The Economics of Labor Coercion: Corrigendum

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The claim in Lemma 2 that $u^l \leq 0$ in any equilibrium contract with $a > 0$ is incorrect as stated.[1] Conceptually, the lemma overlooks the possibility that the limited liability constraint $p^l \geq 0$ may be slack at the optimum. If this is the case, then the moral hazard problem is not binding, the principal can implement the first-best level of effort, and the strict comparative statics results in the paper (e.g., Proposition 2) will not apply, as discussed on p.569 of the text. Thus, for the results in the paper to be valid, we must assume that the limited liability constraint binds. A simple sufficient condition for this to be the case is the following:

Asssumption (Binding Limited Liability) $\bar{u} - g^{FB} - a^{FB} c' \left(a^{FB}\right) + c\left(a^{FB}\right) \leq 0$, where a^{FB} and g^{FB} are given by $Px = c'\left(a^{FB}\right)$ and $1 = \eta \chi' \left(g^{FB}\right)$, respectively.

This assumption will always be valid if using coercion is sufficiently easy for the principal (i.e., if η is sufficiently small), since $g^{FB} \to \infty$ as $\eta \to 0$.

It is straightforward to give a valid proof of Lemma 2 under binding limited liability.

Lemma 2 In any equilibrium contract with $a > 0$, we have $u^l \leq 0$ and $u^h \geq 0$.

Proof. Equilibrium contracts with $a > 0$ are solutions to the principal's problem:

$$\max_{\left(a,g,u^h,u^l\right) \in [0,1] \times \mathbb{R}_+ \times \mathbb{R}^2} a\left(Px - \left[u^h\right]_+\right) - (1-a)\left[u^l\right]_+ - \eta\chi(g) \tag{A-1}$$

[1] The proof in the published version contains several errors, including a simple algebra mistake. We thank Luca Braghieri for bringing these to our attention.

subject to

$$au^h + (1-a)\,u^l - c\,(a) \geq \bar{u} - g \tag{IR$_1$}$$

and

$$u^h - u^l = c'\,(a)\,, \tag{IC$_1$}$$

where the condition $a > 0$ and the Inada condition $\lim_{a \to 1} c\,(a) = \infty$ have let us replace incentive compatibility ((IC$_0$) in the text) with the corresponding first-order condition (IC$_1$).

First, suppose toward a contradiction that $u^l > 0$ at a solution. Then (IR$_1$) must bind, as otherwise reducing u^h and u^l by the same constant would yield an improvement. Using (IR$_1$) and (IC$_1$) to substitute for u^h and u^l in the objective yields the unconstrained problem

$$\max_{(a,g)\in[0,1]\times\mathbb{R}_+} Pxa - a\left[(1-a)\,c'\,(a) + c\,(a) + \bar{u} - g\right]_+ - (1-a)\left[-ac'\,(a) + c\,(a) + \bar{u} - g\right]_+ - \eta\chi\,(g)\,.$$

Both terms in brackets are positive, as the latter term is u^l (assumed to be positive) and the former term is $u^h = u^l + c'\,(a) > u^l$. So the objective equals

$$\max_{(a,g)\in[0,1]\times\mathbb{R}_+} Pxa - c\,(a) - \bar{u} + g - \eta\chi\,(g)\,,$$

and the unique solution is

$$\begin{aligned} a &= a^{FB}, \\ g &= g^{FB}. \end{aligned}$$

This yields

$$u^l = \bar{u} - g^{FB} - a^{FB}c'\left(a^{FB}\right) + c\left(a^{FB}\right).$$

But this is non-positive under binding limited liability, a contradiction. Hence, $u^l \leq 0$ at any solution.

Next, suppose toward a contradiction that $u^h < 0$ at a solution. Using (IC$_1$) to substitute for u^l, any optimal choice of $\left(a, g, u^h\right)$ must solve the subproblem

$$\max_{\left(a,g,u^h\right)\in[0,1]\times\mathbb{R}_+\times\mathbb{R}} a\left(Px - \left[u^h\right]_+\right) - (1-a)\left[u^h - c'\,(a)\right]_+ - \eta\chi\,(g)$$

subject to

$$u^h - (1-a)\,c'\,(a) - c\,(a) \geq \bar{u} - g. \tag{IR$_2$}$$

As $\lim_{a\to 1} c(a) = \infty$, any optimal choice of a is less than 1, so it is possible to increase u^h and a such that u^h remains negative and (IR_2) continues to hold. Such a modification improves the contract, giving a contradiction. So $u^h \geq 0$ at any solution. ■