equivalent wealth, is calculated in the same units as the load factor on an annuity policy. Thus, when the annuity equivalent wealth is 1.25, which translates to a wealth equivalence of 0.80 ($= 1/1.25$), a couple will raise its utility level by purchasing an annuity, provided the load factor on the annuity is less than 20 percent. Although the authors do not report explicit calculations for the annuity equivalent wealth of annuities with load factors, the translation of their results to the case of annuities with loads is straightforward.

CALIBRATING THE OPTIMAL CONSUMPTION ALGORITHM

To evaluate the annuity equivalent wealth from access to a joint annuity market, the authors need to specify parameters for the various functions in the stochastic dynamic programming algorithm described above. There are two key sets of parameters, mortality rates and risk aversion coefficients, for which the authors can rely on a substantial prior literature in selecting parameter values. For other parameters, such as the degree of jointness in household consumption, previous research provides much less guidance and the authors rely on sensitivity analysis in reporting their results.

Mortality Rates and the "Broken Heart" Effect

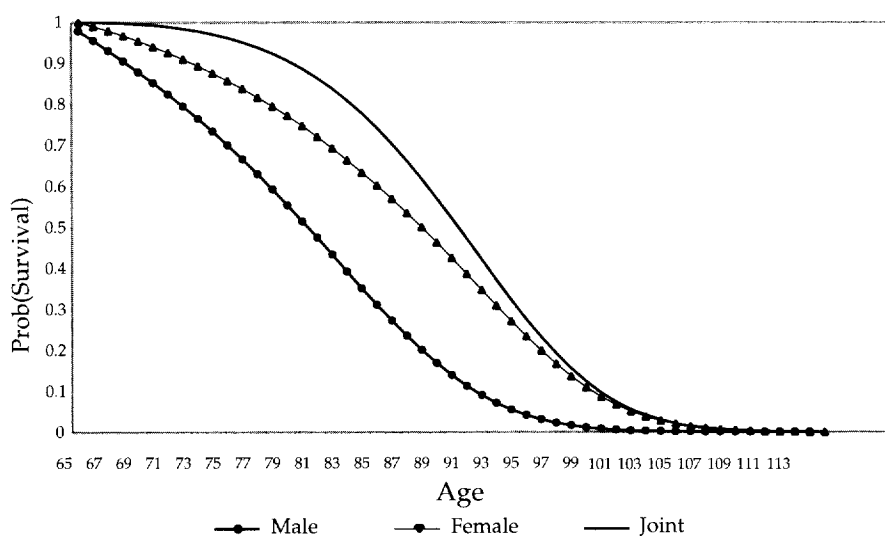
Mortality rates for potential annuitants are a central component of the annuity equivalent wealth calculation. Two mortality tables could be used to value annuity payouts. The first is the population life table, which is compiled by the Social Security Administration Office of the Actuary and corresponds to the population in general. The second is an "annuitant" mortality table, which more accurately captures the mortality experience of individuals who have historically purchased annuity contracts. Mortality rates are systematically lower in annuitant tables than in population tables, as MPWB (1999) demonstrate. In either case, it is essential to use a cohort mortality table, i.e., a mortality table that describes the mortality experience at different ages of individuals who were born in a given year.

Because the authors are primarily interested in the annuity equivalent wealth for representative couples, they use the population cohort mortality tables in the calculations reported below. If the authors chose instead to use annuitant mortality data both in computing actuarially fair annuity payouts and in calculating the couple's lifetime expected utility, the couple's annuity equivalent wealth would be lower than the calculations suggest. This happens because the mortality premium, which arises from the insurance company's ability to offer a rate of return above the market interest rate because not all annuity owners will survive to collect their payments, will be smaller if annuities are priced according to the annuitant life tables.

A key difference between the decision of whether to annuitize for a single person, and for a couple, is that the joint-and-survivor mortality table differs from that for an

individual. Figure 1 illustrates this point. It shows the probabilities that a 65-year-old man, a 65-year-old woman, and either member of a couple composed of two 65-year-olds will survive to various ages. The survivor curve for the couple lies above the individual survival curves, indicating that the probability of at least one member of the couple surviving to a given age is larger than that of a single individual. For example, the probability of at least one member of a 65-year-old couple living to age 80 is .86, compared with .54 for a single man. Life expectancies at age 65 are 15.7 years for the male, 19.4 years for the female, and 23.1 years for the couple.

FIGURE 1
Individual and Joint Survival



The joint survivorship curve in Figure 1 lies above the individual survivor curves, and it also has a different shape. There is less probability mass in the tails of the joint life distribution than in the individual distributions. The standard deviation of the life expectancy of a couple is only 7.8 years, which is lower than the standard deviation of 9 years for the male and 9.5 years for the female. This is potentially quite important because the value of an annuity rises with uncertainty about the number of years of remaining life. A lower standard deviation is not a sufficient condition for the joint survival curve to be "less risky" than the individual one in the sense of Rothschild and Stiglitz (1970). However, the authors' numerical results will show that individuals value annuitization more than couples, as the comparison of the standard deviations suggests might be the case.

Up to this point, the authors have assumed that the mortality risks for husbands and wives are independent. Frees, Carriere, and Valdez (1996) document a "broken heart" effect in the mortality of married couples: conditional on the death of one spouse, the mortality risk of the surviving spouse rises. Whether this is the result of correlated environmental risks for married couples, or of physical changes that occur in the

surviving spouse as a result of the first-to-die's death, is not clear. However, this dependence in mortality rates reduces the expected discounted value of joint and survivor annuity payouts. For a married couple composed of two 65-year-olds, Frees, Carriere, and Valdez (1996) report that dependent mortality reduces the annuity value of a joint-and-survivor annuity to 0.96 times its value assuming independent mortality.

The authors' base case results assume independent mortality. The authors also explore the sensitivity of their results to dependent mortality by using Social Security Administration data on the relative mortality rates of married and widowed individuals. The authors find only modest "broken heart" effects on the annuity equivalent wealth measure.

Risk Aversion

The coefficient of relative risk aversion is the second important parameter input to the authors' stochastic dynamic programming model. It determines the shape of the subutility functions for husbands and wives. A substantial literature, summarized in Laibson, Repetto, and Tobacman (1998), has found levels of risk aversion near unity. Mehra and Prescott (1985) and others have noted, however, that much higher levels of risk aversion are needed to explain the large historical return premium of U.S. equities over riskless bonds. In addition, survey work by Barsky et al. (1997) suggests that household risk aversion levels are higher than unity. In light of this research, the authors consider relative risk aversion values of 1, 2, 5, and 10. Hurd (1989) shows that very risk-averse consumers want to guard against having to consume at a low level if they live long past their life expectancy, so annuities are more valuable for highly risk-averse couples or individuals.

ANNUITY EQUIVALENT WEALTH RESULTS

To illustrate their findings regarding the value of annuities for married couples, the authors begin with a base case in which the couple consists of a 65-year-old man with a 62-year-old wife. The authors assume that there are no consumption complementarities between the spouses ($\lambda = 0$) and that mortality rates are independent. The three-year difference in the age of the husband and wife is roughly representative of current patterns in the United States. The authors' base case assumes that the couple is purchasing a nominal annuity, the real value of which declines by 3.2 percent annually due to inflation. The real interest rate, r , and the utility discount rate, ρ , are both set equal to 3 percent per year. The nominal interest rate is thus 6.2 percent per year, and the authors assume that this is the interest rate at which insurance companies discount their future annuity payout liabilities.

Table 1 presents annuity equivalent wealth results for the base case couple. The top three panels represent cases in which the couple is purchasing a nominal annuity. The bottom panel reports results for the case of a level real annuity. The columns of the table represent alternative assumptions about the ratio of survivor annuity income to the couple's annuity income, which is the parameter ϕ in Equation (2) above. The authors consider the three most common survivor benefit ratios: one-half, two-thirds, and one.

TABLE 1
Annuity Equivalent Wealth for Married Couples
65-Year-Old Husband, 62-Year-Old Wife
(No Consumption Complementarities)

	Survivor Ratio for Joint Life Annuity			65-Year-Old Single Male's Gain From Single Life Annuity
	0.5	0.67	1.0	
NOMINAL ANNUITIES				
No Pre-Existing Annuities				
CRRA = 1	1.175	1.179	1.158	1.465
CRRA = 2	1.244	1.247	1.205	1.576
CRRA = 5	1.339	1.340	1.250	1.679
CRRA = 10	1.407	1.402	1.266	1.703
Pre-Existing Annuity Worth Half of Wealth, Survivor Ratio = 0.5				
CRRA = 1	1.107	1.110	1.099	1.311
CRRA = 2	1.153	1.159	1.138	1.405
CRRA = 5	1.229	1.237	1.189	1.530
CRRA = 10	1.289	1.297	1.223	1.625
Pre-Existing Annuity Worth Half of Wealth, Survivor Ratio = 0.67				
CRRA = 1	1.111	1.106	1.082	1.311
CRRA = 2	1.164	1.153	1.109	1.405
CRRA = 5	1.279	1.252	1.154	1.530
CRRA = 10	1.368	1.301	1.170	1.625
REAL ANNUITIES				
No Pre-Existing Annuities				
CRRA = 1	1.202	1.192	1.143	1.502
CRRA = 2	1.295	1.275	1.181	1.650
CRRA = 5	1.446	1.401	1.243	1.855
CRRA = 10	1.600	1.524	1.319	2.004

Notes: CRRA is the coefficient of relative risk aversion, denoted in the text as parameter β . Further details of the calculations are presented in the text.

The results in the first panel of Table 1 suggest that a married couple with log utility would raise its household utility level if it could purchase a joint and 100 percent survivor annuity with a load factor of less than 13.6 percent ($1/1.158 = 0.864$, which translates to a load of 13.6 percent). This is in the range of actual loads on annuity products that are currently available. MPWB (1999) calculate that in 1995, the load factor on the average joint and 100 percent survivor annuity, using population mor-

tality rates and the Treasury yield curve, averaged 13.2 percent. This indicates that the load factor on these annuities approximately cancels out the utility gain for a couple with log utility. More generally, the load factors on joint and survivor annuities calculated in MPWB (1999) ranged between 7 and 21 percent. Load factors of 7 percent arise when the valuation analysis uses the annuitant mortality table and the yields on riskless Treasury bonds to discount future annuity payouts. Values of 21 percent arise when the analysis uses the population mortality table and the higher yields associated with risky corporate bonds for discounting. A comparison of these load factors to the utility gains reported in Table 1 suggests that some couples, particularly those with low levels of risk aversion or a significant fraction of their wealth pre-annuitized, would not find the utility gains from annuitization large enough to offset the load factors that are present in the private annuity market. This finding may help partially explain the limited demand for annuities in the U.S.

Whether an individual has any pre-existing annuity wealth is an important influence on the value of annuitization in earlier studies, such as MPWB (1999). When considering pre-existing annuity wealth for couples, the authors must not only specify the amount of pre-existing annuity income that the couple receives but must also describe how the income from any pre-existing annuity changes upon the death of one spouse. The authors consider three different scenarios. The top panel of Table 1 corresponds to the case in which the couple has no pre-existing annuity wealth. The second panel assumes the couple begins with half its wealth in a pre-existing real annuity that pays a survivor benefit equal to one-half of the couple benefit. This is a stylized representation of a situation in which half of the couple's wealth consists of benefit promises from a system like the current U.S. Social Security system, with the surviving spouse receiving only his or her own worker benefits. The third panel again assumes that half of wealth is pre-annuitized but assumes that the pre-existing annuity offers a survivor payout equal to two-thirds of the couple's benefit. This corresponds to a stylized case in which the couple's Social Security benefit consists of a primary worker benefit plus an additional 50 percent in dependent benefits. Each panel in Table 1 shows results for four different values of the relative risk aversion coefficient.

The results in the first panel of Table 1 illustrate the authors' findings. When actuarially fair private joint-and-survivor annuities offer a 50 percent survivor's benefit, the base case couple with a log utility function would need a 17.5 percent increase in its nonannuitized wealth to be as well-off without access to an annuity market as with it. Increasing risk aversion from 1, the log utility case, to 2 raises the required wealth increment to 24.4 percent, and with a risk aversion coefficient of 10, the value is 40.7 percent.

The last column of Table 1 presents annuity equivalent wealth calculations for a single man at age 65, following the procedure outlined in Brown, Mitchell, and Poterba (2001). For all levels of risk aversion, the annuity equivalent wealth for couples is significantly lower than that for a single man. Similar comparisons using other parameter values generate similar findings. The difference between individual annuity valuation and annuity valuation by a married couple is partly explained by the fact that risk sharing takes place within couples. If one member of the couple lives an unexpectedly long life, there is some probability that he or she will inherit the re-

maining resources of an earlier-to-die spouse. This provides some mortality insurance, even without a formal annuity contract.

The second and third columns of the top panel in Table 1 show annuity equivalent wealth for survivor ratios of two-thirds and one, rather than one-half as in the first column. All of the annuities that the authors consider provide a constant nominal payout stream, so different survivor payout structures partly affect the degree to which real benefits decline over time. In this case of no consumption complementarities, the survivor ratio of 0.67 provides the highest annuity valuation for most levels of risk aversion, although 0.5 generates a higher annuity equivalent wealth when the risk aversion coefficient is 10. Couples prefer partial rather than complete survivor benefits for all of the parameter values that the authors consider; this is indicated by greater annuity equivalent wealth values for the incomplete survivor benefit cases. This is true because upon the death of the first spouse, the income required to provide each individual with a given level of consumption declines. Providing full survivor benefits places too much income in the state of widowhood, or widowerhood, while lowering annuity payouts and hence consumption in the states of nature in which both spouses are alive.

The second and third panels of Table 1 show how the authors' results change when the couple has a pre-existing annuitized income stream. The authors think of this as Social Security or the benefits from a defined contribution pension plan. The authors assume that the pre-existing annuity benefit is indexed to inflation. Comparing the results to the top panel, the authors see that the couple's annuity equivalent wealth declines when half of its wealth is already annuitized. The required wealth increment that the authors report when the household has some pre-existing annuity income is the fraction of *non-annuitized wealth* that the couple would need in order to be as well off without access to an add-on private annuity market as will full annuitization in such a market. For all levels of risk aversion, and for all combinations of survivor benefits, the annuity equivalent wealth is lower when the household has some pre-annuitized wealth than when it has none. The ratio of the incremental wealth required with pre-annuitized wealth to that without any prior annuity is typically between one-half and three-quarters. Pre-existing annuities reduce the demand for additional annuities because they provide a lower bound on the consumption flow that the couple would receive even if it substantially outlived its life expectancies.

The bottom panel of Table 1 reports results for the base case couple when the private annuity market offers real rather than nominal annuities. A couple's valuation of a real annuity may differ from that of a nominal annuity for two reasons. First, inflation causes the real income stream provided by the fixed nominal annuity to erode over time. Using an inflation rate of 3.2 percent, which is consistent with the mean inflation rate in the U.S. over the period since 1926, the real value of a fixed nominal payment is halved in 22 years. Second, as Brown, Mitchell, and Poterba (2001) emphasize, inflation uncertainty can lead to substantial fluctuations from year to year in the real value of the annuity payment. The authors' comparison of real versus nominal annuities in Table 1 only captures the first of these two effects, since they assume a fixed inflation rate. Incorporating a stochastic inflation process into their dynamic programming algorithm for couples' consumption plans would involve substantial computation burdens, so the authors leave that challenge for future work. The results

reported in Table 1 will understate the difference in the annuity equivalent wealth from real and nominal annuities because they do not capture this second effect.

The results in the bottom panel of Table 1 suggest that couples value real annuities more highly than nominal annuities for values of the survivor ratio near one-half or two-thirds. When the private annuity market offers joint and survivor annuities with a survivor benefit ratio of 50 percent, the annuity equivalent wealth for a couple with log utility is 1.202, up from 1.175 in the case of a nominal annuity. For higher levels of risk aversion, the divergence in the value of real and nominal annuities grows even greater. For risk aversion of 10, the annuity equivalent wealth is 1.600, versus only 1.407 for a nominal annuity.

The results in the third column of the last panel in Table 1 show that for some values of the survivor ratio, the couple may prefer a nominal to a real annuity. This occurs because the real annuity with a full survivor benefit allocates too much of the couple's wealth to states of nature in which only one spouse is alive and too little to states in which both are alive. In the case of a nominal annuity with a fixed inflation rate, the declining value of the annuity over time reduces the extent of this misallocation.

Table 2 explores the sensitivity of the authors' results to alternative assumptions about the degree of "jointness" in consumption. This table continues the analysis of the base case couple, but it now allows the parameter λ to vary. The first column again reports results for the case of no jointness, or $\lambda = 0$. These results correspond to those in the top panel of Table 1. The second column assumes that half of all consumption is joint ($\lambda = 0.5$) and the third column assumes complete jointness in consumption ($\lambda = 1$). The authors do not report results in Table 2 for the risk aversion coefficient of unity since, as noted above, λ does not affect the annuity equivalent wealth in this case. The authors restrict Table 2 to the case of no pre-existing annuities in order to focus on the effect of the consumption jointness parameter, and they consider survivor ratios for the privately purchased annuity of one-half, two-thirds, and unity.

The jointness parameter clearly affects the desired survivor ratio, although it does not have a material effect on the overall level of annuity equivalent wealth. Without consumption externalities, the couple finds a private annuity with a survivor benefit ratio of 0.5 or 0.67 more attractive than an annuity with a full survivor benefit. This is true for all of the risk aversion levels that the authors consider. The preferred survivor ratio rises with the jointness parameter, and when the authors allow for full consumption jointness ($\lambda = 1$), the couple prefers complete survivor benefits to either of the partial alternatives that the authors consider. Full consumption jointness means that a surviving spouse needs just as much income to maintain her utility level as a widow as she and her husband needed when both were alive. With full jointness, two can consume for the price of one. Therefore, the death of a spouse does not reduce a household's expenses, and this explains the attraction of more generous survivor benefits.

Table 3 explores the sensitivity of our annuity equivalent wealth calculations to the ages of the husband and the wife in the annuitizing couple. The table presents results for two values of relative risk aversion, 1 and 5, for the case with no consumption jointness and when the private annuity has a survivor ratio of 0.67. The authors present results both for the case of no pre-existing annuity wealth and for the case in which half of wealth is pre-annuitized with a survivor ratio of one-half.

TABLE 2

The Effect of Consumption Jointness on Annuity Equivalent Wealth for Married Couples No Pre-Existing Annuities, Various Survivor Rates for Joint Annuity and Risk Aversion Coefficients

Survivor Ratio for Joint Annuity	Degree of Jointness in Consumption		
	$\lambda = 0$	$\lambda = .5$	$\lambda = 1$
	CRRA = 2		
.5	1.244	1.213	1.194
.67	1.247	1.229	1.221
1.0	1.205	1.217	1.230
	CRRA = 5		
.5	1.339	1.216	1.142
.67	1.340	1.270	1.223
1.0	1.250	1.264	1.272
	CRRA = 10		
.5	1.407	1.175	1.021
.67	1.402	1.285	1.185
1.0	1.266	1.280	1.278

Notes: Each entry shows the annuity equivalent wealth for a couple purchasing a joint annuity with the survivor ratio indicated in the row label. Entries with no jointness correspond to those in Table 1. See text for further description.

The results in Table 3 suggest that annuity equivalent wealth is an increasing function of the ages of both spouses in a couple. For two 70-year-olds with relative risk aversion coefficients of unity, the annuity equivalent wealth value is 24.2 percent of initial wealth. For two 55-year-olds, the analogous value is only 11.7 percent. Because older individuals face higher mortality probabilities than younger persons, the mortality premium that insurers offer to pay on annuity contracts written for older persons exceeds that on annuity contracts for younger buyers. The older annuity buyers therefore have more to gain from annuitization than do younger couples.

The last issue that the authors consider is the sensitivity of their findings to the possibility that married couples exhibit dependent mortality. To calibrate a mortality table allowing for dependent mortality, the authors adapted Social Security Administration data on mortality rates of married individuals and widows or widowers at various ages. Table 4 shows the basic data used for this calibration.

The first and fourth columns in the table show population cohort mortality rates at various ages for an average man who was 65 in 1999 and for an average woman who was 62 in 1999. These are the mortality rates that the authors used to develop the estimates of annuity-equivalent wealth in tables 1 through 3. The remaining columns of Table 4 report mortality rates for individuals in the same birth cohorts as those in columns one and four, but they disaggregate mortality rates by whether an individual is currently married, is widowed, or is a widower. These mortality rates were calculated using marital status mortality adjustment factors supplied by the Social Security

Administration's Office of the Chief Actuary. The adjustment factors are based on the ratio of the mortality rate for each marital group, to that of all individuals, for each gender, from 1980 through 1981. That was the last year for which the Social Security Administration differentiated its mortality rates by marital status. The authors apply the marital status mortality factors to underlying mortality rates for more recent cohorts.

TABLE 3
Effect of Age Differentials on a Couple's Annuity Equivalent Wealth
(No Consumption Complementarities)

CRRRA = 1				
Privately Purchased Annuity With Survivor Ratio = 0.67	Age of Husband			
	55	60	65	70
No Pre-Existing Annuities				
Wife's Age:				
55	1.117	1.131	1.146	1.160
60	1.133	1.151	1.170	1.188
65	1.150	1.171	1.194	1.215
70	1.168	1.193	1.217	1.242
Half of Wealth Pre-Annuitized With Survivor Ratio = 0.5				
Wife's Age:				
55	1.070	1.082	1.094	1.103
60	1.082	1.095	1.105	1.118
65	1.094	1.104	1.118	1.132
70	1.103	1.117	1.131	1.146
CRRRA = 5				
Privately Purchased Annuity With Survivor Ratio = 0.67	Age of Husband			
	55	60	65	70
No Pre-Existing Annuities				
Wife's Age:				
55	1.216	1.243	1.261	1.283
60	1.253	1.281	1.320	1.339
65	1.286	1.334	1.366	1.411
70	1.322	1.374	1.433	1.475
Half of Wealth Pre-Annuitized With Survivor Ratio = 0.5				
Wife's Age:				
55	1.161	1.178	1.184	1.193
60	1.181	1.193	1.216	1.252
65	1.193	1.221	1.265	1.291
70	1.215	1.265	1.294	1.326

Note: Each entry shows the annuity equivalent wealth for a couple purchasing a joint annuity with the survivor ratio indicated in the row label. See text for further description.

TABLE 4
Mortality Rates by Marital Status at Selected Ages
Social Security Administration Data

Man's Age	MEN			WOMEN		
	All	Married	Widowed	All	Married	Widowed
65	.021	.019	.030	.010	.009	.012
70	.031	.028	.043	.016	.014	.017
75	.047	.043	.059	.023	.020	.025
80	.074	.065	.096	.033	.029	.034
85	.113	.099	.132	.051	.042	.053
90	.168	.151	.178	.084	.053	.088
95	.237	.222	.230	.141	.106	.147

Note: Each entry shows the one-year mortality rate for men or women by marital status. In the case of men, the "all" column shows the overall male mortality rates from the Social Security Administration's 1934 birth cohort table for men turning age 65 in 1999. The entry in the same row for women corresponds to women who might be married to 65-year-old men. The authors assume that such women are three years younger than their husbands, so the "All" female mortality rates are for women turning age 62 in 1999. To obtain marital-status-specific mortality rates, the authors multiply the sex-specific "All" mortality rates by the sex-specific ratio of (mortality rate for that marital group/mortality rate for "all") in 1980-81. The 1980-81 data were provided by the Social Security Administration. See text for further details.

The data in Table 4 suggest significant differences between mortality rates for married persons and widows or widowers. At age 65, for example, the mortality rate for a widower is more than 50 percent higher than the mortality rate for a married man. For women, the differences are more modest, on the order of 25 percent. The effect persists at all ages, and there is generally a larger difference between the married and widower mortality rates than between the married and widow mortality rates.

To implement a "broken heart" model, the authors assume that both members of a couple have mortality rates of married individuals as long as both are alive, but that when one spouse dies, the surviving spouse's mortality rates switch to the significantly higher mortality rates of a widow or widower. The authors assume that all pre-existing annuities and privately purchased annuities are priced to take dependent mortality into account. (The authors find that the present discounted value of annuity payouts is roughly 2 percent lower when the authors assume dependent mortality, calibrated as they have done here, than when they assume independent mortality.) In addition, the authors change the mortality rates in their value functions to recognize that mortality changes when one member of the couple dies.

Table 5 reports the results for the base case couple assuming dependent mortality. The overall effect of allowing for a "broken heart" effect is small. By comparing tables 1 and 5, one can see that the annuity equivalent wealth measure is generally higher with dependent mortality, but it is not much higher than the value with independent mortality risk. For example, with a relative risk aversion coefficient of unity, no pre-

existing annuities, and a private annuity survivor ratio of one-half, annuity equivalent wealth rises from 1.175 to 1.180 when the authors allow for dependent mortality. The wealth equivalent results using dependent mortality are typically within 1 or 2 percent of those derived under the assumption of independent mortality.

TABLE 5
Annuity Equivalent Wealth for Married Couples Allowing for Dependent Mortality
65-Year-Old Husband, 62-Year-Old Wife
No Consumption Complementarities

	Survivor Ratio for the Annuity		
	0.5	0.67	1.0
NOMINAL ANNUITIES			
No Pre-Existing Annuities			
CRRA = 1	1.180	1.183	1.162
CRRA = 2	1.250	1.252	1.209
CRRA = 5	1.347	1.344	1.253
CRRA = 10	1.410	1.398	1.270
Pre-Existing Annuity Worth Half of Wealth, Survivor Ratio = 0.5			
CRRA = 1	1.109	1.112	1.102
CRRA = 2	1.158	1.163	1.143
CRRA = 5	1.233	1.238	1.193
CRRA = 10	1.309	1.312	1.247
Pre-Existing Annuity Worth Half of Wealth, Survivor Ratio = 0.67			
CRRA = 1	1.113	1.108	1.086
CRRA = 2	1.169	1.159	1.117
CRRA = 5	1.284	1.257	1.161
CRRA = 10	1.358	1.302	1.194
REAL ANNUITIES			
No Pre-Existing Annuities			
CRRA = 1	1.207	1.197	1.151
CRRA = 2	1.303	1.284	1.194
CRRA = 5	1.458	1.416	1.264
CRRA = 10	1.606	1.536	1.344

Notes: Calculations correspond to those in Table 1 but assume that mortality rates of one spouse depend on the survival status of the other spouse. The mortality rate for the surviving spouse rises after the death of the first spouse. CRRA is the coefficient of relative risk aversion, denoted in the text as parameter β . Further details of the calculations are presented in the text.

CONCLUSIONS AND FUTURE DIRECTIONS

The authors have presented new evidence on the value that married couples attach to access to an actuarially fair joint-and-survivor annuity market. A couple consisting of a 65-year-old man and a 62-year-old woman who had access to such a market would require between 18 and 30 percent more wealth in order to achieve the same utility level without access to such a market. The required wealth increment would be smaller if the couple had a substantial stock of pre-annuitized resources, and it would be greater if the couple were older than the authors' base case example. The authors' findings are not very sensitive to the treatment of the "broken heart" effect, i.e., the correlation of mortality rates within couples.

MPWB (1999) report that for the U.S. annuity market in the mid-1990s, the expected present discounted value of joint-and-survivor annuity payouts for a 65-year-old couple was about 84 percent of the annuity's initial premium. Thus it is possible that one of the reasons couples do not purchase private annuities is that the effective wealth reduction they would face when they purchased such annuities would be too large to justify purchasing these products.

The authors' results raise a number of issues that warrant further study. One is the effect of bequest motives on the demand for joint-and-survivor annuities. To the extent that couples value wealth that is left to their heirs, annuities will be less valuable than the authors' calculations suggest. Modeling bequest motives would make it possible to analyze the demand for joint-and-survivor annuities that contain "period certain" or "refund" options. Josten (1998) discusses the theoretical issues involved in modeling the utility of gifts and bequests for a life-cycle consumer. Yet the empirical evidence that can be brought to bear on calibrating the utility of bequests function often suggests very small value to bequests. Hurd (1987) estimated some of the relevant parameters, but he found that the marginal utility of bequests was statistically indistinguishable from zero. Brown (2001a, 2001b) has also provided evidence that bequest motives do not appear to affect marginal annuitization decisions. Further work calibrating life-cycle models with bequest motives is clearly needed, and this work could inform the current problem of adding a desire for bequests to the stochastic life-cycle model.

A second issue for further study concerns the nature of the utility functions for men and women in married couples. This is another important issue about which empirical work offers relatively little guidance. There is some evidence, for example from asset allocation patterns in defined contribution plans, that women are more conservative investors than men. If this reflects higher risk aversion, then it may be appropriate to modify the assumption that men and women have the same preferences, and hence subutility functions, in the analysis. It is also not clear how to estimate the degree of consumption jointness within married couples, or the relative weight that each member of the married couple places on the utility of his or her spouse.

A final issue that is not explored in this article is the potentially important role of medical expense uncertainty, or the need for precautionary saving more generally, on annuity valuation. If couples are concerned about the risk of future uninsured medical expenses, or about the cost of entering a nursing home, they may be less likely to annuitize their resources. The link between expenditure shocks and annuity demand is a subject of ongoing research.

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