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Time Dependent Rules, Aggregate Stickiness And Information Externalities

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ABSTRACT

Most of the recent studies of price rigidities using *time* dependent models either impose such a rule or –if optimal– complement it with some sort of extraneous price rigidity. This paper concentrates on the characteristics and implications of time dependent rules when derived from first principles. The source of such rules is some cost of gathering information about the state. In this context, if information externalities are present *sampling* is uniformly staggered. Prices, however, change every time new information arrives, therefore need not be staggered. In spite of this, when this mechanism is combined with signal extraction problems, nominal shocks have more persistent –although smaller– effects than in economies with no information externalities*.

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1. INTRODUCTION

Staggering and slow nominal price (and/or wages) adjustment are central ingredients to most Keynesian macroeconomic models[1]. Perhaps as old as the price "stickiness" hypothesis is skepticism about the rationale for the sources of nominal rigidities (e.g. wage contracts).

One theoretical justification of price stickiness is the presence of small costs of changing prices in the context of an optimizing monopolistic competitive firm (e.g. Mankiw 1985, Blanchard and Kiyotaki 1987). However the dynamic analysis of such a framework leads to what is known as *state* dependent rules[2] (e.g. Barro 1972, Sheshinski and Weiss 1983) as opposed to the *time* dependent rules[3] envisioned in Keynesian models. Even though staggering is the most natural outcome of state dependent rules (Caballero and Engel 1989), the effects of aggregate demand policy seem to be dramatically different from those of the time dependent-staggered models (Caplin and Spulber 1985, Caballero and Engel op. cit.). In the limit, when prices are perfectly staggered (in the state) and rules are state dependent, microeconomic price rigidity does not necessarily render aggregate price rigidity. When contrasted with the data this is an apparent shortcoming of menu cost-state dependent models. Although more realistic at the microeconomic level they do not seem to fully reproduce, alone, the slow adjustment observed in the aggregate data[4].

Recent models have revived interest in time dependent rules. Ball (1987) studies the externalities arising from a change in the contract length (i.e. a change in the time dependent rule). He shows that in the decentralized equilibrium wages are too rigid. Ball's model is one in which the source of contracts (fixed for a period of time) is a fixed cost of contract negotiations. However in this case the optimal rule is *state* not time dependent, therefore his insightful results can only be seen as partial rationale for the more traditional Keynesian models. In the same vein, Parkin (1986)'s paper is perhaps one of the most comprehensive treatments of aggregate price behavior under endogenous time dependent rules, however his price adjustment rule is also suboptimal. Ball and Cecchetti (1988), on the other hand, study the implication of information externalities for staggering in a model with prices fixed for more than one period. In their model –one of information problems– time dependent rules are indeed optimal[5]. However, while a cost of gathering

[1] A recent and representative version of these models is given in Blanchard (1986).

[2] State dependent rules are policies in which an action is taken only occasionally. Furthermore, when to take actions is determined by some state variable hitting a trigger point, as opposed to calendar time.

[3] I.e. rules in which actions are taken at predetermined calendar times.

[4] See Rotemberg (1982).

[5] Models in which the source of the time dependent rule is not made explicit but addressing issues of optimal staggering can be found in Ball and Romer (1987), Felthke and Policano (1986)

information about the optimal price justifies a time dependent rule, it does not imply that prices should be fixed (or even predetermined), as prices are in their paper. Unlike "menu costs" of changing prices, which lead to fixed prices (and state dependent rules), information problems yield time dependent *sampling* periods[6], but not fixed or predetermined prices. In fact, if sampling is staggered and there are information externalities, firms will change prices every time other firms change their prices. This will shorten and reduce the dynamics of optimally staggered *sampling* periods, however, as this paper shows, it is still the case that as long as signals are not fully revealing, prices are stickier and more persistent than in a full information economy.

Certainly full realism should include elements of both, time and state dependent rules. Still, it is fruitful to disentangle the effects of each of these mechanisms. This paper concentrates on the conditions to obtain time dependent rules as well as the implications of such rules when derived from first principles.

The presence of information externalities is shown to lead to uniform staggering in sampling, independently of the degree of interaction of firms' objective functions. Furthermore, the information externality implies that as the number of firms increase the cost of not sampling is reduced, hence each firm samples less frequently. When this mechanism is combined with signal extraction problems, nominal shocks can have more persistent, although smaller, effects than in economies with no information externalities. This same conduit, however, determines that the price level becomes too responsive to shocks that affect only a small set of firms.

The paper is organized in eight sections. Section 2 is a brief overview of the structure of the model as well as the main insights of the paper. Section 3 presents and solves the optimization problem of a single firm that has imperfect information about its optimal price and faces a lumpy cost of acquiring full information. Section 4 extends the previous problem to the collusive and non-collusive problems of two firms. The information problem is separated from the signal extraction problem, and both are carefully analyzed. The insights of this section are used in Section 5 to extend the results to n firms. This section stresses the effect of an increase in the number of firms on the different implications of the model. Section 6 studies the role of non-informational interactions. It shows that although the optimal price rules followed by firms –as well as the optimal sampling intervals– change, uniform staggering of sampling is invariant to the degree of competition among firms. Section 7 presents examples and figures highlighting the results found in the theoretical sections, like the fact that the optimal sampling period is increasing in: the number of firms, the relative importance of common shocks, the information content of the signal received from other

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and (1987), and Matsukawa (1986).

[6] Sampling is defined as the act of incurring in the lumpy cost required to collect full information about the state of nature.

firms' sampling, and the degree of monopoly power. This section also illustrates that as the quality of the signal increases (without reaching perfection), the effects of aggregate shocks become smaller but more persistent. Finally, Section 8 discusses possible extensions and concludes.

2. OVERVIEW

This section discusses briefly, and informally, the general framework and main mechanisms to achieve the results presented below.

Except for Section 7, firms are assumed to be independent monopolist whose optimal frictionless full information prices can be described by some (vector) stochastic process generated by aggregate and idiosyncratic sources. In contrast with standard menu-cost models, there is no cost whatsoever of changing prices, therefore actual prices will differ from the frictionless full information prices only if these stochastic processes are not perfectly observable. The purpose of this paper is to study the behavior of aggregate prices as firms are given different mechanisms to learn about the unobservable components of full information optimal prices. Two information channels are considered: First, firms are allowed to pay a fixed *sampling* cost to learn about the realization of their own optimal prices. And second, firms may learn from other firms' price changes as long as the unobservable[7] components of the optimal prices are not independent.

The first of these channels is explained in detail in Section 3 where a single firm case is studied[8]. The problem of a single firm can be separated into two steps: First, the firm sets its price optimally *given* the information available at each point in time. Under the assumptions of the model, this amounts to set the actual price equal to the expected value of the full information price at all points in time. This has the implication that as long as information flows continuously prices will also change continuously. In the second step, the firm has to decide how often to collect information about the unobservables (sampling). The model is such that optimal sampling is done at fixed time intervals. Furthermore, these time intervals are increasing in the cost of sampling and decreasing in the variance of the unobserved components as well as in the convexity of the loss function. Moreover, whenever the firm samples there is a discrete change in the information set, hence prices are likely to be more volatile at sampling periods.

The second channel is studied by introducing more than one firm (Sections 4 and 5) whose optimal full information prices are generated by stochastic processes that have non-independent

[7] Throughout the paper I use the word "unobservable" to refer to that part of the stochastic processes that cannot be observed without somebody sampling.

[8] Alternatively, many firms but with independent unobserved components.

unobservables. If information is in any sense shared (voluntarily or not), the information set of each firm changes discretely not only when its own sampling occurs but also when other firms sample. This introduces the concept of *information externality*. As long as all firms are not synchronized, the cost of not sampling is reduced for each firm. Hence, they can afford reducing the frequency at which each firm's resources in gathering information are spent. Furthermore, uniform staggering in sampling is the unique Nash equilibrium and coincides with the collusive outcome[9].

Yet, unless very few firms have informational links, these two channels alone are not sufficient to provide slow and smooth adjustment of the aggregate price level to a non perfectly observable aggregate shock. In fact, given the uniform staggering result, when the number of firms is large each firm samples very infrequently but the time intervals at which no firm samples are very short. However, if every time a firm samples it fully reveals the shocks leading to its price change, aggregate stickiness is limited to the length of those time intervals in which nobody samples. The paper solves this problem by allowing for cases in which firms do not fully reveal (or observe) the different components leading to a price change in a sampling firm.

To summarize, the informational problem here constructed leads to discrete sampling but continuous price changes by each firm. The sampling periods are uniformly staggered, and when signal extraction problems are added, the model determines that the response of the economy to some (non-fully observable) aggregate shocks is smooth and slow. Furthermore, Section 7 shows that these results are robust to some simple forms of interactions of firms beyond information sharing. The next sections explain in detail these and other ancillary results.

[9] Notice that firms do not compete in the goods markets. Their only link is through information sharing.

3. SINGLE FIRM

Consider a single firm whose optimal (frictionless) nominal price (all the variables in logarithms), $p^*(t)$, evolves according to[10]:

$$p^*(t) = v(t) + e(t) + \eta(t).$$

The process for $v(t)$ is not restricted, whereas $de(t) = \sigma_e dW_1(t)$ and $d\eta(t) = \sigma_\eta dW_2(t)$, with W_1 and W_2 two independent Standard Brownian Motions, also independent of the process generating $v(t)$. The shock $v(t)$ corresponds to the change in the optimal price due to the observable part of aggregate shocks[11]. The shocks $e(t)$ and $\eta(t)$, on the other hand, correspond to the unobserved changes in the optimal full information price due to firm specific and aggregate shocks respectively. The cost of departing from the optimal price is a quadratic function of the distance between the actual and optimal price[12]:

$$c(p(t), t) = \alpha \left(p(t) - p^*(t)\right)^2,$$

where $\alpha > 0$ determines the degree of convexity of the cost (loss) function, a parameter increasing in the elasticity of demand faced by the firm.

As said before, the firm may depart from the optimal price because there is no costless up to date information about the realizations of $e(t)$ and $\eta(t)$. It is assumed, however, that full revelation of both disturbances can be obtained at a "sampling" cost k. Notice that the main difference with traditional menu cost arguments is that there is no direct cost of changing the price. In fact the price can be changed as many times as the firm wants without incurring in any direct cost. The cost k is only paid at the moment of sampling. Another important difference of this model with respect to state dependent models, is that in the latter higher (steady state) inflation is bad for firms since they have to both spend some time even further away from their optimum price and pay menu cost more often, whereas in pure time dependent models is the variability not the level of inflation that affects firms' cost functions.

This framework allows to solve the optimization problem in two steps. The first one is static and corresponds to the cost minimization at each time t, given the information available at time

[10] Changes in $p^*(t)$ should be thought of as coming from both, aggregate (e.g. monetary shocks) and idiosyncratic (e.g. changes in consumer tastes) sources. This distinction is made clearer in the next sections.

[11] Idiosyncratic shocks are assumed to have no observable component. Adding an observable component is trivial but it does not add any insight and clutters the notation.

[12] As usual, this can be seen as a "crude" Taylor approximation of a more complex specification of foregone profits.

t. Proposition 1 below shows that the solution to this problem is to set the price equal to the conditional expectation of $p^*(t)$. In particular, at sampling time $p(t) = p^*(t)$.

PROPOSITION 1 *Under a quadratic loss function, the optimal price at each time t is: $p(t) = \mathrm{E}_t[p^*(t)]$, where E_t denotes the conditional expectation given the information available at time t.*

PROOF In any given point in time the firm solves the problem:

$$C(t) = \min_{p(t)} \mathrm{E}_t[c(p(t), t)]. \tag{1}$$

The proposition follows trivially from the first order condition of this problem. †

Certainly, this proposition relies heavily on the quadratic nature of the cost function. As such, it is only an application of a standard least square problem. However, it has the important implication that information costs do not lead to fixed prices unless the information structure is itself discrete and all changes in the optimal full information prices in between information arrivals are unforeseen.

Assuming no discounting[13], and given Proposition 1 and the fact that the objective function is stationary, it is possible to pose the second stage of the optimization problem as a simple minimization of long run average cost:

$$L(k) = \min_{\tau} \frac{1}{\tau}\left\{ \mathrm{E}_0 \left[\int_0^{\tau} C(t)\, dt \right] + k \right\}.$$

Furthermore, denoting the sum of $e(t)$ and $\eta(t)$ by $z(t)$, and the standard deviation of this sum by σ_z, the cost function in (1) can be written as $C(t) = \alpha\,(z(t) - z(0))^2$ for $0 \le t \le \tau$. Hence, straight application of Fubini's theorem and Ito's lemma simplify the second stage of the problem to:

$$L(k) = \min_{\tau} \frac{1}{\tau}\left\{ \frac{\alpha \sigma_z^2 \tau^2}{2} + k \right\}. \tag{2}$$

It is important to notice that the fact that $e(t)$ and $\eta(t)$ have been assumed to be non-stationary played an important role in the previous derivations. Furthermore, if the expected cost of not sampling does not grow monotonically with time (since last sampling), there is nothing to guarantee that τ is finite for all vales of the variance of unobserved components, convexity of the cost function, and sampling costs.

[13] Allowing for positive discounting does not change the single firm solution in any fundamental way. It does have more relevance, however, in the non-collusive multi-firm case.

Proposition 2 below shows that the solution to the problem posed in (2) is very similar to that of the one sided state dependent models (time is monotonic!), although it determines the sampling interval instead of the width of the S-s bands.

PROPOSITION 2 *The optimal sampling interval is of the square root form:*

$$\tau = \sqrt{\frac{2k}{\alpha\sigma_z^2}}$$

PROOF It follows trivially from the first order condition of problem (2). †

Whenever there is a cost of gathering the relevant information to determine the optimal price, and no direct cost of changing prices, the rule is *time* as opposed to *state* dependent[14]. The dynamics implied by this model are very different to that of state dependent models. In fact prices change continuously if the firm receives information continuously. However sampling, and therefore full adjustment to changes in the full information optimal price, occurs at a fixed time interval that is increasing in the cost of sampling, and decreasing in the convexity of the loss function and in the variance of the unobserved component of the full information optimal price.

The next sections turn to the case in which more than one firm exist, giving rise to information externalities.

[14] Obviously it cannot be state dependent since the state is not observed unless sampling occurs (see Blanchard and Fischer 1989 p.413). It is interesting to notice, however, that the rule is time dependent regardless of the extent of the correlation between the observable and unobservable shocks (remember that the representation in $p^*(t)$ can be thought as an orthogonal decomposition of "deeper" shocks).

4. TWO FIRMS

The notation of the two firms case is very similar to the previous notation, except for the use of the subindexes 1 and 2. In particular, define the unobservables of each firm as $z_1(t) \equiv e_1(t) + \eta(t)$ and $z_2(t) \equiv e_2(t) + \eta(t)$, with $de_1(t) = \sigma_e dW_1(t)$, $de_2(t) = \sigma_e dW_2(t)$ and $d\eta(t) = \sigma_\eta dW_3(t)$, where $W_1(t)$, $W_2(t)$ and $W_3(t)$ are independent Standard Brownian Motions[15]. The fully observable disturbances, $v_1(t)$ and $v_2(t)$, are not restricted except for their independence from $W_1(t)$, $W_2(t)$ and $W_3(t)$. As a result, the optimal full information prices can be characterized by:

$$p_1^*(t) = v_1(t) + e_1(t) + \eta(t)$$

and

$$p_2^*(t) = v_2(t) + e_2(t) + \eta(t).$$

Below I analyze cases with different degrees of private information, however except for a digression in Section 4.3, a maintained assumption is that the firm doing the sampling only learns about $\eta(t)$ and its own $e_i(t)$. The other firm, on the other hand, at least observes the price change of the sampling firm and the fact that the other firm has sampled[16]. The shocks $v_1(t)$ and $v_2(t)$ are assumed to be public information.

Although the information structure is now more complex, Proposition 1 still applies, hence $p_i(t) = \mathrm{E}_t[p_i^*(t)]$ for $i = 1, 2$. Using this insight, the problems analyzed below simplify significantly.

[15] The assumption $\sigma_{e_1} = \sigma_{e_2}$ is important since it determines, later, that both firms' optimal sampling intervals are equal. If this is not the case both, notation and mathematics, become more complex since remainders have to be taken into account.

[16] In most cases the assumption that firms explicitly reveal the time at which they are sample is not very restrictive. As long as the v_i are public information, firms will identify the firm currently sampling by its "unexplained" price change.

4.1. Collusion

Having solved the problem of how to determine the optimal price of each firm given the information available at each point in time, the cartel has to decide when and how frequently each firm will sample. In doing this two assumptions play an important role here and in the remaining of the paper: First, firms are restricted to take a constant sampling interval. Under the structure of the model this is not a binding restriction for the single firm nor the collusive cases, however it might be important for the non-collusive case. In the latter, if the variance of idiosyncratic shocks is small relative to the variance of common shocks, incentives for mixed strategies could appear as firms may want to free ride on other firms' sampling[17]. However, allowing for mixed strategies may lead to complex multiperiod war of attrition problems that are beyond the scope of this paper. Instead, I restrict the solution space to simple strategies. Once this is done, constant sampling intervals are indeed optimal. Second, even though the cartel optimizes respect to τ_1 and τ_2, the objective function is written as if both were equal. This simplifies "reminders" problems and is shown later to be innocuous.

The purpose of this paper is to isolate the role of information transmission on price staggering and stickiness, therefore it is convenient to assume that firms are only related through the information process. In other words, changes in one firm's price are assumed not to affect the demand or cost conditions of the other firm, but only to reveal some information about unobservables. This assumption is relaxed later in Section 6.

Under these assumptions the problem to be solved by the cartel can be written as:

$$L(k) = \min_{\tau_1,\tau_2,\delta} \frac{1}{\tau_1}\left\{ \mathrm{E}_0\left[\int_0^{\tau_1} \alpha\left(p_1^*(t) - \mathrm{E}_t[p_1^*(t)]\right)^2 dt\right] + k\right\} \tag{3}$$

$$+\frac{1}{\tau_2}\left\{ \mathrm{E}_0\left[\int_\delta^{\tau_2+\delta} \alpha\left(p_2^*(t) - \mathrm{E}_t[p_2^*(t)]\right)^2 dt\right] + k\right\}$$

where the first and second terms correspond to the long run average cost of firm 1 and 2, respectively.

Using Ito's lemma, the expected instantaneous cost faced by firm 1, for given τ_1, δ and the

[17] Notice, however, that as long as idiosyncratic shocks are non-stationary firms will eventually sample, no matter how often other firms are sampling. This may render simple strategies optimal even when the variance of idiosyncratic shocks is small and firms do not collude in their use of information.

fact that the firm has sampled at time 0, is:

$$\mathrm{E}_0\left[\alpha\left(p_1^*(t)-\mathrm{E}_t[p_1^*(t)]\right)^2\right]=\begin{cases}\alpha\sigma_z^2 t & 0\le t<\delta;\\ \alpha(\sigma_z^2 t-\sigma_\eta^2\delta) & \delta\le t<\tau_1.\end{cases}$$

At time 0, when the firm has just sampled, the expected cost is zero since the actual price is equal to the optimal full information price. Expected cost is, however, increasing with time. At time δ firm 2 samples, revealing $\eta(\delta)$, hence the only motive for a discrepancy between the actual and full information price of firm 1 is $e_1(\delta)$, lowering the expected instantaneous cost to $\alpha\sigma_e^2\delta$ or $\alpha(\sigma_z^2-\sigma_\eta^2)\delta$. For $t>\delta$ both shocks accumulate again.

Firm 2 faces a similar cost structure. Assuming that it samples at time delta, its expected cost grows in time proportionally to σ_z. At time τ_1, however, firm 1 samples, revealing $\eta(\tau_1)$. Both shocks accumulate again thereafter. Using the fact that $\mathrm{E}_0\left[\alpha\left(p_2^*(t)-\mathrm{E}_t[p_2^*(t)]\right)^2\right]=\mathrm{E}_\delta\left[\alpha\left(p_2^*(t)-\mathrm{E}_t[p_2^*(t)]\right)^2\right]$, yields:

$$\mathrm{E}_0\left[\alpha\left(p_2^*(t)-\mathrm{E}_t[p_2^*(t)]\right)^2\right]=\begin{cases}\alpha\sigma_z^2(t-\delta) & \delta\le t<\tau_1;\\ \alpha(\sigma_z^2(t-\delta)-\sigma_\eta^2(\tau_1-\delta)) & \tau_1\le t<\tau_2+\delta.\end{cases}$$

Fubini's theorem allows to use the previous expressions and write (3) as follows:

(4)
$$L(k)=\min_{\tau_1,\tau_2,\delta}\ \frac{1}{\tau_1}\left\{\frac{\alpha\sigma_z^2\tau_1^2}{2}-\alpha\sigma_\eta^2\delta(\tau_1-\delta)+k\right\}$$
$$+\frac{1}{\tau_2}\left\{\frac{\alpha\sigma_z^2\tau_2^2}{2}-\alpha\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)+k\right\},$$

where again, the first and second terms represent the long run average cost of firms 1 and 2, respectively.

PROPOSITION 3 *When firms collude in the determination of optimal sampling intervals and share all their available information:*

a) Staggering is optimal, and

b) each firm samples less frequently than in the single firm case.

PROOF The first order conditions of problem (4) respect to τ_1, τ_2 and δ, respectively, are:

(5)
$$\frac{\sigma_z^2}{2}+\frac{\sigma_\eta^2\delta(\tau_1-\delta)-k}{\tau_1^2}-\frac{\sigma_\eta^2\delta}{\tau_1}+\frac{\sigma_\eta^2(\tau_2+2\delta-2\tau_1)}{\tau_2}=0,$$

(6)
$$\frac{\sigma_z^2}{2}+\frac{\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)-k}{\tau_2^2}-\frac{\sigma_\eta^2(\tau_1-\delta)}{\tau_2}=0$$

and

(7)
$$\sigma_\eta^2\left(\frac{2\delta}{\tau_1}+\frac{(\tau_2+2\delta-2\tau_1)}{\tau_2^2}-1\right)=0.$$

The solution to this system of equations is:

$$\tau_1=\tau_2=\tau_c=2\delta,$$

where τ_c represents the value of τ when firms collude in information gathering and is equal to:

$$\tau_c=\sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{1}{2}\theta}\right)}\geq\tau=\sqrt{\frac{2k}{\alpha\sigma_z^2}},$$

and

$$\theta\equiv\frac{\sigma_\eta^2}{\sigma_z^2}.$$

†

Hence this proposition shows that when firms share their information and take joint decisions, it is optimal for them to stagger (in their sampling), and for each of them to sample less often[18]. Each time a firm samples, it produces an information externality (internal in the case of the cartel),

[18] As mentioned before, the problem was written as if the sampling intervals were equal. It is natural to wonder whether this is an important restriction. I claim it is not for the following reason: Assume that starting from τ_c firm 1 slightly increases its own τ. Firm 2 will be worse off since it receives less information per unit of time. If it decides to increase the sampling interval to match firm 1, then obviously both firm will be worse of since (5) and (6) will not be satisfied even though sampling periods are equal. If, on the other hand, τ_2 is kept constant, the analysis is slightly more complex. In this case firm 1 will no longer receive information from firm 2 in the middle of its sampling interval all the time. In fact on average this will be the case, but information externalities will be uniformly distributed across all the sampling intervals of firm 1. This will necessarily raise the average cost of firm 1. On the other hand, the only gain (besides the second order gain of sampling less often in the problem analyzed in Proposition 3) is the fact that now there are periods in which firm 1 receives information externalities twice within a sampling interval, however this is a set of measure zero when the changes in the value of τ_1 considered are infinitesimal, hence cannot compensate the first order loss due to the occasions (multiple) in which the information externality from firm 2 does not come at the optimal time. Certainly these arguments extend easily to the non-collusive case, since both firms are worse off when one of them departs from the monopoly solution. The next subsection studies this case.

therefore reducing the other firm's expected cost of not sampling. Uniform staggering, on the other hand, maximizes this information "externality". It is also apparent from the proposition that as θ rises, sampling is more infrequent. This, again, reflects the fact that the information externality rises as the relative importance of common shocks increases.

In spite of the fact that each firm samples less frequently, full revelation of the information gathered by the sampling firm implies that aggregate price stickiness is reduced. The full response to an aggregate shock occurs when one (anyone) of the firms samples, but *total* sampling is increased when moving from the single firm model discussed in the previous section to the model of this subsection. This is shown in Proposition 4 below.

PROPOSITION 4 *In the collusive full revelation equilibrium there is, on average, less aggregate price stickiness respect to an aggregate shock, $\eta(t)$, than in the single firm case.*

PROOF Revisions with respect to $\eta(t)$ are done every δ periods. However, for any $\sigma_\eta > 0$

$$\delta = \tau \frac{1}{2}\sqrt{\frac{1}{1-\frac{1}{2}\theta}},$$

but

$$1 \leq \sqrt{\frac{1}{1-\frac{1}{2}\theta}} \leq \sqrt{2},$$

hence

$$\delta < \tau.$$

†

In this model, the reason why each firm samples less frequently (relative to the single firm case) is an increase in the overall information availability. The only way this can happen, however, is by increasing the frequency of overall sampling. This is precisely the mechanism exploited in Proposition 4 to conclude that moving from the single to two firms equilibrium reduces aggregate price stickiness when there is full revelation.

Later, when the full revelation assumption is relaxed, complete response of the economy to an aggregate shock is related not only to overall sampling but also to the length of the sampling interval of each firm, increasing aggregate stickiness.

4.2. Non-Collusive Equilibrium: Full Revelation

In this section I assume that it is still the case that when one firm samples, the other one can observe the value of $\eta(t)$. Here, nonetheless, firms do not jointly optimize. Instead the problems faced by firms 1 and 2, respectively, are[19]:

$$L_1(k) = \min_{\tau_1} \frac{1}{\tau_1}\left\{\frac{\alpha\sigma_z^2\tau_1^2}{2} - \alpha\sigma_\eta^2\delta(\tau_1-\delta)+k\right\}, \tag{8}$$

and

$$L_2(k) = \min_{\tau_2,\delta} \frac{1}{\tau_2}\left\{\frac{\alpha\sigma_z^2\tau_2^2}{2} - \alpha\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)+k\right\}. \tag{9}$$

PROPOSITION 5 *Under fixed sampling strategies, the collusive equilibrium is a Nash equilibrium.*

PROOF The first order conditions of (8) and (9) respect to τ_1, τ_2 and δ, respectively, are:

$$\frac{\sigma_z^2}{2} + \frac{\sigma_\eta^2\delta(\tau_1-\delta)-k}{\tau_1^2} - \frac{\sigma_\eta^2\delta}{\tau_1} = 0, \tag{10}$$

$$\frac{\sigma_z^2}{2} + \frac{\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)-k}{\tau_2^2} - \frac{\sigma_\eta^2(\tau_1-\delta)}{\tau_2} = 0 \tag{11}$$

and

$$\sigma_\eta^2\left(\frac{2\delta}{\tau_1} + \frac{(\tau_2+2\delta-2\tau_1)}{\tau_2^2} - 1\right) = 0. \tag{12}$$

It is easy to corroborate that the solution $\tau_1 = \tau_2 = 2\delta$ satisfies condition (12). Furthermore, once this condition –that is also a condition in the collusive case– is replaced in equations (5) and (6), the remaining first order conditions of the collusive and non-collusive problems become identical, therefore the solution is the same. †

In understanding Proposition 5 it is important to remember that only two type of departures are permitted for these firms: they can either decide not to stagger uniformly (i.e. $\delta \neq \tau/2$), or to change their own sampling frequency (i.e. $\tau_1 \neq \tau_2$). Given that there is no discounting (so short run gains are irrelevant), and that the information externality is maximized when the firms are uniformly staggered, there is no incentive for any single firm to move away from uniform staggering. Furthermore, the arguments used in the previous section to rule out differences in the sampling interval also apply here, therefore the cartel solution is a Nash equilibrium.

[19] Note that given that the firms are minimizing long run average, who moves first, or alternatively, who picks δ, is irrelevant.

This insight is important since it applies to all the cases discussed in this paper, allowing us to analyze all the cases presented below either in the collusive or the non-collusive framework at discretion. The only important restriction is that the information structure of the collusive and non-collusive cases has to be the same.

4.3. Signal Extraction

An important assumption of the models described above, is that there is full revelation of the information obtained by each firm when sampling. Furthermore, firms were assumed to be able to perfectly disentangle $z_i(t)$ into their own idiosyncratic shock, $e_i(t)$, and the aggregate shock $\eta(t)$. Even though that structure leads to uniform staggering in sampling, aggregate price stickiness was reduced by the addition of a new firm.

Conversely, in this section it is assumed that whenever a firm samples, it only reveals (or finds out) the value of $z_i(t)$ but not its components. In this context the other firm has to solve a signal extraction problem in order to "guess" how much of the price change by the sampling firm is due to the aggregate shock, $\eta(t)$. This setup leads to a signal extraction problem whose particular form depends upon the specific assumptions about the information structure. At least three cases are possible: First, one in which each firm observes all the shocks of the economy every time it samples. In this case the optimal forecast of $\eta(t)$ for a firm that sampled at time zero, say 1, for the interval $\delta_{se} \leq t \leq \tau_1$ (i.e. after firm 2 has sampled) is $\mathrm{E}_t[\eta(t)] = \theta z_2(\delta_{se}) + (1-\theta)\eta(0)$, where the subindex *se* denotes *signal extraction.* Second, if the same firm only observes the common shock and its own idiosyncratic shock at the time of sampling, the optimal forecast of the common shock for the interval $\delta_{se} \leq t \leq \tau_1$ is $\mathrm{E}_t[\eta(t)] = \frac{\theta}{2-\theta} z_2(\delta_{se}) + \frac{2(1-\theta)}{2-\theta}\eta(0)$. Hence in this case there is less weight given to the other firm's signal since the relative importance of the other firm's idiosyncratic shock on the *news* component of the signal is larger[20]. And third, the case in which the firm only observes $z_i(t)$ but not its components when sampling. Certainly this is the case with least information and implies that the best forecast of the common shock for the interval $\delta_{se} \leq t \leq \tau_1$ is $\mathrm{E}_t[\eta(t)] = \frac{\theta^2}{2\theta^2-\theta^3+2-2\theta} z_2(\delta_{se}) + \frac{2(1-\theta^3+\theta^2-\theta)}{2\theta^2-\theta^3+2-2\theta} z_1(0)$. All these solutions are well behaved in the sense that as $\theta \to 1$, all the weight goes to the most current information.

Perhaps the most realistic case is the second one. For this reason I center my analysis around this case, however all the fundamental (qualitative) results shown below apply for the three scenarios since they all hinge on the fact that the signal is imperfect but non-negligible. This puts the signal extraction problem in between the single firm and the full revelation-two firms cases.

[20] This occurs because firm 1 only observed η and its *own* idiosyncratic shock at the time it sampled.

After some algebraic manipulation, the cost functions of firms 1 and 2, respectively, for the second case mentioned above are:

$$\mathrm{E}_0\left[\alpha\left(p_1^*(t)-\mathrm{E}_t[p_1^*(t)]\right)^2\right]=\begin{cases}\alpha\sigma_z^2 t & 0\le t<\delta_{se};\\ \alpha(\sigma_z^2 t-\frac{\theta}{2-\theta}\sigma_\eta^2\delta_{se}) & \delta_{se}\le t<\tau_1.\end{cases}$$

and

$$\mathrm{E}_0\left[\alpha\left(p_2^*(t)-\mathrm{E}_t[p_2^*(t)]\right)^2\right]=\begin{cases}\alpha\sigma_z^2(t-\delta_{se}) & \delta_{se}\le t<\tau_1;\\ \alpha(\sigma_z^2(t-\delta_{se})-\frac{\theta}{2-\theta}\sigma_\eta^2(\tau_1-\delta_{se})) & \tau_1\le t<\tau_2+\delta_{se}.\end{cases}$$

The main difference with the full revelation case is that now each firm does not get full information about the value of the common shock when the other firm samples. This is reflected in the fact that $\frac{\theta}{2-\theta}<1$, hence the information externality is reduced for any given θ.

The problem to be solved now is:

(13)
$$L(k)=\min_{\tau_1,\tau_2,\delta_{se}}\frac{1}{\tau_1}\left\{\frac{\alpha\sigma_z^2\tau_1^2}{2}-\frac{\alpha\theta\sigma_\eta^2\delta_{se}(\tau_1-\delta_{se})}{2-\theta}+k\right\}$$
$$+\frac{1}{\tau_2}\left\{\frac{\alpha\sigma_z^2\tau_2^2}{2}-\frac{\alpha\theta\sigma_\eta^2(\tau_1-\delta_{se})(\tau_2+\delta_{se}-\tau_1)}{2-\theta}+k\right\}.$$

PROPOSITION 6 *In the signal extraction equilibrium:*

a) Staggering is optimal,

b) each firm samples less frequently than in the single firm case, but

c) more often than in the two firms-full revelation case.

PROOF The proposition follows easily from realizing that the first order conditions of problem (13) are identical to those of the full revelation case, except for the fact that $\frac{\theta}{2-\theta}\sigma_\eta^2$ replaces σ_η^2 everywhere. Therefore,

$$\tau_1=\tau_2=\tau_{se}=2\delta_{se},$$

with

$$\tau_{se}=\sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{\theta^2}{2(2-\theta)}}\right)}\ge\tau=\sqrt{\frac{2k}{\alpha\sigma_z^2}}.$$

Part c) of the proposition is proved by the fact that $0<\theta<1$ (i.e. $0<\frac{\theta}{2-\theta}<1$). †

This economy differs from that of the previous subsection in that the information externality is smaller (for comparable parameter values). Proposition 6 shows, however, that uniform staggering does not depend on the size of the information externality but on its existence. Nonetheless, the smaller amount of information sharing does raise the sampling frequency of each firm (relative to the full information case). The most important innovation introduced by signal extraction, however, is the smoothness of the aggregate price adjustment to a an innovation in the common (aggregate) unobservable shock. The next proposition formalizes this statement.

PROPOSITION 7 *In the signal extraction equilibrium:*

a) the economy takes longer than in the single firm and two firms-full revelation cases to complete the adjustment to an aggregate shock, $\eta(t)$, although

b) it adjusts more smoothly than in the single firm case.

PROOF In order to compare the response-length of the economy it is appropriate to assume that the shock occurs just after one of the firms has sampled.

In the signal extraction case, full adjustment will not be completed until *both* firms have sampled, hence it takes τ_{se} periods for this economy to fully adjust, as opposed to τ and δ in the single firm and two firms-full revelation cases, respectively. It is clear from the previous propositions, nonetheless, that:

$$\tau_{se} = \sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{\theta^2}{2(2-\theta)}}\right)} \geq \tau = \sqrt{\frac{2k}{\alpha\sigma_z^2}} \geq \delta = \frac{1}{2}\sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{\theta}{2}}\right)}.$$

This proves part a) of the proposition. Part b) is proved by the fact that every time a firm samples, there is an updating in the forecast of $\eta(t)$ in both firms. The sampling firm fully adjusts to the new $\eta(t)$, whereas the other firm incorporates only a fraction $\frac{\theta}{(2-\theta)}$ of the change (until its own time to sample). Therefore some adjustment to changes in $\eta(t)$ occurs every δ_{se} periods, clearly less than τ since:

$$\delta_{se} = \frac{\tau_{se}}{2} = \frac{1}{2}\sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{\theta^2}{2(2-\theta)}}\right)} = \tau\frac{1}{2}\sqrt{\frac{1}{1-\frac{\theta^2}{2(2-\theta)}}},$$

and

$$1 \leq \sqrt{\frac{1}{1-\frac{\theta^2}{2(2-\theta)}}} \leq \sqrt{2},$$

hence

$$\delta_{se} < \tau.$$

†

Although simulation experiments will be shown later in Section 7, it is apparent by now that the model with information externalities and signal extraction problems leads to a behavior of the aggregate price index not very far from conventional wisdom. When aggregate shocks are not directly observable, the response of the aggregate price index to them is smooth and long lasting. At the same time, however, the model is well behaved in the sense that the response to observable shocks is very fast. The next sections discuss these issues extensively.

5. N FIRMS

None of the proofs of the previous section hinged in an important way on the fact that just two firms were present. Except for additional algebraic manipulation and notation complexity, the proofs of propositions equivalent to those of the two firms case for the n firms case are similar. Given this, I will only comment briefly on the extensions of the previous results, to concentrate on the implications of an increase in the number of firms for the optimal sampling period of each firm and degree of aggregate price stickiness[21].

5.1. Full Revelation

Denote by δ_i for $i = 1, \ldots, n-1$, the staggering position (relative to firm 1) of firms 2 to n. Also let $\delta_0 = 0$ and $\delta_n = \tau_1$ to simplify the notation. So $0 < \delta 1 < \ldots < \tau_1$. In this case the instantaneous expected cost at each time t (for a representative period) faced by firm 1, can be written as follows:

$$\mathrm{E}_0\left[\alpha(p_1^*(t) - \mathrm{E}_t[p_1^*(t)])^2\right] = \alpha\left(\sigma_z^2 t - \sigma_\eta^2 1[\delta_i \leq t < \delta_{i+1}]\delta_i\right),$$

where $1[\delta_i \leq t < \delta_{i+1}]$ is an indicator function that takes value one when $\delta_i \leq t < \delta_{i+1}$ and zero otherwise.

Lengthier but similar steps to those followed in the two-firms case yield the following problem for firm 1:

$$L_1(k) = \min_{\tau_1} \frac{1}{\tau_1}\left\{\frac{\alpha\sigma_z^2\tau_1^2}{2} - \alpha\sigma_\eta^2\sum_{i=1}^{n-1}\delta_i(\delta_{i+1} - \delta_i) + k\right\},$$

with first order condition:

$$\frac{\alpha\sigma_z^2}{2} + \frac{\alpha\sigma_\eta^2}{\tau_1^2}\sum_{i=1}^{n-1}\delta_i(\delta_{i+1} - \delta_i) - \frac{\alpha\sigma_\eta^2\delta_{n-1}}{\tau_1} - \frac{k}{\tau_1^2} = 0. \tag{14}$$

[21] Remember that here competition does not change as the number of firms change. All the effects are due to information externalities.

The same can be done for every firm (adding the first order conditions respect to the different δ_i). Tedious but straight forward steps lead to the extension of the results of the previous section with $\tau_i = \tau_1$ and $\delta_i = \tau_i i/n$, for $i = 1, \ldots, n$. As said before, this section concentrates now on issues that could not be fully addressed by the single and two firms model.

PROPOSITION 8 *The optimal sampling period of each firm is increasing in the number of firms.*

PROOF Plugging into (14) the uniform staggering result and the fact that all τ_i are equal, yields the condition:

$$\frac{\alpha\sigma_z^2}{2} - \frac{\alpha\sigma_\eta^2(n-1)}{2n} = \frac{k}{\tau_1^2},$$

therefore (using the fact that all firms have the same τ),

$$\tau_i = \sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1-\frac{n-1}{n}\theta}\right)}.$$

Trivial differentiation proves the proposition. †

Overall sampling rises with the number of firms, enlarging the information externality. Such environment lowers the expected cost of not sampling, reducing the frequency at which firms pay the explicit cost to fully observe their state. Proposition 9 shows that under full revelation the increase in overall sampling reduces price stickiness.

PROPOSITION 9 *In the full revelation case aggregate price stickiness is always reduced as the number of firms increase.*

PROOF As indicated in Section 4, in the full revelation case, aggregate price stickiness can be measured by δ_1. The proposition is not trivial only because there is a tradeoff involved since $\delta_1 = \tau_1/n$ and τ_1 is increasing in n (see Proposition 8). However,

$$\frac{\partial\delta_1}{\partial n} = \frac{\tau_1}{n^2}\left(\frac{1}{2n}\left(1-\theta+\frac{\theta}{n}\right)^{-1} - 1\right).$$

Hence,

$$sign\left(\frac{\partial\delta_1}{\partial n}\right) = sign\left(\theta - \frac{2n-1}{2n-2}\right) < 0.$$

†

Relaxing the full revelation assumption not only introduces aggregate price stickiness and smoothness, but also links the aggregate dynamics to the number of firms in interesting ways. The next subsection turns into this.

5.2. Signal Extraction

As seen in subsection 4.3, the dynamics of the aggregate price level are modified in an important way when signal extraction issues are considered. Not surprisingly, the aggregate (and individual) price level becomes smoother[22] and stickier. Below I analyze the effects of changing the number of firms on the dynamics of prices.

The first step in finding the optimal sampling interval of each firm, is to determine how firms use the information emerging from other firms sampling, as well as the quality of this information. In doing this, it is important to remember that firms observe their full information optimal price (and components) when sampling. This feature makes the signal extraction problem more complex than the standard Muth-type problem, since the role of initial conditions (i.e. the conditions when the firm samples) never washes away. On the contrary, each firm's problem restarts every time that the firm samples. When a firm has just sampled, it gives a high weight to its own (past) information in forecasting the current position of the common shock, $\eta(t)$, however as time passes the noise of other firms' signals becomes less important than potential departures of $\eta(t)$ from the value of η observed by the firm a long time ago (at sampling). It is precisely the change in the weight given to the different signals what makes the signal extraction problem more complex. Nonetheless, the model has a state space representation that allows using the updating equations of a Kalman Filter[23]. This is the approach followed below.

Recalling that information arrives discretely (when some firm samples) and at uniform intervals (given uniform staggering) it is possible to write a discrete time state space model in which the units of time (s) are normalized by δ_1 (or τ/n). Hence the *transition* and *measurement* equations can be written as:

$$\eta(s) = \eta(s-1) + \sigma_1 w(s)$$

and

$$z(s) = c(s)\eta(s) + \sigma_2(s)u(s),$$

respectively, where $w(s)$ and $u(s)$ are independent white noises with unitary variances.

The signal extraction problem for any firm starts again every time the firm samples, hence there is no loss of generality in setting the time in which firm 1 samples equal to 0. Furthermore, $z(s)$ and $\sigma_2(s)u(s)$ correspond to the realization of $z(s\tau/n)$ and $e(s\tau/n)$, respectively, of the firm sampling at time s ($s = 1, \ldots, n-1$). Given that unobservable idiosyncratic shocks, e_i, are independent across firms, the white noise assumption on $u(s)$ is justified, whereas $w(s)$ is serially

[22] Respect to aggregate shocks.

[23] See Anderson and Moore (1979) for an excellent exposition of filtering techniques.

uncorrelated and independent of e_i for all i by the assumptions made earlier in the paper. In addition $c(0) = z(0)/\eta(0)$, $\sigma_2(0) = 0$, $c(s) = 1$ and $\sigma_2(s) = \sigma_e\sqrt{\tau/n}$ for $s = 1, \ldots, n-1$.

The Kalman filter can now be written as follows:

$$\hat{\eta}(s) = \hat{\eta}(s-1) + \left(\Omega(s-1) + \sigma_1^2\right) c(s) F(s)^{-1} \left(z(s) - c(s)\hat{\eta}(s-1)\right)$$

and

$$\Omega(s) = \Omega(s-1) + \sigma_1^2 - \left(\Omega(s-1) + \sigma_1^2\right)^2 c(s)^2 F(s)^{-1},$$

with $F(s) \equiv c(s)^2 \left(\Omega(s-1) + \sigma_1^2\right) + \sigma_2^2(s)$ and $\hat{\eta}$ denoting the mean square error forecast of η.

All the parameters, as well as $\eta(0)$, are assumed to be known. The mean square error Ω at time 0, $\Omega(0)$, is equal to zero since firm 1 obtains full information about η (and e_1) when it samples (time 0).

In this context, the expected cost at each time t (for a representative period) faced by firm 1 is:

$$\mathrm{E}_0\left[\alpha(p_1^*(t) - \mathrm{E}_t[p_1^*(t)])^2\right] = \alpha\left(\sigma_e^2 t + 1[\delta_i \leq t < \delta_{i+1}]\left(\sigma_\eta^2(t-\delta_i) + \Omega(i)\right)\right).$$

Solving the filter backward it is possible to rearrange this equation in a form consistent with the rest of the paper:

$$\mathrm{E}_0\left[\alpha(p_1^*(t) - \mathrm{E}_t[p_1^*(t)])^2\right] = \alpha\left(\sigma_z^2 t - \sigma_\eta^2 \pi_i[\delta_i \leq t < \delta_{i+1}]\delta_i\right),$$

with

$$\pi_i = \frac{1}{i}(\theta_1 + \ldots + \theta_i)$$

for $i = 1, \ldots, n-1$,

$$\theta_i = \frac{\theta(i - \theta_1 - \ldots - \theta_{i-1})^2}{\theta(i - \theta_1 - \ldots - \theta_{i-1}) + (1-\theta)n}$$

for $i = 2, \ldots, n-1$, and[24]

$$\theta_1 = \frac{\theta}{n - (n-1)\theta}.$$

Hence the problem to be solved by firm 1 is:

$$L_1(k) = \min_{\tau_1} \frac{1}{\tau_1}\left\{\frac{\alpha\sigma_z^2\tau_1^2}{2} - \alpha\sigma_\eta^2 \sum_{i=1}^{n-1} \pi_i\delta_i(\delta_{i+1} - \delta_i) + k\right\},$$

with first order condition:

$$\frac{\alpha\sigma_z^2}{2} + \frac{\alpha\sigma_\eta^2}{\tau_1^2}\sum_{i=1}^{n-1}\pi_i\delta_i(\delta_{i+1} - \delta_i) - \frac{\pi_{n-1}\alpha\sigma_\eta^2\delta_{n-1}}{\tau_1} - \frac{k}{\tau_1^2} = 0. \quad (15)$$

[24] Remember that $\theta = \frac{\sigma_\eta^2}{\sigma_z^2}$.

As in previous sections, the same can be done for every firm (adding the first order conditions respect to the different δ_i). Therefore, $\tau_i = \tau_1$ and $\delta_i = \tau_1 i/n$, for $i = 1, \ldots, n$.

PROPOSITION 10 *The optimal sampling period of each firm is increasing in the number of firms and, ceteris paribus, the quality of the signal (indexed by θ).*

PROOF Plugging into (15) the uniform staggering result and the fact that all τ_i are equal, yields the condition:

$$\frac{\alpha\sigma_z^2}{2} - \alpha\sigma_\eta^2 \sum_{i=1}^{n-1} \theta_i \frac{i}{n^2} = \frac{k}{\tau_1^2},$$

therefore (using the fact that all firms have the same τ),

$$\tau_i = \sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{1 - 2\theta \sum_{i=1}^{n-1} \theta_i \frac{i}{n^2}}\right)}. \tag{16}$$

Trivial differentiation of (16) respect to θ (notice that $\frac{\partial\theta_i}{\partial\theta} \geq 0$) proves the second part of the proposition.

The first part of the proposition can be proved heuristically. Suppose that there is an equilibrium with $n-1$ firms. If a new firm is added without changing τ, there will be one more firm sampling, reducing each firm's incentives to sample. If firms were previously in equilibrium, τ must increase to restore it, otherwise the (long run) marginal benefit of changing prices would be lower than the marginal cost of doing it. †

COROLLARY 1 *Aggregate price stickiness is increasing in the number of firms, in the sense that it takes longer for the price level to* fully *adjust to an aggregate shock.*

PROOF When signal extraction problems are present, the adjustment is fully completed when *all* firms have sampled, but τ_i is increasing in the number of firms (Proposition 10). †

Corollary 1 does not fully characterize the path of the aggregate price level, though. For this, it is necessary to uncover how is the smoothness of this path affected by the number of firms, however Proposition 11 shows that information arrives more frequently as the number of firms increases, therefore firms also revise their price more often yielding a smoother path of prices as less news accumulate before each price change.

PROPOSITION 11 *The quality of each signal deteriorates as the number of firms increases, but the overall availability of information increases since the frequency at which signals are received, $\frac{1}{\delta_1}$, rises.*

PROOF The first part of the proposition is proved by the fact that the sampling interval of each firm is longer, therefore the relative importance of σ_e rises (see definition of θ_1). The second part, on the other hand, is a direct implication of the increase in τ, since the only reason for this to happen -given α, σ_z and k- is because of an improvement in the information received by each firm. †

6. COUPLED FIRMS

The previous sections have highlighted the role of information externalities in the determination of firm's optimal sampling intervals and in yielding uniform staggering of sampling. For this, the information problem was isolated by assuming that firms were only linked through the presence of unobservable (except when sampling) shocks, but not through competition. This section shows that uniform staggering in sampling is robust to the presence of firms' interactions that go beyond the information externality. In fact, in the simple extension studied below the uniform sampling result depends *only* on the presence of the information externality.

This section also shows, however, that the optimal sampling interval and, not surprisingly, the optimal price response to other firms' price changes, do depend on the market structure assumed.

A very simple but sufficiently rich framework to discuss these issues is one in which two firms reveal all their information at sampling and do not collude in any sense (except for information sharing). This is the model used here.

The interaction between firms can be accounted for by a simple generalization of the instantaneous cost function (for firm 1[25]):

$$c_1(p_1(t), p_2(t), t) = \alpha\left(p_1(t) - p_1^*(t)\right)^2 + \beta\left(p_1(t) - p_2(t)\right)^2, \tag{17}$$

with $\alpha > 0, \beta \geq 0$ and $p_i^*(t)$ the same as before, although it no longer represents the full information price[26].

This model can be thought of as a local approximation of the profit function of monopolistically competitive firm[27]. It is apparent that when $\beta \to 0$ interactions beyond information externalities

[25] Firm 2's cost function is similar to firm's 1.

[26] In fact it represents the monopolist full information price.

[27] Certainly this assumption is not fully consistent with the fact that only two firms exist, however this is not important for the main arguments of this section. Furthermore, allowing for a "cross" term in the cost function yields a different interpretation of the coefficients obtained but it does not change the main insight of this section; the fact that uniform staggering is robust to some

disappear. Conversely, ad β rises, the importance of information relative to competition issues is reduced. What follows of this section studies the implications of this new term for the conclusions derived in the previous sections. The next section quantifies the impact of changes in β on optimal sampling periods.

PROPOSITION 12 *The optimal price of firm 1 at each time t is a convex combination of $\mathrm{E}_t[p_1^*(t)]$ and $p_2(t)$.*

PROOF In any given point in time firm 1 solves the problem:

$$C_1(t) = \min_{p_1(t)} \mathrm{E}_t[c_1(p_1(t), p_2(t), t)]. \tag{18}$$

The proposition follows trivially from the first order condition of this problem. Furthermore, the weight of $\mathrm{E}_t[p_1^*(t)]$ is ψ, with $\psi = \frac{\alpha}{\alpha+\beta}$, and that of $p_2(t)$ is $1-\psi$. †

The problem is symmetric so that the same arguments apply to firm 2.

Given that both firms have identical information sets (full revelation), it is straight forward to compute the equilibrium prices from the results of Proposition 12:

$$p_1(t) = \phi \mathrm{E}_t[p_1^*(t)] + (1-\phi)\mathrm{E}_t[p_2^*(t)] \tag{19}$$

and

$$p_2(t) = \phi \mathrm{E}_t[p_2^*(t)] + (1-\phi)\mathrm{E}_t[p_1^*(t)], \tag{20}$$

where $0.5 \leq \phi \equiv \frac{1}{2-\psi} \leq 1$.

Plugging (19) and (20) into the cost function (18), and taking expected value at time 0 yields:

$$\mathrm{E}_0\left[C_1(t)\right] = \begin{cases} \alpha\sigma^2 t + \alpha\phi(1-\phi)l(0) & 0 \leq t < \delta; \\ \alpha(\sigma^2 t - \sigma_\eta^2 \delta + \phi(1-\phi)\sigma_e^2(\tau_2 - \delta)) + \alpha\phi(1-\phi)l(0) & \delta \leq t < \tau_1. \end{cases}$$

where $l(0) \equiv (v_1(0) + e_1(0) - v_2(0) - e_2(\delta - \tau_2))^2$ and $\sigma^2 \equiv \sigma_z^2 + 2\phi(1-\phi)\sigma_v^2$. The same can be done for firm 2:

$$\mathrm{E}_\delta\left[C_2(t)\right] = \begin{cases} \alpha\sigma^2(t-\delta) + \alpha\phi(1-\phi)l(\delta) & \delta \leq t < \tau_1; \\ \alpha(\sigma^2(t-\delta) - \sigma_\eta^2(\tau_1 - \delta) + \phi(1-\phi)\sigma_e^2(\tau_1 - \delta)) + \alpha\phi(1-\phi)l(\delta) & \tau_1 \leq t < \tau_2 + \delta. \end{cases}$$

where $l(\delta) \equiv (v_2(\delta) + e_2(\delta) - v_1(\delta) - e_1(0))^2$.

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forms of non-informational competition.

The minimization problems of firms 1 and 2 can now be written as follows:

$$L_1(k) = \min_{\tau_1} \frac{1}{\tau_1}\left\{\frac{\alpha\sigma^2\tau_1^2}{2} - \alpha\sigma_\eta^2\delta(\tau_1-\delta) + \alpha\phi(1-\phi)l(0)\right\} \tag{21}$$

$$+\frac{1}{\tau_1}\left\{\alpha\phi(1-\phi)\sigma_e^2(\tau_2-\delta)(\tau_1-\delta)+k\right\},$$

and

$$L_2(k) = \min_{\tau_2,\delta} \frac{1}{\tau_2}\left\{\frac{\alpha\sigma^2\tau_2^2}{2} - \alpha\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1) + \alpha\phi(1-\phi)l(\delta)\right\} \tag{22}$$

$$+\frac{1}{\tau_2}\left\{\alpha\phi(1-\phi)\sigma_e^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)+k\right\}.$$

PROPOSITION 13 *a) Uniform sampling is robust to the presence of non-informational interactions, b) the sampling interval is decreasing in the degree of non-informational interaction between firms, and c) the sampling interval shrinks as the relative importance of the* observable *idiosyncratic shock rises.*

PROOF The first order conditions of (21) and (22) respect to τ_1, τ_2 and δ, respectively, are:

$$\frac{\sigma^2}{2} - \frac{\sigma_\eta^2\delta^2 + k - \phi(1-\phi)\sigma_e^2(\tau_2-\delta)\delta}{\tau_1^2} = 0, \tag{23}$$

$$\frac{\sigma^2}{2} - \frac{\sigma_\eta^2(\tau_1-\delta)(\tau_2+\delta-\tau_1) + k - \phi(1-\phi)\sigma_e^2(\tau_1-\delta)(\tau_2+\delta-\tau_1)}{\tau_2^2} = 0 \tag{24}$$

and

$$\sigma_\eta^2\left(\frac{2\delta+\tau_2-2\tau_1}{\tau_1}\right) - \phi(1-\phi)\sigma_e^2\left(\frac{2\delta+\tau_2-2\tau_1}{\tau_1}\right) = 0. \tag{25}$$

It is easy to corroborate that the solution $\tau_1 = \tau_2 = 2\delta$ satisfies condition (25). Furthermore, it also satisfies conditions (23) and (24) with a sampling interval equal to:

$$\tau = \sqrt{\frac{2k}{\alpha\sigma_z^2}\left(\frac{1}{\rho^{-1}+\phi(1-\phi)-\frac{\theta}{2}(1+\phi(1-\phi))}\right)},$$

where $\rho \equiv \frac{\sigma_z^2}{\sigma^2}$.

Part $b)$ of the proposition is proved by noting than an increase in the degree of firms interaction β, lowers ψ and ϕ. But τ is decreasing in $\phi(1-\phi)$ and the latter is decreasing in ϕ.

Part $c)$ follows from the fact that τ is increasing in ρ. †

Perhaps the most important result in Proposition 13 is that firms tend to stagger in their information gathering independently of the degree of competition among them. The latter only affects the optimal response of one firm to other firms signals (Proposition 12).

Proposition 13 also shows that as competition increases firms tend to sample more often. This can be understood more easily by replacing (19) in (17). Then the first term of (17) (identical to the monopolist case studied before) is:

$$\alpha\left((\mathrm{E}_t[p_1^*(t)] - p_1^*(t)) + (1-\phi)(\mathrm{E}_t[p_2^*(t)] - \mathrm{E}_t[p_1^*(t)])\right)^2 . \tag{26}$$

Given the full revelation assumption, the two terms in (26) are orthogonal, therefore the variance of this sum is increasing in $(1-\phi)$, the index of competition[28]. Therefore, for any given σ_z, an increase in $(1-\phi)$ yields a higher expected cost of waiting to sample, hence reducing the optimal sampling period. The same argument justifies the fact that the sampling interval shrinks with increments in the variance of the *observed* idiosyncratic shock –a variable that does not play any role in the decoupled case– since the second term in (26) is a function of both idiosyncratic shocks.

The next section quantifies these effects.

7. EXAMPLES

This section presents examples to illustrate some of the results derived in the theoretical part of the paper. In order to facilitate comparisons, the units of time are normalized by the size of τ in the single firm case. As a reference, if the expected cost of not sampling for a whole year is twice as large as k, the cost of sampling, the firm will sample once a year. Figures 1 to 9 refer to the case in which firms are decoupled.

Figure 1 shows the value of τ (relative to the single firm case) for different number of firms and relative importance of the unobservable common shock ($\eta(t)$), when there is full revelation of the different components inducing firm's price changes (after sampling). Hence the $\theta = 0.5$ line shows that when there are 50 firms the length of time between sampling (for each firm) is about 30% longer than in the single firm. The first feature to notice is that when the relative importance of the common shock is small ($\theta = 0.2$), the length of the optimal sampling interval is only weakly affected by the number of firms. In fact, in this case the size of the information externality is small, hence it is too "risky" for firms to spend too much time without sampling since their own

[28] An alternative interpretation is to notice that when the firm is not expected to be at the optimal monopoly price, the profit function is steeper, hence equally (respect to the monopoly case) important unobserved changes in z lead to much larger changes in profits (or costs).

idiosyncratic shocks may be too out of line. Conversely, when θ is large, the information externality is important[29], hence each firm can rely heavily on other firms' sampling. The figure shows, for example, that when $\theta = 0.8$ and n is large, the sampling interval of each firm can easily exceed twice that of the single firm case.

Figure 2 illustrates how the time interval at which sampling occurs throughout the economy, δ, varies with the number of firms and the relative importance of common shocks, θ. This figure shows that the overall sampling time is decreasing in the number of firms, for any value of θ. In the full revelation case this implies that aggregate shocks become more neutral as the number of firms increases, since it is enough with one firm uncovering the aggregate shock for the rest to find out.

Figures 3 and 4 are the analogue of figures 1 and 2 but in the case in which signal extraction problems are added. Figure 3 shows the same patterns of figure 1 but now τ increases more slowly with the number of firms, especially when θ is small. When signal extraction issues are involved, not only sampling firms do not reveal information about their own idiosyncratic shock (as in the full revelation case) but also the signal about the common shock comes scrambled with other firms' idiosyncratic shocks. This "scrambling" is decreasing in θ, therefore an increase in the relative importance of the common shock has the double effect of improving the quality of the signal, as well as reducing the relative importance of firm specific shocks. Figure 4, on the other hand, shows the effect of n on δ_1. Even though the figure is similar to figure 2, the implications are different since firms do not fully observe the common shock until they sample. The staggering interval, δ_1, is important to determine the smoothness of the path as well as the time interval until firms first react to the common shock, but it does not determine when the economy fully adjusts to the shock. This is better depicted in the next three figures.

Figure 5a shows that the percentage of the adjustment completed (y-axis) after different periods of time (x-axis) for the case in which an unobservable aggregate shock occurs and $\theta = 0.2$. Given that δ_1 is decreasing in n, it takes longer for firms to react to the common shock as n falls, although the full adjustment is completed earlier as indicated by figure 4 (smaller τ). It can also be seen in figure 5a that as the number of firms increases the aggregate price path becomes smoother and better informed (closer to one) for most of the transition.

Figure 5b shows the path of the price of a firm that sampled just before the aggregate shock occurs. It is apparent that the firm does not wait until its next sampling period to change its price.

[29] In the sense that firm's idiosyncratic shocks are not the main contributors to expected cost. Also remember that in the full revelation cost there is, by definition, no signal extraction problem respect to other firms price changes.

For example, the figure shows that even when the importance of common shocks, θ, is small, more than 90% of the adjustment to the aggregate shock is completed before the firm actually samples. This is an important difference with recent version of time dependent models in which prices are assumed to be fixed between sampling periods (e.g. Ball and Cecchetti 1988).

Figures 6 and 7 are analogous to figure 5a but for the cases in which θ is equal to 0.5 and 0.8, respectively. Given that the quality of each signal improves as θ rises, it takes longer (than when $\theta = 0.2$) for the economy to react, but when the first firm reacts, the rest reponds more strongly since it is more likely that the sampling firm's price change reflects a common shock. After the large initial response, however, the rest of the adjustment is implemented more slowly (as compared with the $\theta = 0.2$ case) as less firms per unit of time sample (δ_1 larger). Furthemore, as the complete adjustment requires all firms sampling, and that τ is increasing in θ, the complete adjustment is slower in the case in which θ is large.

These figures bring to light an additional issue. If a sampling firm changes its price due to an idiosyncratic shock, then other firms will overreact to it as they will confuse it with a common shock. This confusion tends to disappear slowly as new firms sample and find out the true $\eta(t)$, generating the right signal to the firms that have not yet sampled. Overall, however, the signal extraction problem will induce large amounts of spurious fluctuations in the aggregate price level. This is illustrated in Figures 8 and 9. There it is possible to see the response of the aggregate price level after the firm sampling at time 0 suffers a unitary idiosyncratic shock (before sampling). As the price change has a signal component to other firms, the initial response of the aggregate price level overshoots (i.e. responds by more than the share of the sampling firm suffering from the idiosyncratic shock). Not surprisingly, the overshooting is larger the larger is the relative importance of aggregate shocks. Although it is also true that firms will undo their mistakes faster when θ is large.

Finally, Figure 10 illustrates the fact that when firms are related by factors beyond pure information, sampling periods tend to be shorter. This effect is more important as the ratio of the variance of observable idiosyncratic shocks to the variance of non-observable shocks, φ, rises.

8. CONCLUSION

Economist are used to thinking in terms of time dependent rules. Furthermore, these type of rules are deeply rooted in most of the traditional explanations of the effects of monetary shocks on the real sectors of the economy. In spite of this, most of the recent microeconomic rationale for price rigidities have been given in terms of models in which state but not time dependent rules are optimal. Very likely, both types of rules have an element of truth in explaining pricing and wage setting behavior. This paper has tried to isolate the implications of time dependent rules, in a framework in which these rules are truly optimal.

The paper has presented a model in which costs of gathering information about the optimal price makes sampling at fixed intervals optimal for firms. Furthermore, when one firm's gathering of information reveals some information to other firms, an information externality appears. This externality leads each firm to sample less frequently, and to uniform staggering of sampling. However, the fact that *sampling* is uniformly staggered does not imply that price changes are uniformly staggered. In fact, prices change every time information arrives. Uniform staggering guarantees that information is received more frequently than the sampling frequency, therefore prices are changed more frequently than the sampling interval[30].

In this framework, the typical response of the aggregate price level to an unobservable aggregate shock conveys a period –inversely related to the number of firms and the lack of information externalities– in which no firm adjusts its price, followed by the full adjustment of the sampling firm and the signal extraction response of the rest. The combination of both leads to an initial jump in the price level that is increasing in the importance of the information externality contained by *each* signal. Afterwards the price level changes smoothly and slowly until the completion of the adjustment. The smoothness and slowness of the after-jump price adjustment is increasing in the size of the *overall* information externality.

The application of this model go beyond pricing problems. In fact it is more the rule rather than the exception that state variables are not costless to observe. The next step in this research agenda is to extend this model to the case in which there is also a fixed cost of controlling the state variable. As mentioned before, this is likely to yield a state dependent rule built into a time dependent rule in which the sampling intervals are a function of past observations. Among other things, a model like this can complement the realism of the microeconomic lumpy adjustment S-s model with the observed macroeconomic slowness of new durables purchases (Caballero 1988).

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30 One possible solution is to add a direct cost of changing prices. This, however, is likely to yield a complex state dependent rule built in a time dependent rule in which sampling intervals depend on previous observations.

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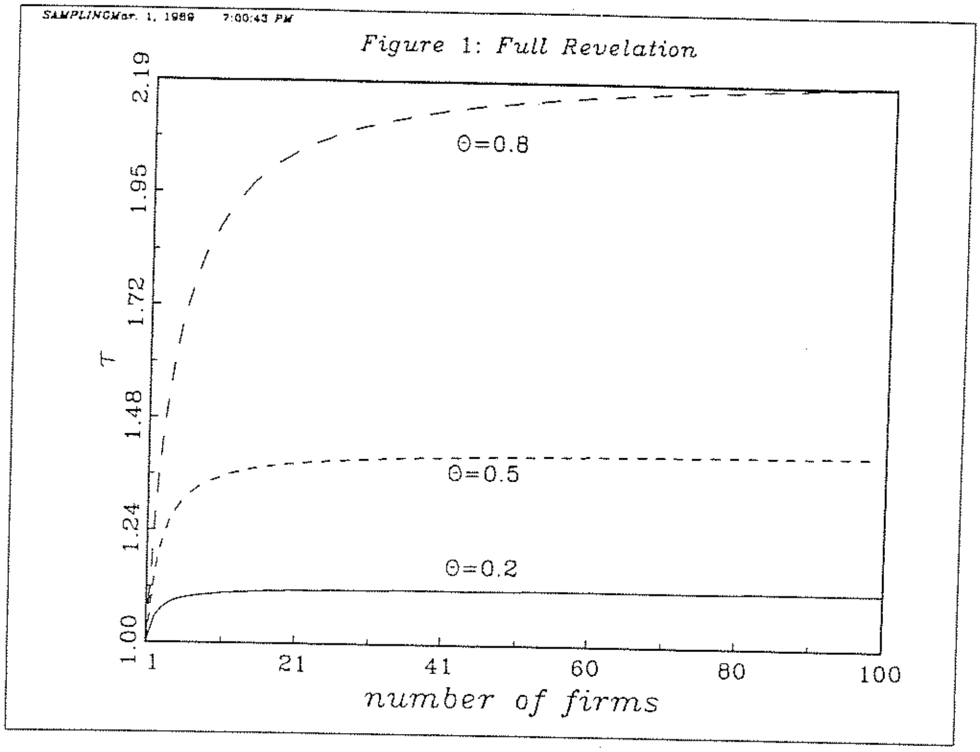
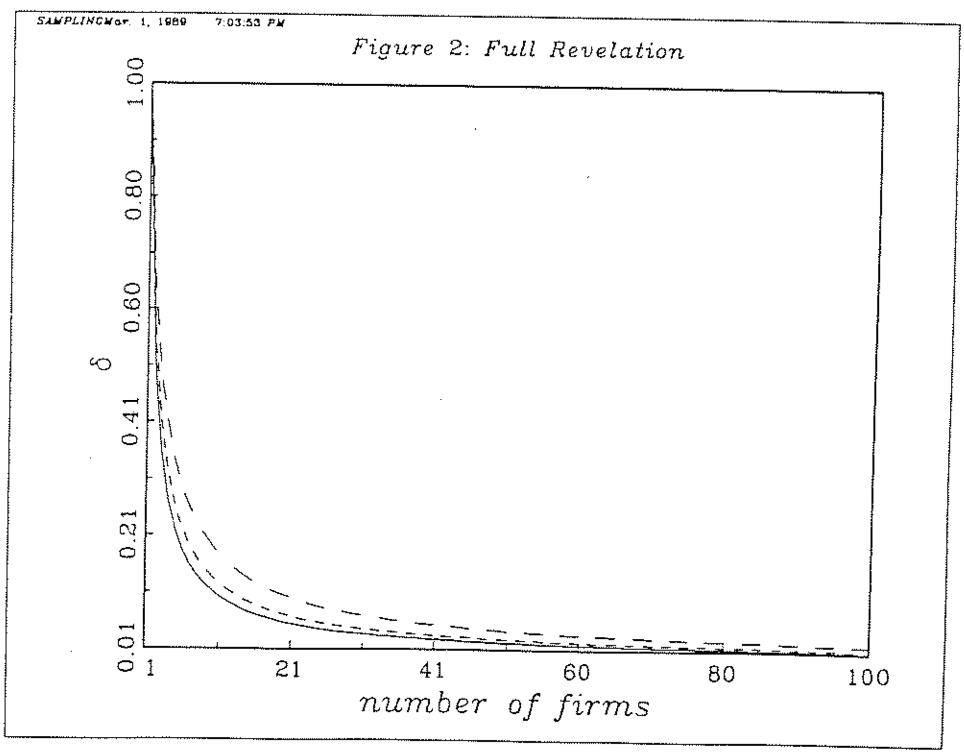


SAMPLINGMar. 1, 1989 7:00:43 PM
Figure 1: Full Revelation
2.19
1.95
1.72
1.48
1.24
1.00
τ
Θ=0.8
Θ=0.5
Θ=0.2
1
21
41
60
80
100
number of firms



SAMPLINGMar. 1, 1989 7:03:53 PM
Figure 2: Full Revelation
1.00
0.80
0.60
0.41
0.21
0.01
δ
1
21
41
60
80
100
number of firms

Figure 3: Signal Extraction

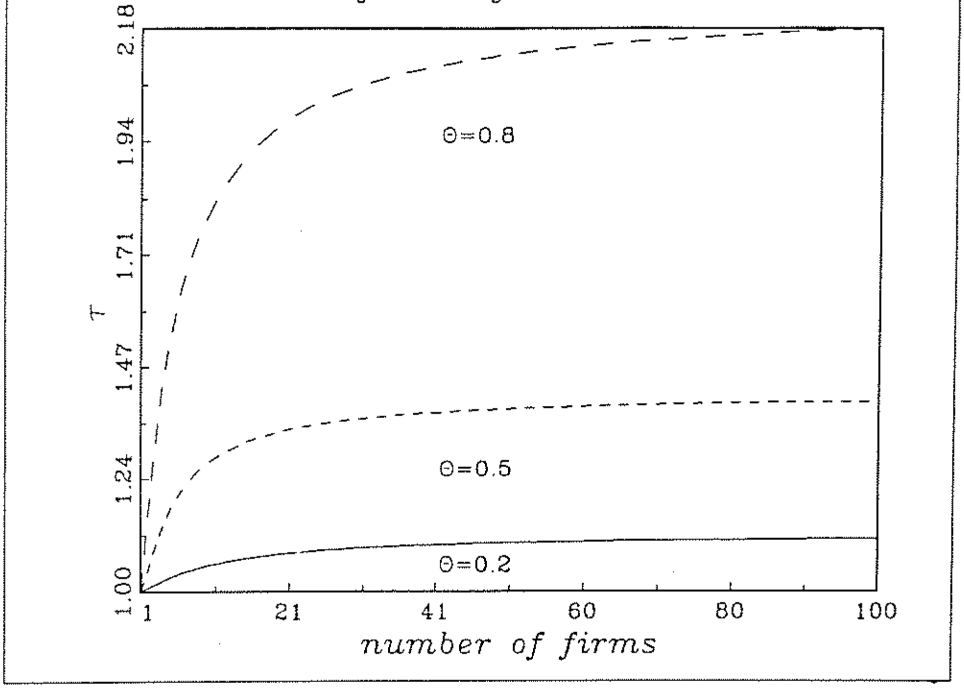
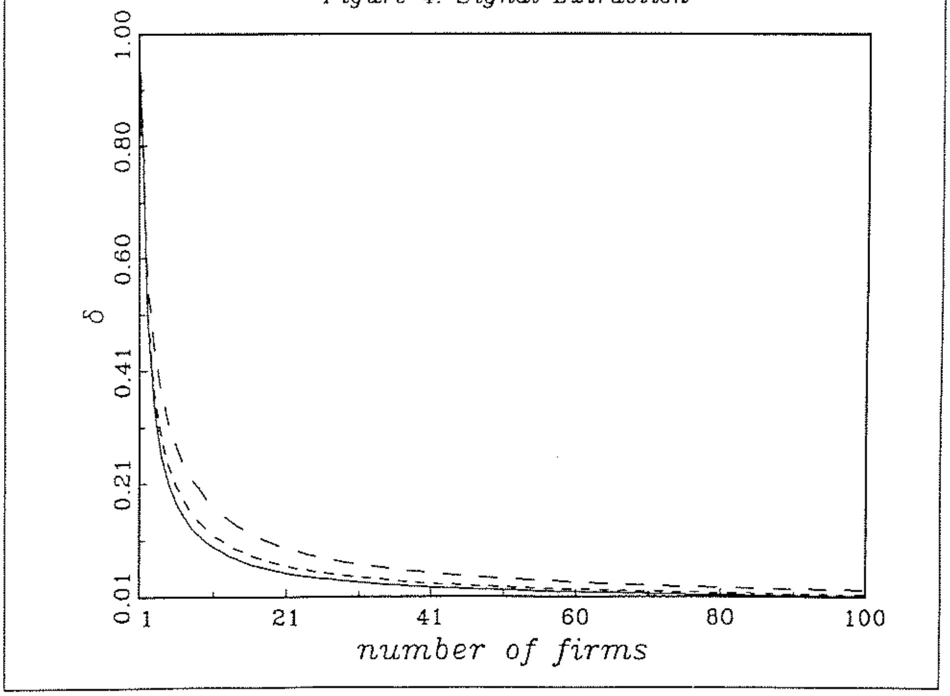


Figure 4: Signal Extraction



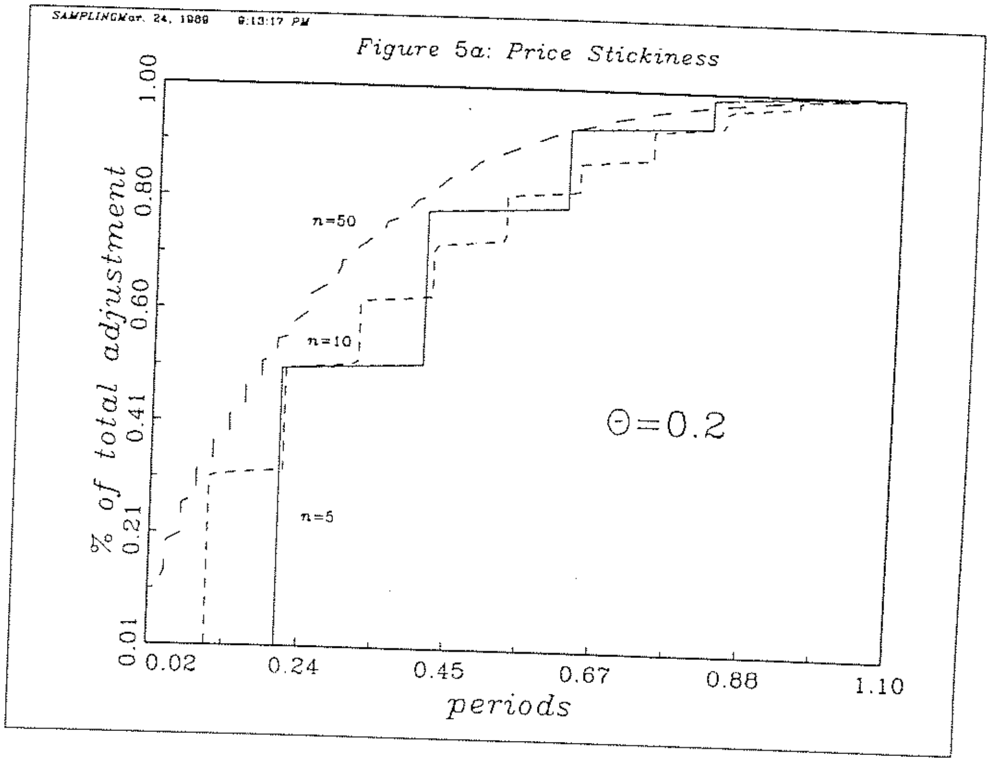


Figure 5a: Price Stickiness

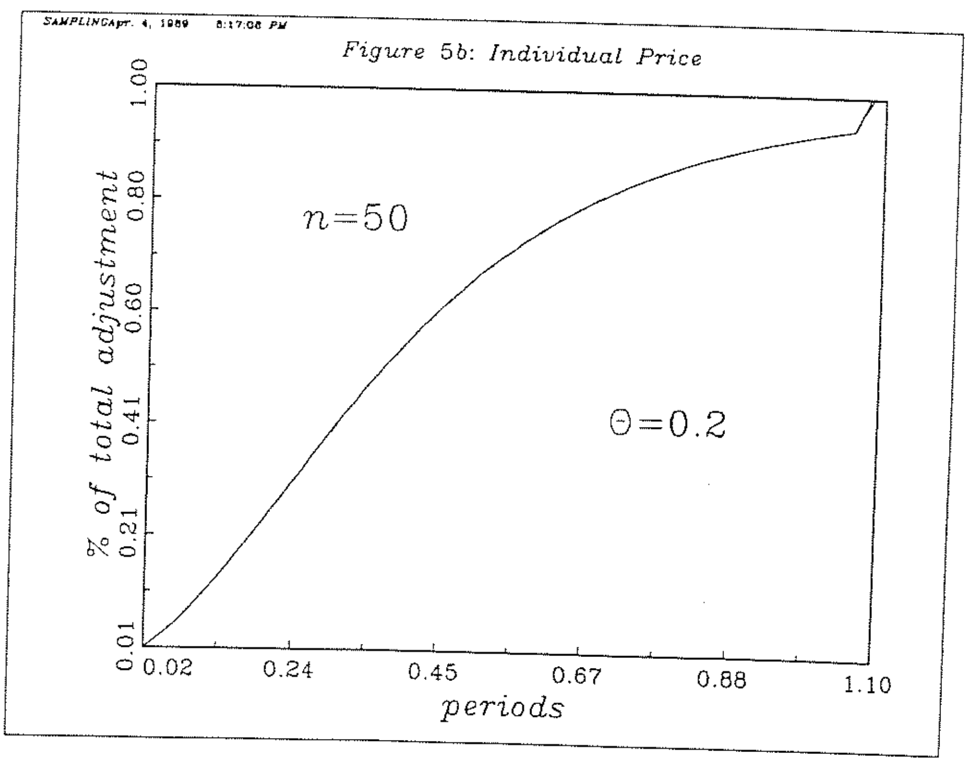


Figure 5b: Individual Price

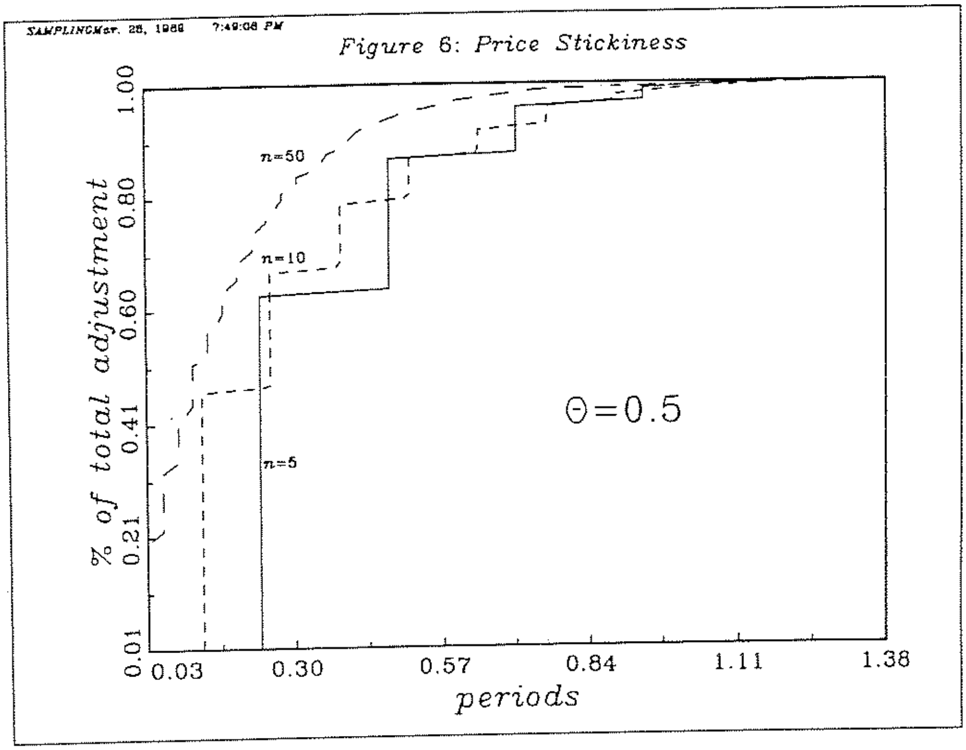


Figure 6: Price Stickiness
1.00
0.80
0.60
0.41
0.21
0.01
% of total adjustment
n=50
n=10
n=5
Θ=0.5
0.03
0.30
0.57
0.84
1.11
1.38
periods

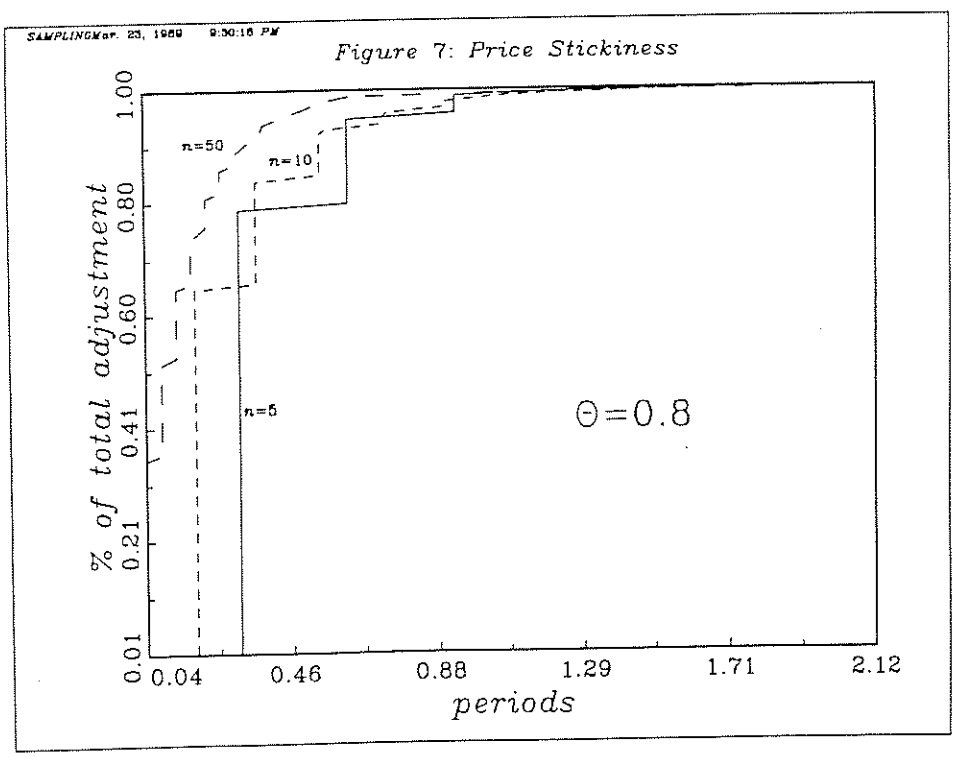


Figure 7: Price Stickiness
1.00
0.80
0.60
0.41
0.21
0.01
% of total adjustment
n=50
n=10
n=5
Θ=0.8
0.04
0.46
0.88
1.29
1.71
2.12
periods

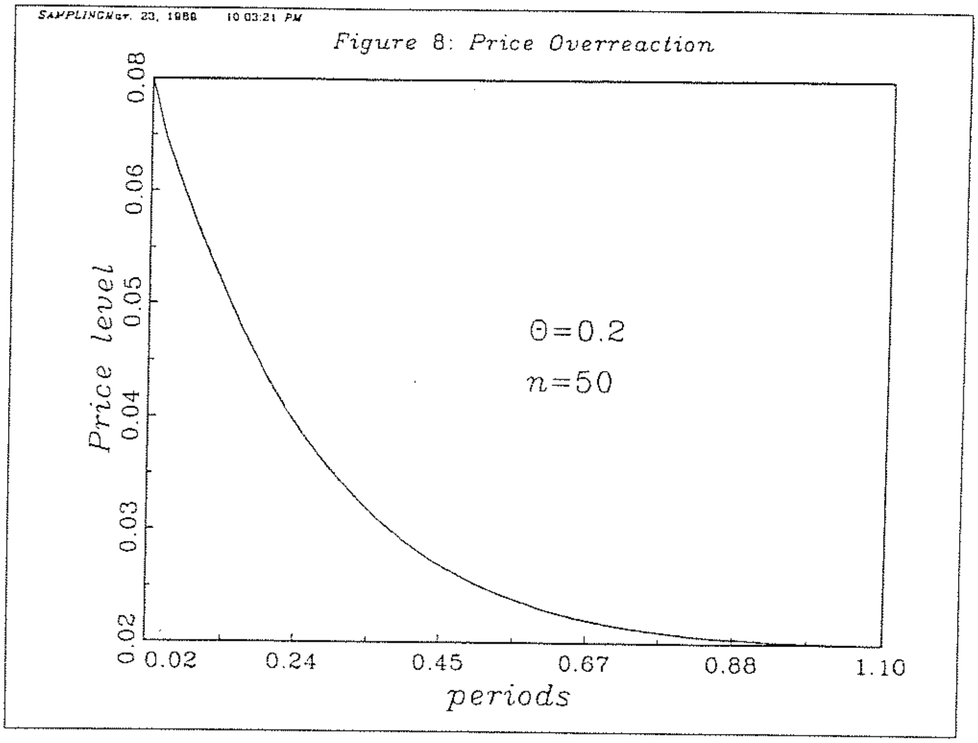

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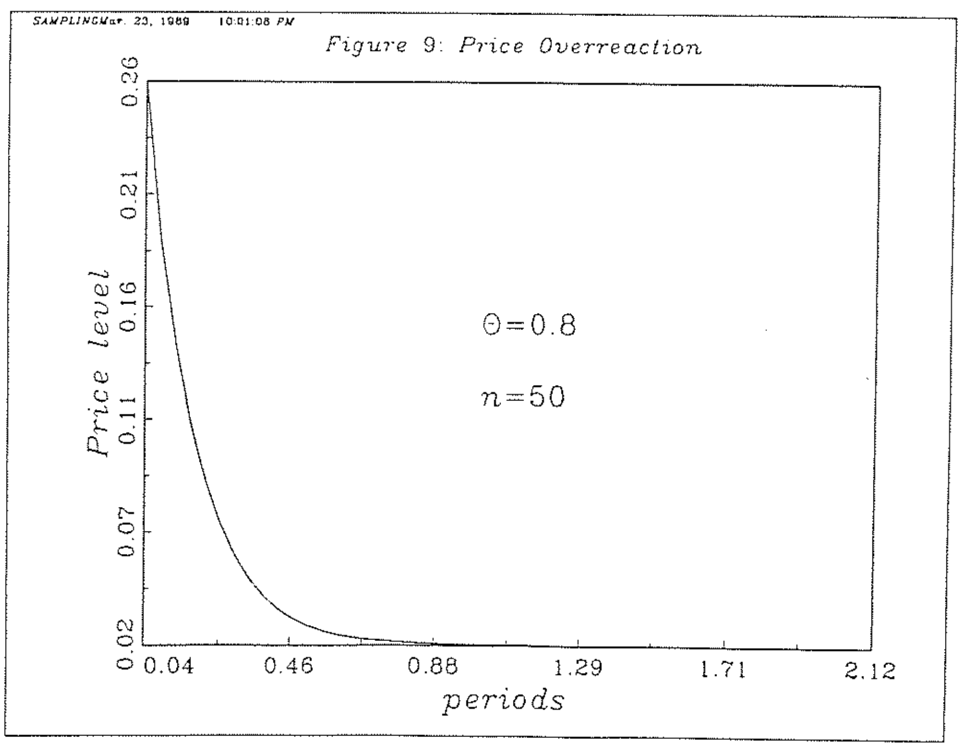
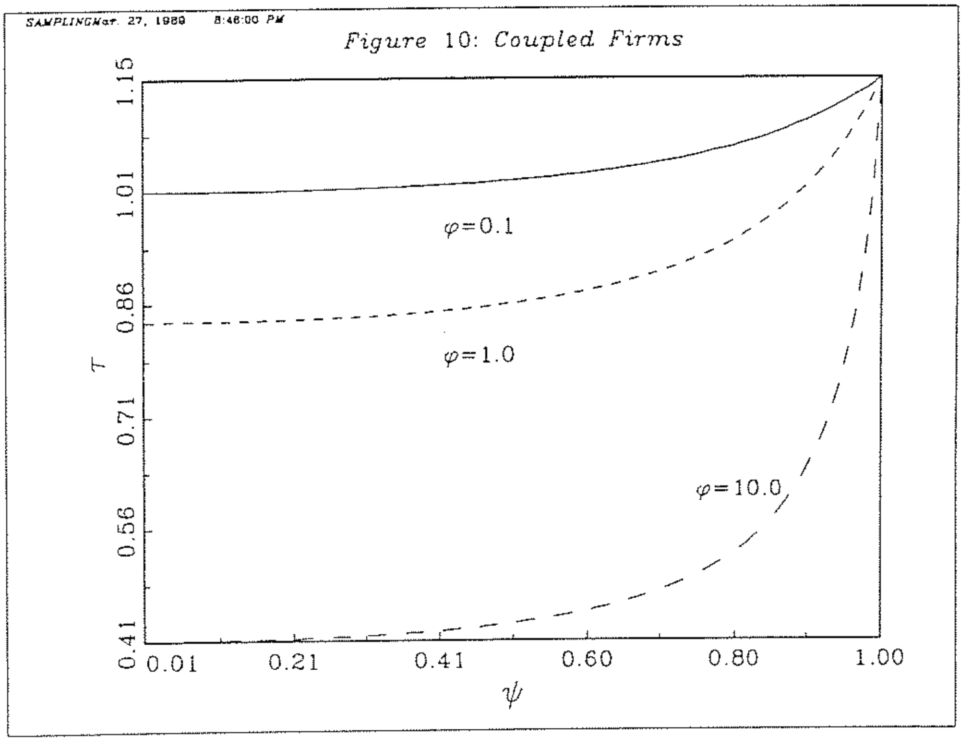
Figure 8: Price Overreaction
Price level
0.08
0.06
0.05
0.04
0.03
0.02
Θ=0.2
n=50
0.02
0.24
0.45
0.67
0.88
1.10
periods


SAMPLINGMar. 23, 1989 10:01:08 PM
Figure 9: Price Overreaction
Price level
0.26
0.21
0.16
0.11
0.07
0.02
Θ=0.8
n=50
0.04
0.46
0.88
1.29
1.71
2.12
periods



SAMPLING Mar. 27, 1989 8:46:00 PM
Figure 10: Coupled Firms
1.15
1.01
0.86
0.71
0.56
0.41
τ
φ=0.1
φ=1.0
φ=10.0
0.01
0.21
0.41
0.60
0.80
1.00
ψ

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