Public Debt as Private Liquidity: Optimal Policy

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10/8/2016

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The Main Issue

- **Financial frictions** $\Longrightarrow$ public debt can be non-neutral
- **Public debt as collateral/buffer stock/outside liquidity** $\Longrightarrow$ alleviate frictions
  - Woodford, 1990, Aiyagari-McGrattan, 1998, Holmström-Tirole, 1998.

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- **Relevant policy implications** in the aftermath of the Great Recession
  - **Mitigate a financial crisis:** Level vs portfolio composition
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  - **Relax the ZLB constraint on monetary policy** (Eggertson and Krugman, 2011, Guerrieri and Lorenzoni, 2011)
- **Similarities but also differences** with Friedman rule literature (Chari, Christiano and Kehoe, 1996, Correia and Teles, 1999)

What we do

Missing: Theoretical study of **optimal fiscal policy** is a Ramsey setting in which

- **public debt is non-neutral**, because it influences the virulence of financial frictions
- but **does not generate a free lunch** for the government, because taxation is distortionary

Contribution of this paper: Fill the gap, offer new lessons for

- optimal long-run quantity of public debt
- desirability of tax smoothing
- optimal policy response to shocks (including financial crises)

How do we do it?

We characterize optimal provision of debt in 3 steps:

1. **Setup a micro-founded Ramsey policy problem** (as in Barro, 1979, Lucas and Stokey, 1983, and Aiyagari et al., 2002) that allows public debt to affect the bite of a financial friction (as in Woodford, 1990 and Holmström-Tirole, 1998).

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2. Obtain a **convenient reduced-form representation** of the planners problem in terms of a standard optimal control problem over the rate of taxation and the level of debt.

3. Characterize **analytically** the solution to a class of reduced-form problems that nests the one obtained from our model.

Parenthesis: Absent Financial Frictions

- Without financial friction, the model reduces to a deterministic version of Barro and AMSS
- Optimal policy satisfies
  - ✓ **Tax smoothing:** the tax rate (the shadow value of tax revenue) is equated across periods;
  - ✓ **Steady-state indeterminacy:** *any* sustainable level of debt is consistent with steady state.

Main Findings: Deterministic

- When the friction is present, a **tension** emerges between
  - (i) Easing the friction so as to improve market efficiency/allocation of resources
  - (ii) Exacerbating the friction so as to raise premia and reduce the government's cost of borrowing

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  - more generally: possibly multiple steady states, but each one is locally-determinate

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- Depending on primitives, **2 scenarios** can emerge
  1. Financial distortion vanishes as $t \to \infty \implies$ Friedman rule applies in LR but not SR
  2. Financial distortion preserved as $t \to \infty \implies$ **Friedman rule never applies**

Main Findings: Stochastic

- **Mean reversion**
- Optimal policy response to **“wars”**: less persistent and less volatile
- Optimal policy response to **“financial recessions”**:
  - aforementioned tension $\Rightarrow$ ambiguous effect on planner’s incentives
  - response driven by fiscal considerations, not apparent desire to ease the aggravated friction
  - a financial crisis presents an opportunity for **“cheap borrowing”**
  - ultimately: **optimal deficit is larger** than in a comparable traditional recession.

The Baseline Model

Micro-Founded Model

- **Similar to Barro (1979), AMSS (2002), Lucas-Stokey (1983):**
  - Infinitely lived agents,
  - competitive markets and flexible prices,
  - Government issues debt and collect taxes by distorting labor supply decisions only.

- **Financial friction as in Kiyotaki-Moore (1997) or Holmström-Tirole (1998):**
  - Agents are hit by idiosyncratic shocks $\Longrightarrow$ reallocation of goods across agents.
  - The reallocation requires borrowing, borrowing requires collateral.
  - Private supply of collateral is limited as so is the pledgeable income of the private sector.

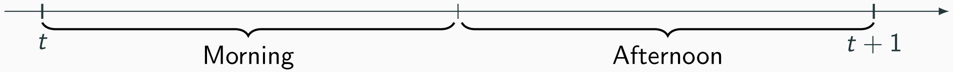
$\Longrightarrow$ **Public debt can serve as collateral and alleviate financial frictions.**

Micro-Founded Model

- Helps clarify
  - role of liquidity: risk sharing
  - why debt matters: lack of pledgeable income
- Gives a language to talk about collateral and buffer stock
- Can accommodate effects such as
  - fire sales externalities,
  - inside/outside money,
  - crowding out of capital

- **But Importantly:** show that although each of these effects is relevant in its own right, none is key for our results.

Micro-Founded Model: Timing



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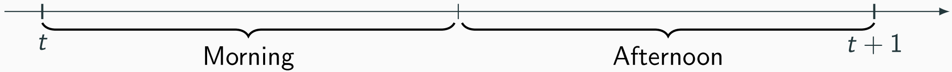
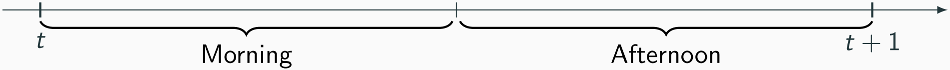
- Endowment (e) + assets (a_{it})
- Taste Shock (θ_{it})
- Decide consumption (x_{it}) and IOU issuance (z_{it})



Micro-Founded Model: Timing

- Endowment (e) + assets (a_{it})
- Taste Shock (θ_{it})
- Consumption (x_{it}) and IOU issuance (z_{it})

- Decide labor supply (h_{it})
- Receive labor income ($(1\text{-}\tau_t)w_t h_{it}$)
- Decide savings (a_{it+1}) and consumption (c_{it})
- IOU repayment



Micro-Founded Model

- Households:

$$\begin{aligned} \max \quad & \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(c_{it} + \theta_{it} u(x_{it}) - \nu(h_{it}) \right) \right] \\ s.t. \quad & c_{it} + p_t x_{it} + q_t a_{it+1} = a_{it} + (1 - \tau_t) w_t h_{it} + p_t \bar{e} \\ & p_t (x_{it} - \bar{e}) \leq \xi w_t h_{it}^{def} + a_{it} \\ & - a_{it+1} \leq \xi w_{t+1} h_{it+1}^{def} \end{aligned}$$

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- Firms: $y_t = A h_t$
- Government: $q_t b_{t+1} + \tau_t w_t h_t = b_t + g$
- Market clearing: $y_t = c_t + g, \quad \int x_t(\theta) \mathrm{d}\mu(\theta) = \bar{e}, \quad \int a_{t+1}(\theta) \mathrm{d}\mu(\theta) = b_{t+1}.$

A Convenient Reduced-Form Ramsey Problem

Proposition

The optimal policy path for the tax rate and the level of public debt solves:

$$\max_{\{s_t, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[U(s_t) + V\left(b_t\right)\right] \quad \textit{subject to} \quad Q(b_{t+1})b_{t+1} = b_t + g - s_t$$

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2. **$V(b)$:** Value of cross-sectional allocation of asset holdings and morning-good consumption for a given level of public debt, b, with $V'(\cdot) > 0$.

▸ Program

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3. $Q(b)$**:** Price of a bond that supports this allocation, $Q'(\cdot) < 0$.
($\pi(b) \equiv Q(b) - \beta$ is associated liquidity premium)

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4. b_{bliss}: There exists b_{bliss} s.t. $\forall b \geqslant b_{bliss}$, $V'(b) = 0$ and $Q(b) = \beta$ $(\pi(b) = 0)$

An (even more) Convenient Reduced Form Representation

- The planner chooses a path for $(s, b) \in (0, \bar{s}) \times [\underline{b}, \bar{b}]$ that solves

$$\begin{aligned} \max \quad & \int_0^{+\infty} e^{-\rho t}[U(s) + V(b)]\mathrm{d}t \\ \text{s.t.} \quad & \dot{b} = R(b)b + g - s \\ & b(0) = b_0 \end{aligned}$$

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- **Dual role of public debt** (as in Woodford, Aiyagari-McGrattan or Holmström-Tirole):

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- **Non convex problem** due to pecuniary externality.

- **Key:** Dependence of V and R (id. π) on b, not the exact reason of this dependence.

Assumptions

Main Assumptions

We consider economies in which the following properties hold:

A1. *U, V, and π are continuously differentiable.*[1]

A2. *U is concave in s, with maximum attained at $s = 0$.*

A3. There exists a threshold $b_{bliss} \in (0, \bar{b})$ such that $V'(b) > 0$ and $\pi(b) > 0$ for all $b < b_{bliss}$, and $V'(b) = 0$ and $\pi(b) = 0$ for all $b > b_{bliss}$.

A4. *$\pi(b) \leq \rho$ for all b.*

[1] To be precise, we allow V and π to be non-differentiable at $b = b_{bliss}$.

Characterizing Optimal Debt Provision

Necessary Conditions for Optimality

- Set of necessary conditions

$$\begin{cases} \dot{\lambda} = V'(b) - \lambda\pi(b)\left(\sigma(b) - 1\right) \\ \dot{b} = g + \left(\rho - \pi(b)\right) b - s(\lambda) \end{cases}$$

$+$ transversality condition: $\lim_{t\to\infty} e^{-\tau t}\lambda(t)b(t) = 0.$

Euler Equation: $\dot{\lambda} = V'(b) - \lambda\pi(b)\left(\sigma(b) - 1\right)$

- Think of a static problem: max social value+seigniorage revenue for a given λ

$$\max_{b} \Omega(b, \lambda) \equiv V(b) + \lambda\pi(b)b$$

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$$\underbrace{V'(b)}_{\substack{\text{Marginal social gain from easing the financial friction} \\ \textbf{(Allocation Efficiency)}}} - \underbrace{\lambda\pi(b)\left(\sigma(b) - 1\right)}_{\substack{\text{Marginal cost in terms of seigniorage revenue} \\ \textbf{(Interest Rate Manipulation)}}} = 0$$

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- **Tempting:** interpret the above condition as the optimal steady state debt provision decision $\Longrightarrow$ **Misleading!**
- Tax Smoothing acts as an adjustment cost.

Optimality Conditions

- Set of necessary conditions

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+ transversality condition: $\lim_{t\to\infty} e^{-\tau t}\lambda(t)b(t) = 0.$

- **Non-convex problem** $\Longrightarrow$ **not sufficient** $\Longrightarrow$ **Skiba (1978), Brock-Dechert (1983)**

- Study the global dynamics.

A Useful Benchmark

Benchmark

(i) *the elasticity* σ *is monotone;*

(ii) *the ratio* V'/π *is constant.*

A Useful Benchmark

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(i) *the elasticity* σ *is monotone;*

(ii) *the ratio* V'/π *is constant.*

(i) Guarantees that $\pi(b)b$ is single-peaked (Laffer curve for seigniorage);

(ii) Any social gain following an increase in liquidity is exactly compensated by an increase in borrowing cost.

A Useful Benchmark

Result

Under the previous assumptions, there exists a unique pair $(b^\star, \lambda^\star)$*, such that for any* $b_0 \leqslant b_{bliss}$ *the economy converges to* $(b^\star, \lambda^\star)$.

A Useful Benchmark

Result

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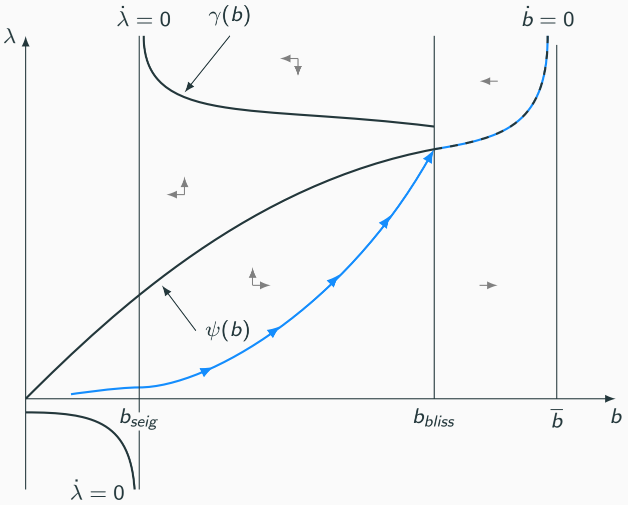
Result

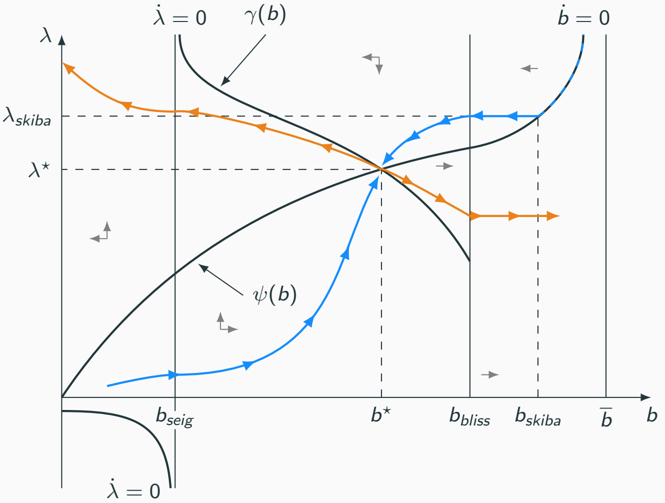
There exists a threshold level, $\widehat{g}$, for government spending such that

- *For any $g \leqslant \widehat{g}$, $b^\star = b_{bliss}$: An analogue to the Friedman rule holds in the long run;*
- *For any $g > \widehat{g}$, $b^\star < b_{bliss}$: it is optimal for the government to squeeze liquidity to increase seigniorage revenue to finance expenditures.*

Phase Diagram: Benchmark I: $g \leqslant \widehat{g}$.



Phase Diagram: Benchmark II: $g > \widehat{g}$.

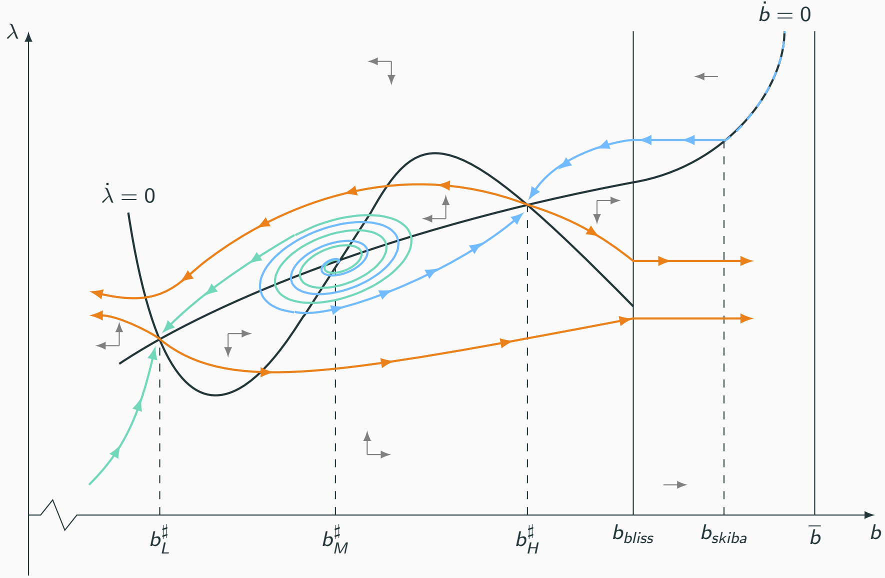


Useful Lemma

Beyond the Benchmark

- Relax monotonicity of σ and the invariance of $V'(b)/\pi(b)$ to b.
- Things get more intricate.
- Many situations can occur $\Longrightarrow$ Only illustrate a typical one

Beyond the Benchmark: A Typical Situation



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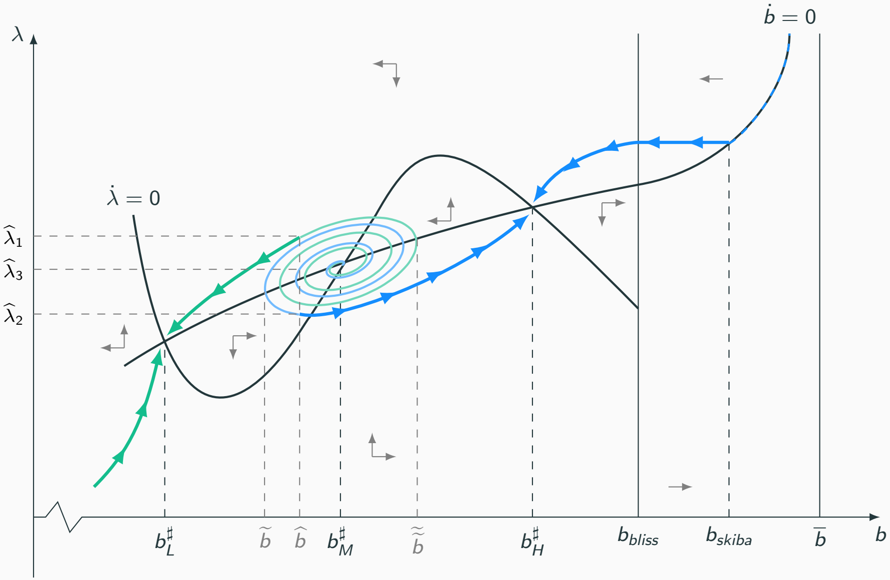


Optimal Policy

Theorem

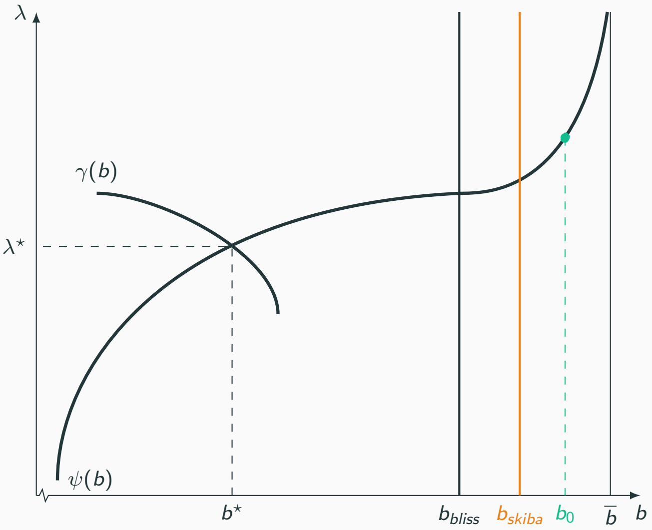
Let $B^{\#} \equiv \{b \in (\underline{b}, b_{bliss}] : \gamma(b) = \psi(b)$ *and* $\gamma'(b) \leq \psi'(b)\}$ *be the set of the points at which* γ *intersects* ψ *from above. In every economy, there exists a threshold* $b_{skiba} \in [\underline{b}, \overline{b}]$ *and a set* $B^* \subseteq B^{\#}$ *such that the following are true along the optimal policy:*

(i) *If either* $b_0 \in B^*$ *or* $b_0 > \max\{b_{bliss}, b_{skiba}\}$, *debt stays constant at* b_0 *for ever.*

(ii) *If* $b_0 < b_{skiba}$ *and* $b_0 \notin B^*$, *then debt converges monotonically to a point inside* B^*.

(iii) *If* $b_{skiba} < b_{bliss}$ *and* $b_0 \in (b_{skiba}, b_{bliss})$, *debt converges monotonically to* b_{bliss}.

Optimal Policy



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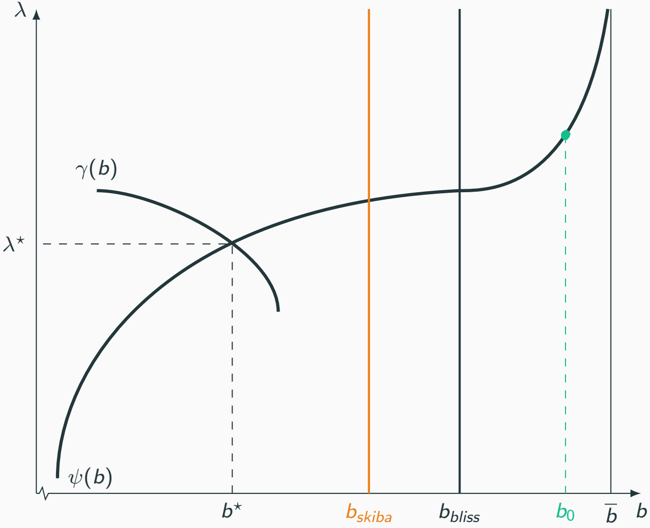

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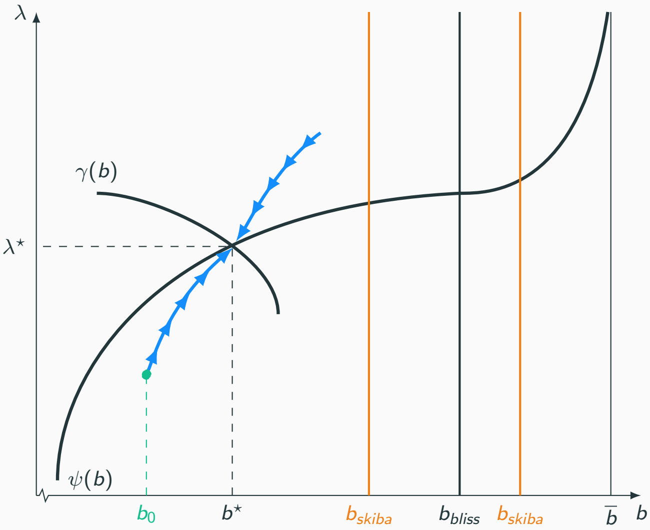

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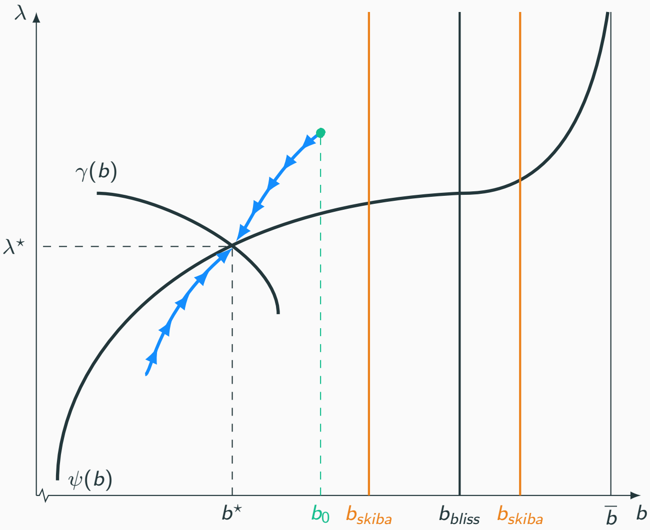

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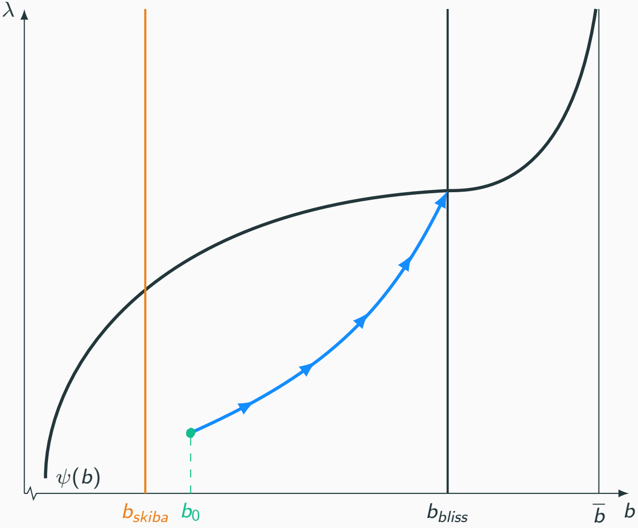


Taxonomy of Economies

Theorem

Any economy belongs to one of the following three non-empty classes:

(i) *Economies in which* $B^* = \emptyset$ *and* $b_{skiba} = \underline{b}$.

(ii) *Economies in which* $B^* \neq \emptyset$ *and* $b_{skiba} \in (\underline{b}, b_{bliss})$.

(iii) *Economies in which* $B^* \neq \emptyset$ *and* $b_{skiba} \geq b_{bliss}$.

Furthermore, $\psi_{bliss} > \gamma_{bliss}$ *is sufficient for an economy to belong to the last class.*

Optimality of Steady State Public Debt

Proposition

Let $\Omega(b, \lambda) = V(b) + \lambda\pi(b)b$ be the liquidity plus seignorage. Consider an economy in which the set $B^\star \neq \emptyset$, then take any $b^\star \in B^\star$ and let $\lambda^\star = \psi(b^\star)$.

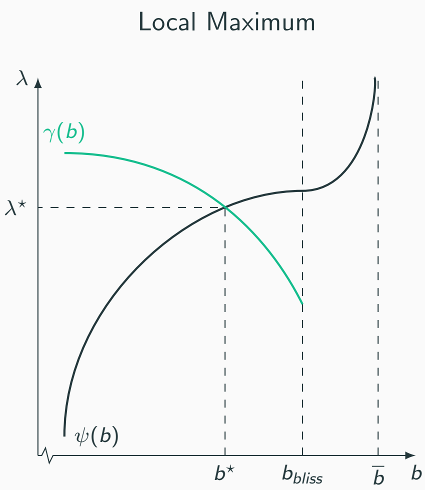
- *If $\gamma'(b^\star) < 0$, $b^\star$ attains a local maximum of $\Omega(b, \lambda^\star)$.*
- *If instead $\gamma'(b^\star) > 0$, $b^\star$ attains a local minimum of $\Omega(b, \lambda^\star)$.*

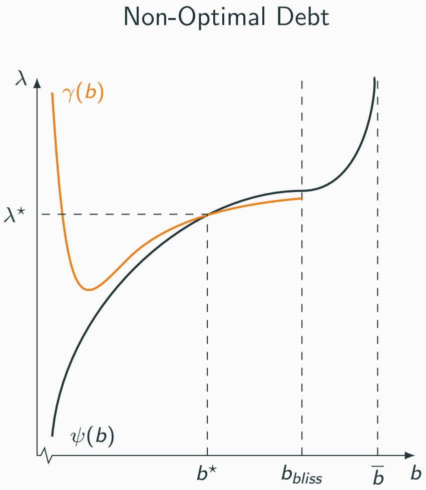
$\Longrightarrow$ Economies do not necessarily converge towards the optimal level of debt.

- There exist economies in which $B^\star$ is a singleton and, nevertheless, $b^\star$ attains a local minimum of $\Omega(b, \lambda^\star)$.

▸ Aiyagari McGrattan

Optimality of Public Debt





▸ Jump to Conclusion

Additional Insights

On the Friedman Rule

- **Key difference from FR literature:** All government liabilities offer liquidity services.

On the Friedman Rule

- **Key difference from FR literature:** All government liabilities offer liquidity services.
- Assume the Government can issue money like assets (Bonds), m, and hold a position in **non-money** asset, $n \implies b = m + n$

$$\dot{m} + \dot{n} = [\rho - \pi(m)]m + \rho n + g - s \qquad \iff \qquad \dot{b} + s = \rho b - \pi(m)m + g$$

- For a given level of **total** liabilities ($\dot{m} + \dot{n} = 0$), the government can change liquidity (m) w/o affecting neither its fiscal position (b) nor the interest rate (ρ) just by varying the composition of liabilities (m/n).

$\Longrightarrow$ **Complete separation between liquidity provision and fiscal position**

On the Friedman Rule

- **Formally:**

$$\max \int_0^{+\infty} e^{-\rho t}[U(s) + V(m)]\mathrm{d}t \qquad \text{s.t.} \qquad b_0 = \int_0^{+\infty} e^{-\rho t}[\pi(m)m + s - g]\mathrm{d}t$$

- s and m are constant over time $\Longrightarrow$ back to tax smoothing.
- $m^\star$ may or may not coincide with the Friedman rule
- If it does, unlike in our model, it does in each and every period.

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- s and m are constant over time $\Longrightarrow$ back to tax smoothing.
- $m^\star$ may or may not coincide with the Friedman rule
- If it does, unlike in our model, it does in each and every period.

- **Why?** The supply of liquidity is isolated from fiscal policy
- Assume instead that n has (even a tiny) role as liquidity then fiscal and liquidity considerations are intertwined and we are back to our trade-off,

Crowding Out Effect of Debt

- Aiyagari-McGrattan (1998): Public debt crowds out capital because debt is a substitute for capital as a buffer stock
- Model la Holmstöm-Tirole (1998): firms have to borrow to finance their capital need
- Agents can relax future financial constraints by saving in the form of capital and debt $\Longrightarrow$ Possible **crowding out**

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- Agents can relax future financial constraints by saving in the form of capital and debt $\Longrightarrow$ Possible **crowding out**

- **But** another conflicting effect:
  - More debt that serves as collateral alleviate the financial friction
  - Improves the allocation of capital and raises the ex-ante return to capital

  $\Longrightarrow$ **Crowding in**

- Overall effect depends on micro details and calibration

Borrowing Cheap?

- **Recent great recession:** low interest rates = signal that it is cheap for the government to borrow.
- Krugman and DeLong: US government should have run an expansionary fiscal policy
  - for Keynesian-stimulus reasons
  - **but also** because interest rates were extraordinarily low.

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  - **but also** because interest rates were extraordinarily low.

- **Misleading claim!**

- **Barro/AMSS:** Adding deterministic variations in the discount rate does not justify changes in the optimal fiscal mix!
- **Why?** The interest rate captures the cost of borrowing **but also** what a society desires (No wedge between the interest rate and the CP's discount rate)

Borrowing Cheap?

- **Our framework:** More borrowing may be optimal during a crisis
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Borrowing Cheap?

- **Our framework:** More borrowing may be optimal during a crisis
- **But** not simply because of low interest rates!
- **Because** low interest rate is a manifestation of
  - the aggravated financial friction
  - the associated increase in the liquidity premium the Govt can extract.

$\Longrightarrow$ Optimal response to adverse financial shocks?

Effect of shocks

- War shocks: Model departs from standard tax smoothing (both across time and states) and exhibits mean reversion. See Figures

- Here look at a financial shock that causes
  - income to fall
  - tax basis to shrink
  - private and social value of liquidity to increase

Effect of shocks

- Both the $\dot{b} = 0$ and $\dot{\lambda} = 0$ shift.
- 3 effects drive movements in the debt
  - Tax Smoothing (shock is temporary)
  - Increase in the marginal value of providing liquidity (Increase liquidity)
  - Increase in the opportunity cost of liquidity (Squeeze liquidity)
- Net effect is ambiguous and depends on the calibration and the micro details.

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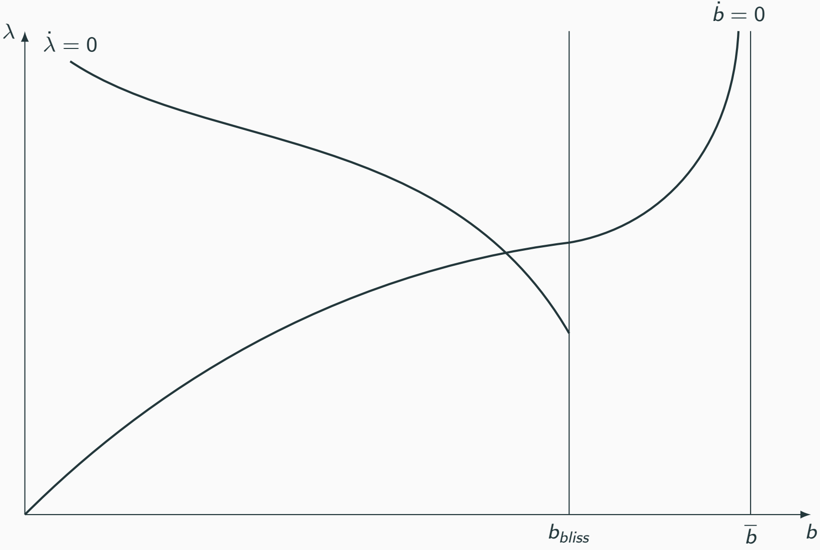
- Consider the case: $\frac{V'(b)}{\pi(b)}$ and $\sigma(b)$ remain constant $\Longrightarrow \dot{\lambda} = 0$ locus does not shift

$\Longrightarrow$ Response of b is dictated by fiscal considerations only (increase in tax burden)

Financial Shock: No change in $\dot{\lambda} = 0$



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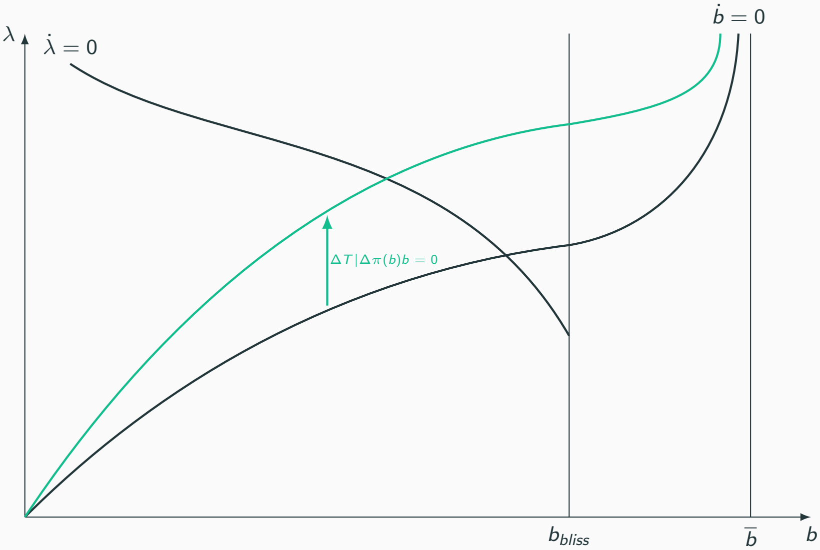

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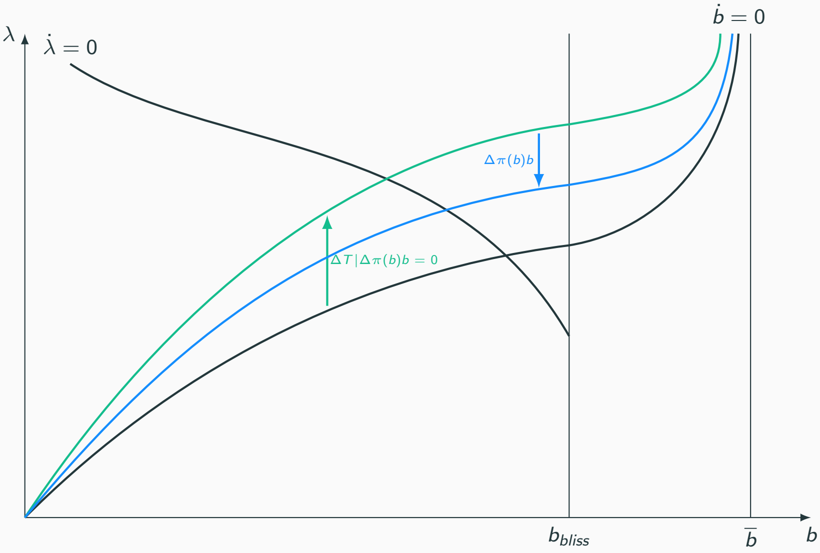

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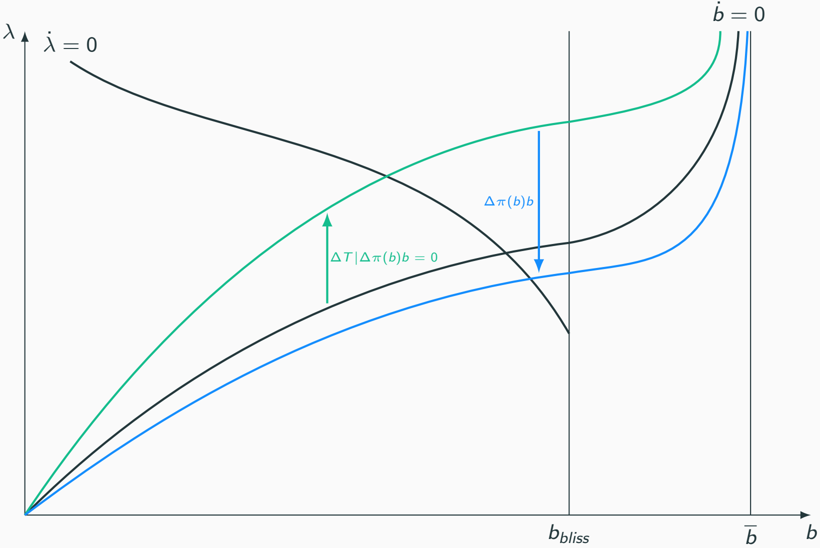

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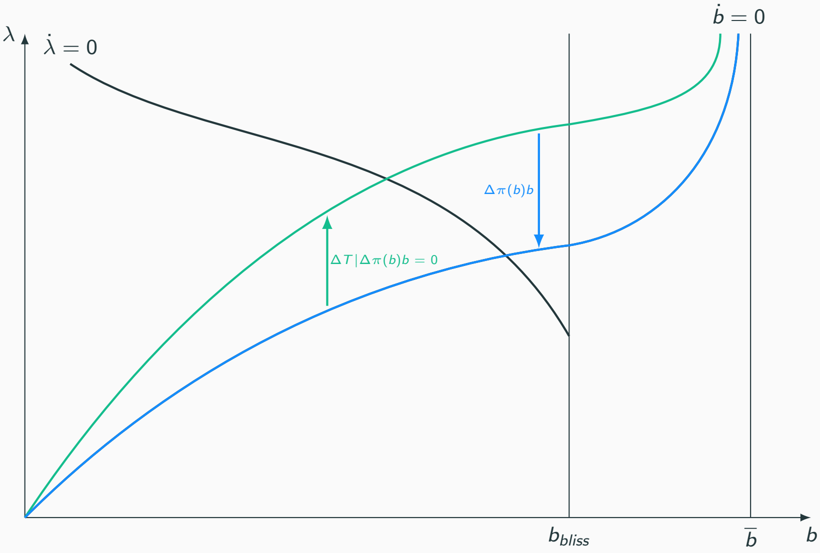
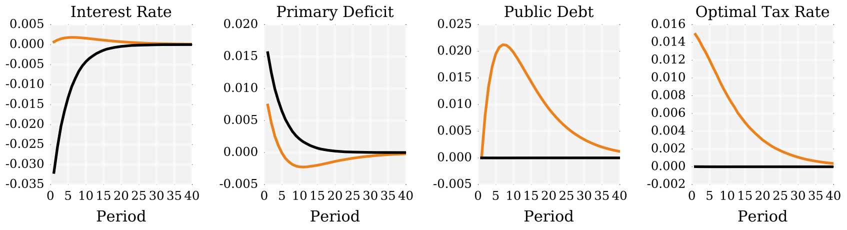


Effect of shocks

Case 1: No direct interest rate effect $\Longrightarrow$ traditional recession (raise debt and taxes)

Case 2: Change in interest rate compensates for decrease in tax $\Longrightarrow$ no change in b and τ. The drop in tax revenue is debt financed ($\dot{b} = 0$ and $\dot{\lambda} = 0$ unaffected)

IRF to a Financial Shock



$\Delta\pi(b)b = 0$; $\Delta\pi(b)b = -\Delta T$

Summary

- **Question:** Is it always optimal to supply debt to alleviate financial frictions?
- **Not always!** The government may wish to exploit its collateral producing capacity in order to earn rents from the private sector and thus reduce its reliance on distortionary tax revenue sources.
- Provide a full characterization of
  - ✓ long term properties and
  - ✓ the global dynamics

  of optimal policy in a model where public debt can alleviate the financial frictions created by lack of sufficient collateral for asset trades.

2 Topical Insights

- **Do low interest rates during recessions make it "cheap" for the Government to borrow?**
    - Argument has no place in the standard Ramsey framework.
    - **Why?** no wedge between interest rate and discount rate of planner
    - May make sense in our framework (if the low interest rate reflects the financial friction.)

2 Topical Insights

- **Do low interest rates during recessions make it "cheap" for the Government to borrow?**
  - Argument has no place in the standard Ramsey framework.
  - **Why?** no wedge between interest rate and discount rate of planner
  - May make sense in our framework (if the low interest rate reflects the financial friction.)

- **Optimal policy response to a financial crisis**
  - crisis raises the marginal value of providing liquidity $\Longrightarrow$ Increase debt
  - but it also raises the opportunity cost of doing so $\Longrightarrow$ Squeezing liquidity

$\Longrightarrow$ Not clear!

  - In a benchmark (one where these 2 effects cancel each other), optimal policy response dictated by budgetary considerations (λ) rather than by the apparent increase in the social value of easing the friction

THANK YOU!

Social Value of Debt

- $V(b)$ is the value of the following problem:

$$
\begin{aligned}
\max_{(p,q)\in\mathbb{R}^2_+ \& (x,a):[\underline{\theta},\bar{\theta}]\to\mathbb{R}_+\times[-\phi,+\infty)} & \int \theta u(x(\theta))\varphi(\theta)\mathrm{d}\theta \\
\text{subject to} \quad & \int x(\theta)\varphi(\theta)\mathrm{d}\theta = \bar{e} \\
& \int a(\theta_-)\varphi(\theta_-)\mathrm{d}\theta_- = b \\
& \phi + a(\theta_-) - p(x(\theta) - \bar{e}) \geq 0 \quad \forall(\theta,\theta_-) \\
& \theta u'(x(\theta)) \geq p \quad \forall\theta \\
& \left[\theta u'(x(\theta)) - p\right]\left[\phi + a(\theta_-) - p(x(\theta) - \bar{e})\right] = 0 \quad \forall(\theta,\theta_-) \\
& a(\theta_-) + \phi \geq 0 \quad \forall\theta_- \\
& \beta + \mathcal{U}_a(a(\theta_-),\theta_-,p) \leq q \quad \forall\theta_- \\
& \left[\mathcal{U}_a(a(\theta_-),\theta_-,p) - \pi\right]\left[a(\theta_-) + \phi\right] = 0 \quad \forall\theta_-
\end{aligned}
$$

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APPENDIX

Define $\mathcal{H}(b,\lambda) = \max_s H(s,b,\lambda) \equiv U(s) + V(b) + \lambda(s - [\rho - \pi(b)]b - g)$, we have

Lemma (Skiba, 1978, Brock and Dechert, 1983)

For any b_0 and any $\lambda_0 \in \Lambda(b_0)$, the path in $\mathcal{P}(b_0)$ that starts from initial point (b_0, λ_0) yields a value that is equal to $\mathcal{H}(b_0, \lambda_0)/\rho$.

Lemma

$\mathcal{H}(b,\lambda)$ is convex in λ (upper envelop of linear functions of λ).

Back

Departure from Aiyagari-McGrattan (1998)

- Aiyagari-McGrattan use a shortcut to avoid the computational challenge of studying optimal debt in incomplete market economies

$\Longrightarrow$ Restrict debt and taxes to be constant, abstract from transition and study Steady State welfare

- Using our notations, amounts to maximize $U(s) + V(b)$ s.t. $r(b)b = g - s$

$$\Longrightarrow b^{AMG} = \underset{b}{argmax}\left[V(b) - \lambda^{AMG}(\rho - \pi(b))b\right] = \underset{b}{argmax}\left[\Omega(b, \lambda^{AMG}) - \lambda^{AMG}\rho b\right]$$

- FOC: $\Omega_b = \lambda^{AMG}\rho > 0$ while we get $\Omega_b = 0 \Longrightarrow b^{AMG} < b^{\star}$.

Back

Departure from Aiyagari-McGrattan (1998)

- Aiyagari-McGrattan's exercise
  - underestimates the optimal long-run level of debt
  - results in a debt level below b_{bliss} even when long-run satiation would be optimal
- **Why?** Because this exercise treats the entire payments on debt, $r(b)b$, as a cost, while the social planner should view debt issuance as a profit generating exercise (seignorage) to the tune of $\pi(b)b$.

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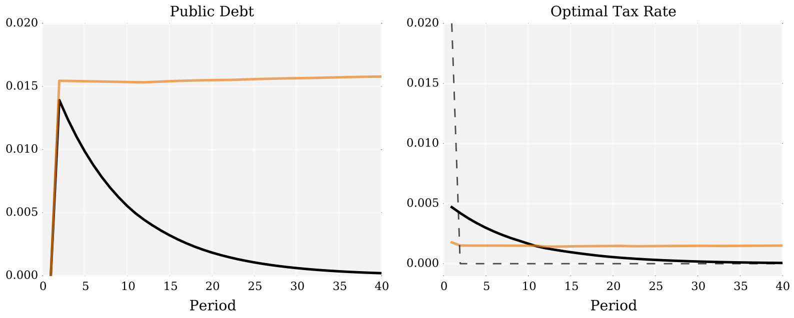
Effect of a WAR – AMSS (2002) Version

i.i.d. Case



Debt and Taxes in our Model; Debt and Taxes in AMSS; _ _ _ _ Government Spending.

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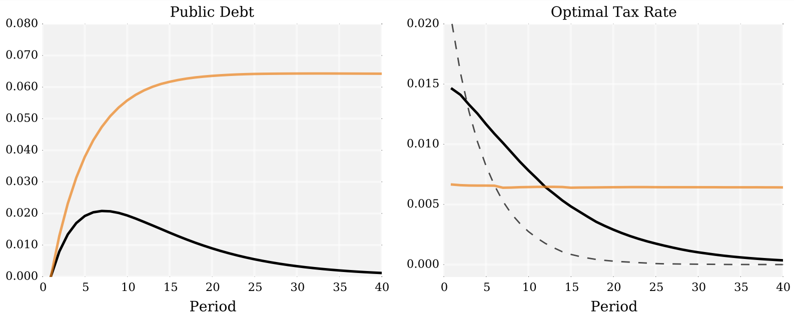
Effect of a WAR – AMSS (2002) Version

Persistent Case



Debt and Taxes in our Model; Debt and Taxes in AMSS; Government Spending.

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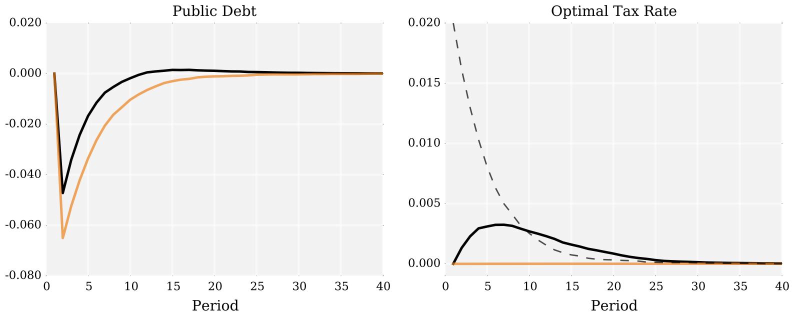
Effect of a WAR – Lucas-Stokey (1983) Version



Our Model; Lucas-Stokey; Government Spending Shock

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