Efficient Unemployment Insurance

Daron Acemoglu

Massachusetts Institute of Technology

Robert Shimer

Princeton University

This paper constructs a tractable general equilibrium model of search with risk aversion. An increase in risk aversion reduces wages, unemployment, and investment. Unemployment insurance has the opposite effect: insured workers seek high-wage jobs with high unemployment risk. An economy with risk-neutral workers achieves maximal output without any unemployment insurance, but an economy with risk-averse workers requires a positive level of unemployment insurance to maximize output. Therefore, moderate unemployment insurance not only improves risk sharing but also increases output.

I. Introduction

This paper studies the impact of risk attitudes and unemployment insurance on the composition of jobs. It develops a general equilibrium model of search and matching with risk-averse agents and incomplete insurance. Firms make irreversible investments and post

wages. Workers optimally search among posted wages. Risk-averse workers wish to avoid unemployment, and in response, the labor market offers its own version of insurance: an equilibrium with lower unemployment and wages. Because lower unemployment raises the vacancy risk for firms, it reduces the utilization of, and the returns to, ex ante investments, leading to lower capital/labor ratios and a worse quality of jobs.

Unemployment insurance encourages workers to apply to high-wage jobs with high unemployment risk. The impact of unemployment insurance on worker and firm behavior is driven by a form of moral hazard. Because insurers cannot directly control workers' actions, the increased utility of unemployment induces them to search for higher wages. Firms respond by creating high-wage, high-quality jobs, with greater unemployment risk.

When agents are risk-averse, the equilibrium without unemployment insurance fails to maximize output because capital/labor ratios are too low. We find, however, that there is a level of incomplete unemployment insurance that maximizes ouput; hence the title of our paper, "Efficient" Unemployment Insurance. Although an allocation that maximizes output does not maximize ex ante utility, our results imply that moderate unemployment insurance not only creates risk-sharing benefits but also increases the level of output. As a result, an economy with optimal—utility-maximizing—unemployment insurance may have *higher* output than an economy with no unemployment insurance. This contrasts with existing results on optimal insurance that emphasize risk-sharing/output (equity/efficiency) trade-offs. In our basic model, this contrast is sharp because conventional moral hazard is absent; that is, unemployment insurance does not reduce search effort. We show, however, that for plausible parameter values, unemployment insurance still raises the level of output when search effort is endogenous.

Despite the dynamic nature of the decisions in our economy, the analysis can be carried out in a static model. In particular, all the qualitative results are the same as in a dynamic model with precautionary savings. Since the equilibrium of the static model is equivalent to the solution of a constrained maximization problem and can be analyzed diagrammatically, it can easily be used for other applications.

Our model is closely related to the general equilibrium search literature (e.g., Diamond 1982; Mortensen 1982; Pissarides 1990) and to the wage-posting models of Montgomery (1991) and Peters (1991). We extend these papers by including risk aversion and capital investments. In the process, we also generalize the efficiency results of Hosios (1990), Shimer (1996), and Moen (1997).

Perhaps the most surprising result of our analysis is that an appropriate level of unemployment insurance restores the economy to the output-maximizing allocation. We are aware of no other paper that has obtained this result. Other papers have also demonstrated that unemployment benefits may improve the allocation of resources in search models (e.g., Diamond 1981; Acemoglu 1997; Marimon and Zilibotti 1999); but all these papers assume risk neutrality, so unemployment insurance is simply a subsidy to search. In contrast, we work with a model in which unemployment insurance reduces output when workers are risk-neutral, demonstrating that the role of unemployment insurance is not to subsidize search but to undo the distortions introduced by uninsured risks. Many of our results, including the comparative static results with respect to attitudes toward risk and the finding that unemployment insurance is the right instrument to achieve productive efficiency, rely explicitly on modeling risk aversion. Our model also implies that the impact of unemployment insurance on search behavior depends on an individual's wealth, an empirical prediction that distinguishes our mechanism from models in which unemployment benefits are simply a subsidy to search.

The result that unemployment insurance is welfare improving is also related to the implicit contract literature, where firms provide insurance to risk-averse workers by increasing employment above the first-best level (e.g., Baily 1974; Gordon 1974; Azariadis 1975). Introducing unemployment insurance would make workers more willing to take the job loss risk and improve welfare. There are important differences between this story and ours, however. First, in our model, unemployment insurance raises the level of output and improves the composition of jobs; in the standard implicit contract model, unemployment insurance reduces output, and there are no implications about job composition. Second, our result is derived in a general equilibrium search model, which has two advantages: (i) the source of the inefficiencies is fully specified, and we show that when frictions disappear, the economy is efficient and there is no room for unemployment insurance; (ii) firms cannot improve on the equilibrium, which contrasts with the implicit contract setting, where firms can increase their profits by introducing severance payments.

Analyses of optimal unemployment insurance in the presence of asymmetric information (e.g., Shavell and Weiss 1979; Hansen and Imrohoroglu 1992; Atkeson and Lucas 1995; Hopenhayn and Nicolini 1997) and partial equilibrium analyses of unemployment insurance, such as Mortensen (1977), also relate to our work. But because they treat the distribution of jobs and wage offers as exogenous, they

do not share our result that unemployment insurance may increase output.

We start in Section II with a static model that illustrates most of our main points. To highlight our innovations, we abstract from conventional moral hazard. In Section III, we analyze the output implications of risk aversion and unemployment insurance. In Section IV, we characterize the optimal level of unemployment insurance. Section V studies a dynamic model with conventional moral hazard. Section VI discusses extensions and empirical implications of our analysis. Section VII presents conclusions. The Appendix contains the major proofs. The remaining proofs are available on request.

II. A Model of Job Search by Risk-Averse Agents

A. *Preferences and Technology*

There is a continuum 1 of identical workers, each with the von Neumann–Morgenstern utility function $u(c)$ over final consumption; u is twice continuously differentiable, strictly increasing, and weakly concave. All workers are endowed with initial wealth A_0, which they may either store or invest in a mutual fund. The absence of aggregate risk ensures that the mutual fund's gross rate of return is equal to the return from storage, $R = 1$. Worker i's consumption is therefore equal to his assets A_0, minus lump-sum taxes τ, plus net income from wages or unemployment benefits y_i, so his utility is $u(A + y_i)$, where $A \equiv A_0 - \tau$ is after-tax assets.

There is a larger continuum of potential firms, each with access to a production technology $f: (0, \infty) \rightarrow (0, \infty)$ that requires one worker and capital $k > 0$ to produce $f(k)$ units of the consumption good. The function f is continuously differentiable, strictly increasing, and strictly concave and satisfies the standard conditions for an interior solution: $\lim_{k \to 0} f(k) = 0$, $\lim_{k \to 0} f'(k) > 1$, and there exists $\bar{k}$ such that $f'(\bar{k}) \equiv 1$. The price of capital is normalized to R. Since workers own a diversified portfolio, firms maximize expected profit. The large number of potential firms ensures free entry, so aggregate profits are zero in equilibrium.

Workers and firms come together via search. At the beginning of the period, each firm j decides whether to buy capital $k_j > 0$. If it does, it is active and posts a wage w_j.[1] In the next stage, each worker

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[1] Observe that firms choose their capital *irreversibly* before they hire a worker as in Acemoglu (1996) and Acemoglu and Shimer (1998). This implies that when buying capital, they take into account how easy it will be to find a worker. Partially irreversible capital decisions are empirically plausible; most equipment is purchased in advance, and many specific costs are incurred before workers are recruited.

observes all the wage offers and decides where to apply. That is, worker i seeks jobs with a wage $w_i \in \mathcal{W}$, where $\mathcal{W} = \{w_j, \text{ for all } j \text{ active}\}$ is the set of wage offers. If he is hired, he earns $y_i = w_i$. Otherwise he is unemployed and obtains unemployment insurance $y_i = z$.[2] A firm that hires a worker produces $f(k)$; an unfilled vacancy produces nothing, and its capital remains idle.

Depending on workers' application decisions, there may be more competition for some jobs than for others. To capture this, we let $q_j \in [0, \infty]$ be the ratio of workers who apply for jobs at firms offering wage w_j to the number of firms posting that wage. We refer to this as the job's *expected queue length,* an endogenous measure of the extent of competition for jobs offering w_j. We assume that a worker applying to wage w_j is hired with probability $\mu(q_j)$, where $\mu: [0, \infty] \to [0, 1]$ is continuously differentiable and decreasing; if many workers apply for one type of job, each has a low probability of employment. Symmetrically, the probability that firm j hires a worker is $\eta(q_j)$, where $\eta: [0, \infty] \to [0, 1]$ is continuously differentiable and increasing;[3] when the number of workers applying for a given wage is held constant, as more firms post that wage, each has a lower probability of hiring. We impose the boundary conditions $\eta(0) = \mu(\infty) = 0$ and $\eta(\infty) = \mu(0) = 1$.

This formulation of the matching technology encompasses many reasonable possibilities. One can think of firms opening jobs in different geographic regions or industries and workers directing their search toward one of these labor markets (see Acemoglu 1997). In labor market j, all firms offer a common wage w_j, and the ratio of workers to firms, q_j, determines the matching probabilities. Standard matching frictions ensure that, within an individual labor market, unemployment and vacancies coexist. Moen (1997) offers a model in which labor markets with different wages are created by a competitive "market-making" sector. Montgomery (1991), Peters (1991), and Burdett, Shi, and Wright (1997) offer another story. Workers use identical mixed strategies in making their applications. A firm may receive multiple applications, in which case it hires one applicant and the others remain unemployed; or it may receive no

applications, in which case its capital remains idle. One can prove that if in expectation q workers apply to each firm posting a wage of w, then $\eta(q) = 1 - \exp(-q)$ and $\mu(q) = \eta(q)/q$. In this case, search frictions arise from a lack of coordination among workers and firms.

Another possibility is the "frictionless" matching process, where the shorter side of the market is fully employed: $\eta^F(q) = \min\langle 1, q\rangle$ and $\mu^F(q) = \min\langle 1/q, 1\rangle$. This matching process does not satisfy the differentiability assumption at $q = 1$, but serves as a useful limiting case. Although our results obtain under any smooth approximation to this matching process, they *do not hold* in this frictionless limit, demonstrating that search frictions are crucial to our analysis.

B. Definition of Equilibrium

An allocation is a tuple $\{\mathcal{K}, \mathcal{W}, Q, U\}$, where $\mathcal{K} \subset \mathbb{R}_+$ is a set of capital investment levels,[4] $\mathcal{W}: \mathbb{R}_+ \rightrightarrows \mathbb{R}_+$ is the set of wages offered by firms making particular capital investments (i.e., $\mathcal{W}(k)$), $Q: \mathbb{R}_+ \to \mathbb{R}_+ \cup \infty$ is the queue length associated with each wage, and $U \in \mathbb{R}_+$ is workers' utility level. We also define the set of wages offered by some firm, $\overline{\mathcal{W}} = \{w | w \in \mathcal{W}(k) \text{ for some } k \in \mathcal{K}\}$. Note that if $w \notin \overline{\mathcal{W}}$, $Q(w)$ is not actually observed. Instead, these "off-the-equilibrium-path actions" represent conjectures that help determine equilibrium behavior.

DEFINITION 1. An equilibrium is an allocation $\{\mathcal{K}^*, \mathcal{W}^*, Q^*, U^*\}$ with the following properties: (1) Profit maximization: For all w, k,

$$\eta(Q^*(w))[f(k) - w] - k \geq 0,$$

with equality if $k \in \mathcal{K}^*$ and $w \in \mathcal{W}^*(k)$. (2) Optimal application: For all w,

$$U^* \geq \mu(Q^*(w))u(A + w) + [1 - \mu(Q^*(w))]u(A + z)$$

and $Q^*(w) \geq 0$, with complementary slackness, where

$$U^* = \sup_{w' \in \overline{\mathcal{W}}^*} \mu(Q^*(w'))u(A + w') + [1 - \mu(Q^*(w'))]u(A + z),$$

or $U^* = u(A + z)$ if $\overline{\mathcal{W}}^*$ is empty.

Profit maximization ensures that, given the queue length associated with each wage, firms choose wages and capital investments to

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[4] For notational simplicity, we omit from the definition of an allocation the fraction of firms making different investments. This matters only if $\mathcal{K}$ is not a singleton, which is only a nongeneric possibility (see n. 5). Even then, the actual distribution of capital is unimportant for computing the equilibrium. A similar statement holds for $\mathcal{W}$.

maximize profits. Also, free entry drives the maximized value of profits to zero. Optimal application ensures that workers make their application decisions to maximize utility and imposes a form of subgame perfection: queue lengths adjust to make workers earn the maximal level of utility U^* at any wage, including wages not offered along the equilibrium path. This rules out situations in which firms may not deviate to a potentially profitable wage, incorrectly conjecturing that very few workers would apply. Finally, the complementary slackness condition ensures that if a wage w does not deliver utility U^* to a worker hired with probability one, then no one applies for that wage; that is, $Q(w) = 0$.

The queue length function Q^* also contains two other pieces of useful information. First, if w^* is the unique equilibrium wage, the number of active firms is $1/Q^*(w^*)$. Second, the unemployment rate of workers applying to a wage w' is $\upsilon(w') = 1 - \mu(Q^*(w'))$. Clearly $\upsilon(w')$ is increasing in $Q^*(w')$ since workers who apply to jobs with longer queues suffer a higher probability of unemployment.

C. Existence and Characterization

PROPOSITION 1. There always exists an equilibrium. If $\{\mathcal{K}, \mathcal{W}, Q, U\}$ is an equilibrium, then any $k^* \in \mathcal{K}$, $w^* \in \mathcal{W}(k^*)$, and $q^* = Q(w^*)$ solve

$$U = \max_{k,w,q} \mu(q)u(A + w) + [1 - \mu(q)]u(A + z) \qquad (1)$$

subject to

$$\eta(q)[f(k) - w] - k = 0 \qquad (2)$$

and

$$w \geq z. \qquad (3)$$

Conversely, if some $\{k^*, w^*, q^*\}$ solves this program, then there exists an equilibrium $\{\mathcal{K}, \mathcal{W}, Q, U\}$ such that $k^* \in \mathcal{K}$, $w^* \in \mathcal{W}(k^*)$, and $q^* = Q(w^*)$.

An equilibrium allocation maximizes workers' utility subject to firms' earning zero profits, and an allocation that does so is part of an equilibrium. In view of this result, we shall refer to a triple $\{k^*, w^*, q^*\}$ satisfying the requirements in proposition 1 as an "equilibrium."

Define $\bar{z} \equiv f(\bar{k}) - \bar{k}$, where $f'(\bar{k}) \equiv 1$. Whenever $z > \bar{z}$, the constraint set, given by (2) and (3), is empty, and there are no active firms in equilibrium (i.e., $\mathcal{K} = \overline{\mathcal{W}} = \varnothing$). If $z < \bar{z}$, then $\mathcal{K}, \overline{\mathcal{W}} \neq \varnothing$ and any equilibrium wage is strictly greater than z, so (3) is slack.
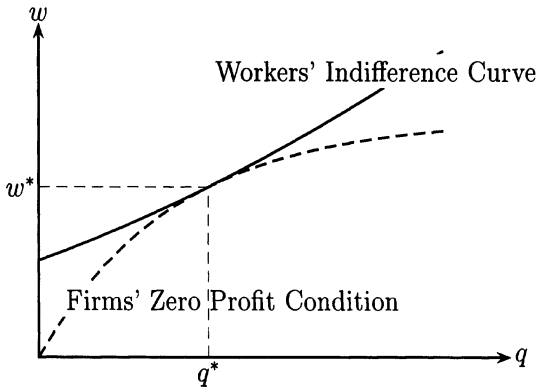
FIG. 1.—An equilibrium maximizes workers' utility subject to firms' earning zero profits. Higher wages and shorter queues raise utility and lower profits.

In the rest of the paper, we restrict attention to the case of $z < \bar{z}$ and ignore constraint (3).

Using proposition 1, we can characterize firms' investment decisions. Since k affects only (2), an equilibrium k^* must satisfy the first-order condition for capital choice, $\eta(q^*)f'(k^*) = 1$. Combining this with zero profits gives

$$w^* = f(k^*) - k^*f'(k^*). \tag{4}$$

Despite the search frictions, capital earns its marginal product and labor keeps the residual output. Since f is concave, (4) defines w^* as an increasing function of k^*.

Proposition 1 and equation (4) allow us to depict an equilibrium graphically. The optimal capital choice and (4) imply that the constraint, firms' zero-profit condition, is an upward-sloping curve in $\{q, w\}$ space, as shown in figure 1. Similarly, workers' indifference curves are upward sloping in $\{q, w\}$ space. An equilibrium is a point of tangency between these two indifference curves. Either curve, however, could be nonconvex, so uniqueness of the equilibrium is not guaranteed.

D. Comparative Statics

PROPOSITION 2. (1) Let $\{k_i, w_i, q_i\}$ be an equilibrium when the utility function is u_i. If u_1 is a strictly concave transformation of u_2, then $k_1 < k_2$, $w_1 < w_2$, and $q_1 < q_2$. (2) Let $\{k_i, w_i, q_i\}$ be an equilibrium when the initial asset level is A_i. If $A_1 < A_2$ and the utility function

has decreasing absolute risk aversion (DARA), then $k_1 < k_2$, $w_1 < w_2$, and $q_1 < q_2$. With increasing absolute risk aversion (IARA), the inequalities are reversed. With constant absolute risk aversion (CARA), asset levels do not affect the set of equilibria. (3) Let $\{k_i, w_i, q_i\}$ be an equilibrium when the unemployment benefit is z_i. If $z_1 < z_2$, then $k_1 < k_2$, $w_1 < w_2$, and $q_1 < q_2$.

Part 1 states that, with more risk-averse workers, wages are lower, job queues are shorter, and firms invest less. Part 2 shows that, with DARA, poorer workers are effectively more risk-averse. Part 3 shows that higher unemployment insurance increases wages, unemployment, and capital/labor ratios.

The Appendix contains a formal proof of proposition 2, but the main idea can be seen graphically in figure 2. Firms' zero-profit condition (2) is unaffected by preferences, assets, and unemployment insurance. The comparative statics are therefore due exclusively to changes in preferences. More risk-averse workers have indifference curves that are everywhere steeper. In contrast, when unemployment insurance increases, the new set of indifference curves becomes everywhere flatter. As indifference curves becomes flatter, the point of tangency must shift to the right, to a point of higher wages and longer job queues. The comparative static results are always unambiguous because the sets of indifference curves cross only once.

The intuition for proposition 2 illustrates the key innovations of our analysis. Frictional matching introduces an inherent risk as workers may remain unemployed and suffer low consumption. In the absence of unemployment insurance, the labor market offers *its own brand of insurance*. Firms see the profit opportunities in creating jobs with lower unemployment risk and charge an "insurance premium" to risk-averse workers by offering lower wages. This market insurance also affects the form of production in the economy, however, because opening a large number of jobs creates high "vacancy risk" for firms, reducing investment and capital intensity. The reason why there is no separation between the consumption and production sides of this economy is incomplete insurance. In the presence of complete insurance, workers would maximize their expected income, and capital/labor ratios would not respond to changes in risk aversion.

When unemployment insurance increases, workers wish to apply to higher wages, which are associated with higher unemployment risk. Firms once more cater to these preferences, and wages, unemployment, and capital intensity increase. Underlying this labor market adjustment is *market-generated moral hazard* (distinct from conventional moral hazard discussed in Sec. V). The inability of the insurer to directly prevent agents from taking on more risk is the essence
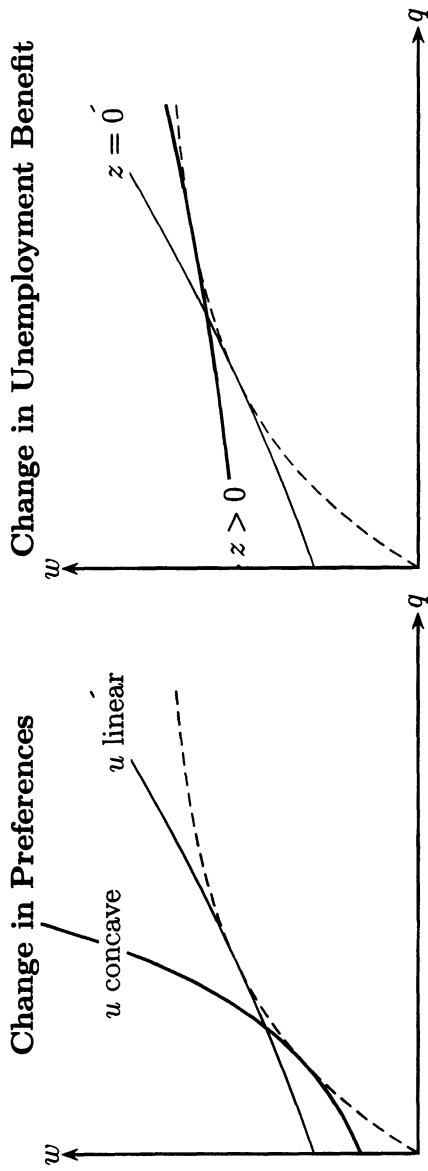
FIG. 2.—More risk aversion makes indifference curves steeper, lowering the equilibrium wage and queue length. Higher unemployment insurance makes indifference curves flatter, having the opposite effect.
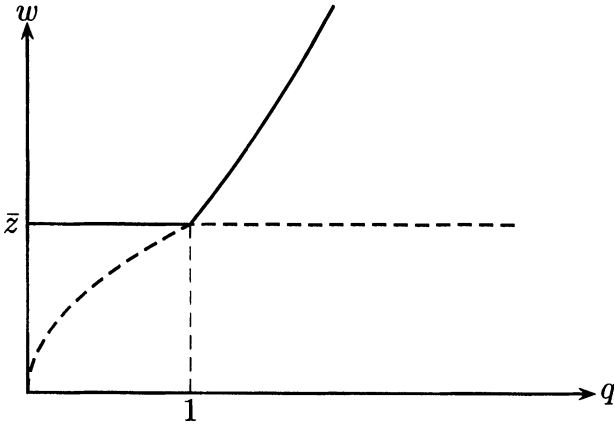
FIG. 3.—The equilibrium under the frictionless matching technology η^F, μ^F. It is unaffected by unemployment benefits and preferences.

of the moral hazard problem (e.g., Holmstrom 1979). In our model, the insurer would like the worker not to apply to higher wages when there is unemployment insurance. But since we assume that the worker's application decision is private information, insurance cannot be conditioned on it. We refer to this as market-generated moral hazard because the response of the labor market is crucial: if firms did not change their wage offers, workers would be unable to apply to higher wages.

We conclude by observing that with the frictionless matching technology defined at the end of subsection A, proposition 2 does not hold. If $\eta^F(q) = \min\langle 1, q \rangle$ and $\mu^F(q) = \min\langle 1/q, 1 \rangle$, the unique equilibrium is $\{k, w, q\} = \{\bar{k}, \bar{z}, 1\}$ for any degree of risk aversion and any unemployment insurance $z < \bar{z}$. Workers gain nothing by applying for a job with a queue length less than one, and firms gain nothing by offering a wage that yields a queue length greater than one. Therefore, indifference curves and the zero-profit condition are kinked at $q = 1$, ensuring that this is the point of "tangency" (see fig. 3). Then $q = \eta^F(q) = 1$ implies $f'(k) = 1$, and hence $k = \bar{k}$. Finally, (4) yields $w = f(\bar{k}) - \bar{k} \equiv \bar{z}$. Nevertheless, with any continuously differentiable approximation to η^F and μ^F, the comparative statics results in proposition 2 obtain. The fact that our results limit to the competitive equilibrium and fail to hold at this limit point (with η^F and μ^F) is reassuring: frictions, as well as incomplete insurance, are crucial for our conclusions.

E. Worker Heterogeneity

We have simplified our analysis by assuming that all workers have the same level of assets and the same utility function and receive the

same level of unemployment benefits. Our results generalize to an environment in which workers differ with respect to all these features.

Suppose that there are $s = 1, 2, \ldots, S$ types of workers, where type s has utility function u_s, after-tax asset level A_s, and unemployment benefit z_s. Let $\mathbf{U}$ now be a vector in $\mathbb{R}^S$, and assume, for simplicity, that $z_s < \bar{z}$ for all s. Then we obtain the following proposition.

PROPOSITION 3. There always exists an equilibrium. If $\{\mathcal{H}, \mathcal{W}, Q, \mathbf{U}\}$ is an equilibrium, then any $k_s^* \in \mathcal{H}$, $w_s^* \in \mathcal{W}(k_s^*)$, and $q_s^* = Q(w_s^*)$ solve

$$U_s = \max_{k,w,q} \mu(q) u_s(A_s + w) + [1 - \mu(q)] u(A_s + z_s)$$

subject to (2) for some $s = 1, 2, \ldots, S$. If $\{k_s^*, w_s^*, q_s^*\}$ solves this program for some s, then there exists an equilibrium $\{\mathcal{H}, \mathcal{W}, Q, \mathbf{U}\}$ such that $k_s^* \in \mathcal{H}$, $w_s^* \in \mathcal{W}(k_s^*)$, and $q_s^* = Q(w_s^*)$.

Any triple $\{k_s^*, w_s^*, q_s^*\}$ that is part of an equilibrium maximizes the utility of one group of workers, subject to firms' making zero profits. The market *endogenously* segments into S different submarkets, each catering to the preferences of one type of worker and receiving applications only from that type. The proof of the proposition is analogous to that of proposition 1 and is omitted.

We can repeat the analysis of proposition 2 for each submarket. If type s workers are more risk-averse or receive less unemployment insurance, their submarket will create lower-wage jobs with shorter queues and less investment, but the rest of the submarkets will be unaffected.

This analysis implies that workers with higher unemployment insurance will apply for higher-wage jobs with longer queues and suffer longer unemployment spells, an empirically verified prediction (Ehrenberg and Oaxaca 1976; Katz and Meyer 1990). It also shows that, except in the case of CARA, workers' asset levels will systematically affect their search behavior and will alter the impact of unemployment insurance. This distinguishes the model from one in which unemployment insurance is simply a subsidy to search by risk-neutral workers.

III. Risk Aversion, Unemployment Insurance, and Output

This section demonstrates that if workers are risk-averse, a moderate amount of unemployment insurance funded by lump-sum taxation will raise output. Since this says nothing about the riskiness of consumption, it is not a normative statement. Nevertheless, it distin-

guishes our approach from existing theories of unemployment insurance.

Suppose that workers are homogeneous, and all firms invest k, offer a common wage w, and attract an average queue length of q. Then output is

$$Y(q, k) \equiv \mu(q)f(k) - \frac{k}{q}.$$

The number of jobs is equal to the measure of workers who find a job, $\mu(q)$, times the output of each filled job, $f(k)$. From this we subtract investment expenditure, which is the measure of firms, $1/q$, times capital expenditure, k. If firms choose different queue lengths and capital, output would be a weighted average of each firm's expected output. We suppress this possibility to simplify our notation.[5]

DEFINITION 2. The triple $\{k^e, w^e, q^e\}$ is output-maximizing if $Y(q^e, k^e) = \max_{q,k} Y(q, k)$. Unemployment insurance z^e and tax τ^e are output-maximizing if any associated equilibrium $\{k_{z^e,\tau^e}, w_{z^e,\tau^e}, q_{z^e,\tau^e}\}$ is output-maximizing and has a balanced government budget, $\tau^e = [1 - \mu(q_{z^e,\tau^e})]z^e$.

Notice that unemployment insurance is output-maximizing only if all equilibria achieve the highest output level that is technologically possible. We begin with an important benchmark.

PROPOSITION 4. If agents are risk-neutral, the unique output-maximizing level of unemployment insurance is $z^e = \tau^e = 0$.

Intuitively, when there is no unemployment insurance, risk-neutral agents maximize the expected value of their wages, which is identical to net income. This result is a generalization of Shimer (1996) and Moen (1997) to the case in which firms choose their physical capital investments.

PROPOSITION 5. If agents are risk-averse and $z^e = \tau^e = 0$, output is below its maximum.

The proof of this proposition is straightforward and is omitted: changes in preferences do not affect the output-maximizing allocation, but unambiguously change the equilibrium (proposition 2). Since $z^e = \tau^e = 0$ maximizes output with risk-neutral agents, it yields a different, hence lower, level of output when workers are risk-averse.

[5] We do this with little loss of generality since it follows from proposition 2 that the equilibrium is generically unique. For example, with all other parameters held fixed, the equilibrium is unique for almost every level of unemployment insurance. To see why, consider the correspondence $K(z)$, defined as the set of all k that solve the constrained optimization problem in proposition 1. Part 3 of proposition 2 implies that K is strictly monotonic, and so there are at most a countable number of z's for which K is not a singleton. At all other z, the equilibrium is unique. A similar argument implies generic uniqueness in an appropriate space of utility functions.
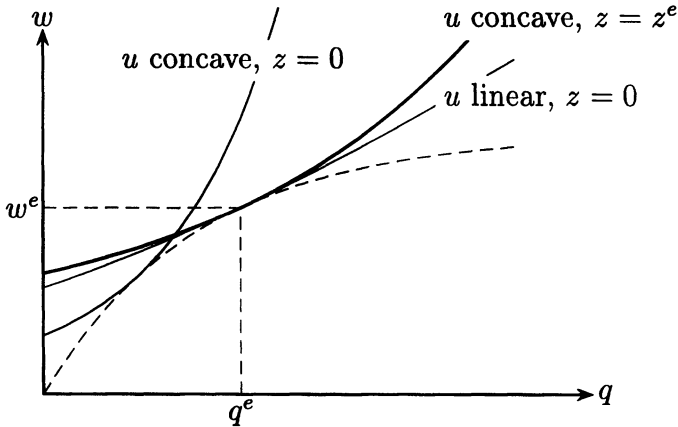
FIG. 4.—Risk aversion makes indifference curves steeper, whereas unemployment insurance makes them flatter. The curve for u concave and $z = z^e$ lies everywhere above the u linear curve.

A different way of expressing the intuition is to relate it to the efficiency results in search models with bargaining (e.g., Diamond 1982; Hosios 1990; Pissarides 1990; Sattinger 1990). In these models, when bargaining dictates that workers receive too small a fraction of the output they produce, unemployment is inefficiently low. In our model, risk-averse workers without unemployment insurance prefer to receive a small fraction of output in order to reduce unemployment risk, and firms cater to these preferences by offering low-wage jobs. This makes the level of frictional unemployment too low compared to the efficient level.[6]

We now show that output can be restored to its maximum through moderate unemployment insurance.

PROPOSITION 6. Let $\{k^e, w^e, q^e\}$ be an output-maximizing allocation, where $w^e = f(k^e) - k^e f'(k^e)$. Then $\{z^e, \tau^e\}$ is output-maximizing if $\tau^e \equiv [1 - \mu(q^e)]z^e$ and $z^e \in (0, w^e)$ satisfies

$$w^e \equiv \frac{u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)}{u'(A_0 - \tau^e + w^e)}. \tag{5}$$

The technical intuition behind this result is given in figure 4. With

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[6] To emphasize the relation to existing efficiency results, note that all our results hold in the limiting case in which capital investment k is fixed at some level $\underline{k}$, which can be interpreted as the cost of a vacancy. The reason why an equilibrium with risk aversion and no unemployment insurance would fail to maximize output in this case is that, as in traditional search models with excessively low worker bargaining power, firms would open too many vacancies. We concentrate on the case with the variable k because we believe that the distortion in capital intensity is more interesting and relevant.

risk-neutral workers and no unemployment insurance, any equilibrium maximizes output (proposition 4). If instead workers are risk-averse, indifference curves are steeper, leading to a lower equilibrium wage and queue length.[7] Unemployment insurance at the level z^e undoes the effects of risk aversion, flattens indifference curves, and restores tangency between workers' indifference curves and the constraint set at the output-maximizing allocation. The result is in fact more subtle and surprising than this intuition suggests. In essence, proposition 6 establishes that a simple and linear intervention takes the economy back to productive efficiency, which is generally not possible in nonconvex economies like ours. We prove this by establishing that the thick indifference curve in figure 4 (u concave and $z = z^e$) lies everywhere above the risk-neutral indifference curve (u linear and $z = 0$) and is tangent to it at $\{q^e, w^e\}$.

IV. Optimal Unemployment Insurance

This section analyzes "optimal" unemployment insurance, which maximizes the ex ante expected utility of the representative worker. To begin, suppose that a benevolent social planner is constrained by the search technology μ and η but can choose the measure of active firms and the wage and capital level of each firm. The planner would maximize total output and divide it equally among all workers, providing full insurance to each. This is trivially the unconstrained optimum.

A more interesting exercise is to ask what level of unemployment insurance maximizes workers' expected utility, with entry, investment, and wages determined in equilibrium (rather than directly chosen by the planner). To this end, we define optimal unemployment insurance as follows.

DEFINITION 3. Let $\{k_{z,\tau}, w_{z,\tau}, q_{z,\tau}\}$ be an equilibrium with unemployment insurance and taxes $\{z, \tau\}$. Then the policy $\{z^o, \tau^o\}$ is optimal if it maximizes $\mu(q_{z,\tau})u(A - \tau + w_{z,\tau}) + [1 - \mu(q_{z,\tau})]u(A - \tau + z)$ subject to $\tau = [1 - \mu(q_{z,\tau})]z$.

CONJECTURE 1. Optimal unemployment insurance $\{z^o, \tau^o\}$ exists. For all u strictly concave, $z^o \in (z^e, \bar{z})$.

Intuitively, at the point of maximal output, decentralized by $\{z^e, \tau^e\}$, a further increase in unemployment insurance leads to a second-order loss of net output, and it increases the income of unemployed

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[7] In proving propositions 5 and 6, we compare the output-maximizing allocation, which is the equilibrium when workers are risk-neutral, to the equilibrium with risk-averse workers. We do not wish to imply that the same allocation is optimal in both economies. Instead, the comparison is simply a method of proof.

workers and decreases the (after-tax) income of employed workers. This suggests that workers will prefer the new, high–unemployment insurance "lottery." Although we are unable to prove it,[8] simulations support this conjecture. Our simulations also suggest that an economy with optimal unemployment insurance has higher output than an economy without any unemployment insurance.[9] Therefore, contrary to conventional wisdom, moderate unemployment insurance increases ex ante utility not only due to better risk sharing but also due to higher output.

V.   A Dynamic Model without Wealth Effects

Forward-looking agents can self-insure by saving even in the absence of insurance markets. To address this, we show that our results generalize to a dynamic environment. We also demonstrate the flexibility of our model by introducing a more conventional form of moral hazard, allowing unemployment insurance to affect search effort. This analysis may have a number of useful applications since it is much more tractable than existing general equilibrium models of search with risk aversion.[10]

   We assume that workers have constant absolute risk aversion. Although we believe that our results generalize to a dynamic environment with arbitrary preferences, CARA is a useful simplification. With other utility functions, a worker's wealth affects his preferences over wages and queue lengths (see part 2 of proposition 2). The labor market segments into submarkets as in proposition 3, but a worker still recognizes that his consumption decision today influences the probability distribution of jobs in the future. Since this determines the budget constraint, it feeds back and affects his con-

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[8] The problem that prevents a simple proof is that the high–unemployment insurance lottery does not second-order stochastically dominate the output-maximizing lottery. The reason is that equilibrium search behavior implies that a worker is more likely to be unemployed when there is more unemployment insurance. This implies that some risk-averse utility functions prefer the output-maximizing lottery to the high–unemployment insurance lottery. However, since both lotteries are determined by the utility function, this does not disprove the conjecture.

[9] For example, let $f(k) = k^{1/2}$, $u(c) = -1/c$, $\eta(q) = 1 - \exp(-q)$, and $A = 0.1$. Without any unemployment insurance, output is 0.090. When unemployment insurance is 0.075, output is maximized at 0.102. Optimal unemployment insurance is about 50 percent higher, 0.111, yielding output 0.100. We obtain similar results with other parameterizations.

[10] For other models of search with risk aversion, see Andalfatto and Gomme (1996), Gomes, Greenwood, and Rebelo (1997), and Costain (1996), all of which are solved numerically. Also, the first two papers assume an exogenous distribution of wages, and the third imposes a restrictive structure to ensure that unemployment insurance and preferences do not affect wages.

sumption choice today. Constant absolute risk aversion allows us to avoid these complexities.

A. Preferences and Technology

Consider an infinite-horizon economy in discrete time. Each worker i makes consumption and job search decisions to maximize

$$\sum_{t=0}^{\infty} \beta^t \frac{1 - \exp(-\theta c_{it})}{\theta}, \tag{6}$$

where c_{it} is consumption at time t, $\beta < 1$ is the discount factor, and $\theta > 0$ is the coefficient of absolute risk aversion. Firms maximize the expected value of profits, discounted at the constant gross interest rate $R > 1$.

The search and production technologies are generalizations of the static model. At the start of a period, firms may be inactive or vacant or have a filled job. Workers may be either unemployed or employed. Inactive firms can create a vacancy by buying perfectly durable capital at a unit cost conveniently normalized to $R/(R - 1)$. After this, firms with newly created and existing vacancies post wages.[11] Next, unemployed workers see the menu of wages and decide which wage, if any, to seek. The matching technology is the same as above. If the expected number of applicants is q, a worker is hired with probability $\mu(q)$, and a firm's hiring probability is $\eta(q)$. All filled jobs produce $f(k)$ units of output. If the firm cannot find a worker, its capital remains idle for the period. An unemployed worker receives unemployment insurance equal to z. Unfilled vacancies and unemployed workers search again in the following period. A productive relationship between a worker and a firm never ends. Instead, we maintain a steady-state population of unemployed workers by assuming that the labor force L_t grows at rate δ and all the δL_{t-1} new workers are unemployed at the start of period t, with initial assets $A_{it} = A_0$.[12]

Workers face the dynamic budget constraint $A_{i,t+1} = R(y_{it} + A_{it} - \tau_t - c_{it})$, where y_{it} is the gross income of worker i at time t, τ_t is the lump-sum tax, and c_{it} is consumption at time t. We also impose the

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[11] We assume that firms post flat-wage contracts rather than more complex wage profiles (Shimer 1996). It can be shown that, in this setting, more complex contracts are no better than flat-wage contracts.

[12] This formulation, rather than one with separations, is adopted for simplicity. With random separations, it is not optimal for firms to offer workers constant wages. Instead, risk-neutral firms would optimally bear the risk of future random separations. This could be done, e.g., by paying workers large signing bonuses and then holding them to their reservation wage.

standard no–Ponzi game condition, $\lim_{s \to \infty} R^{-s} A_{i,t+s} = 0$. We further simplify our analysis by assuming that this is a small open economy with an interest rate equal to the rate of time preference, $R\beta = 1$, so workers will equalize the expected marginal utility of consumption across time. This assumption is unimportant.

We introduce *conventional moral hazard* as follows. Each worker i decides whether to participate in the market economy described above or not to participate, receiving an income x_i per period (say from home production). The distribution of x within each generation is the same and is denoted by $G(x)$. Whether an individual is searching in the market is not observed publicly, so nonparticipating agents also receive unemployment insurance. Hence, as in the conventional moral hazard models, unemployment insurance discourages search. This formulation implies that market opportunities for all agents are identical, yielding a simple recursive structure. We first solve for the utility of a representative participating agent. Since the matching technology has constant returns to scale, the number of agents who participate in the market is unimportant for this analysis. We then compare this market utility to the utility from not participating to determine equilibrium participation.

B. Optimal Consumption Decisions and Value Functions

We begin by defining Bellman values for employed, unemployed, and nonparticipating workers. We focus on steady-state equilibria in which wages, capital stocks, queue lengths, and unemployment rates are constant. Let $E(A, w)$ be the lifetime utility of a worker who is employed at wage w with assets A at the start of the time period. Similarly, let $J(A, w, q)$ be the expected value of an unemployed worker if he applies for wage w with queue length q in this period and follows an optimal consumption rule and application strategy thereafter. Let $U(A)$ denote the Bellman value of an unemployed participant with assets A. Finally, define $N(A, x)$ as the utility of a nonparticipating agent with asset level A, receiving home-produced income x.

These Bellman values depend on workers' consumption and savings decisions. To solve unemployed workers' consumption problem, we assume that they apply for the same wage-queue combination in every period, regardless of their asset level. We later verify that they use such a policy.

LEMMA 1. (1) An employed worker who starts a period with assets A_t, pays taxes τ, and earns wage w in every period has optimal con-

sumption $c_t^e = w + (1 - \beta)A_t - \tau$, so $A_{t+1} = A_t$. His lifetime utility is

$$E(A_t, w) = \frac{1 - \exp\{-\theta[w + (1 - \beta)A_t - \tau]\}}{\theta(1 - \beta)}. \tag{7}$$

(2) An unemployed individual who starts with assets A_t, pays taxes τ in every period, applies for a job offering wage w and queue length q, and earns benefit z in every period in which he fails to get a job has optimal consumption $c_t^u = \beta\psi + (1 - \beta)A_t + z - \tau$, where $\psi \in (0, w - z)$ is implicitly defined by

$$0 = \mu(q) \frac{1 - \exp[-\theta(w - z - \psi)]}{\theta} \tag{8}$$

$$+ [1 - \mu(q)] \frac{1 - \exp[-\theta(1 - \beta)\psi]}{\theta},$$

so $A_{t+1} = A_t - \psi$. His lifetime expected utility is

$$J(A_t, w, q) = \frac{1 - \exp\{-\theta[\psi + (1 - \beta)A_t + z - \tau]\}}{\theta(1 - \beta)}. \tag{9}$$

(3) A nonparticipating worker i, who starts period t with assets A_t, pays taxes τ, earns income x_i, and receives unemployment benefit z in every period has optimal consumption $c_t^n = x_i + z + (1 - \beta)A_t - \tau$, so $A_{t+1} = A_t$. His lifetime utility is

$$N(A_t, x_i + z) = \frac{1 - \exp\{-\theta[x_i + (1 - \beta)A_t + z - \tau]\}}{\theta(1 - \beta)}. \tag{10}$$

Because $R\beta = 1$, workers who face no uncertainty—employed and nonparticipating workers—consume their current net income and interest from savings, maintaining a constant asset level. Unemployed workers consume their current income and dissave a constant amount ψ of their assets each period, resulting in decreasing asset and consumption levels while unemployed. This reflects the fact that each additional period of unemployment is a bad shock. When they find a job, consumption jumps up.

Equation (9) implies that all workers have the same von Neumann–Morgenstern preferences over wage-queue combinations, regardless of their asset levels, and verifies our hypothesis that workers apply for the same wage in every period. This is related to part 2 of proposition 2, which states a similar result in the static model.[13]

[13] This feature of our search environment is not shared by the search and bargaining models of Diamond (1982), Mortensen (1982), and Pissarides (1990), even if workers have CARA preferences. The reason is that, here, the preferences of workers over *lotteries* determine wages, whereas with bargaining what matters is the marginal utility of consumption, which always varies with wealth when agents are risk-averse.

C. Analysis

We define an equilibrium using the value functions derived above. Let $\mathcal{P}$ be the set of market participants.

DEFINITION 4. A steady-state equilibrium is an allocation $\{\mathcal{H}, \mathcal{W}, Q, \mathbf{U}, E, J, N, \mathcal{P}\}$ subject to the following conditions: (1) Profit maximization: For all w, k,

$$\frac{\eta(Q(w))[f(k) - w]}{1 - \beta[1 - \eta(Q(w))]} - k \leq 0, \tag{11}$$

with equality if $k \in \mathcal{H}$ and $w \in \mathcal{W}(k)$. (2) Optimal application: For all w, $\mathbf{U}(\cdot) \geq J(\cdot, w, Q(w))$ and $Q(w) \geq 0$ with complementary slackness; $\mathbf{U}(\cdot) = \sup_{w' \in \bar{\mathcal{W}}} J(\cdot, w', Q(w'))$ or $\mathbf{U}(\cdot) = N(\cdot, z)$ if $\bar{\mathcal{W}} = \varnothing$. (3) Optimal participation: Worker $i \in \mathcal{P}$ if and only if $N(\cdot, x_i + z) \leq \mathbf{U}(\cdot)$.

This is a generalization of the definition of equilibrium in the static model. Optimal application and participation use the fact that the relationship between $\mathbf{U}, J$, and N does not depend on asset levels A. To understand the expression for firm profits, observe that, from free entry, the expected present value of a vacant firm's profits equals the cost of creating a new vacancy, $k/(1 - \beta)$. The value of a vacant firm comes from the possibility of creating a job, yielding profit $f(k) - w$ for the infinite future and the continuation value if it fails to create a job. Then in the steady state, if $k \in \mathcal{H}$, $w \in \mathcal{W}(k)$, and $q = Q(w)$, $k = \eta(q)[f(k) - w] + [1 - \eta(q)]\beta k$.

If there is a unique queue length q in the steady state, then the end-of-period unemployment rate is

$$\upsilon = (1 - p) + p\frac{\delta[1 - \mu(q)]}{\delta + \mu(q)}, \tag{12}$$

where $p \equiv \int_{i \in \mathcal{P}} di$. The first term is the fraction of nonparticipants, who are counted as unemployed. The second is the fraction of unemployed participants, a group consisting of new births and previously participating workers who have not found a job yet.

PROPOSITION 7. (1) If $\{\mathcal{H}, \mathcal{W}, Q, \mathbf{U}, E, J, N, \mathcal{P}\}$ is an equilibrium, then any $k^* \in \mathcal{H}$, $w^* \in \mathcal{W}(k^*)$, and $q^* = Q(w)$ solve

$$0 = h(\psi^*) \equiv \max_{k,w,q} \mu(q)\frac{1 - \exp[-\theta(w - z - \psi^*)]}{\theta} \tag{13}$$

$$+ [1 - \mu(q)]\frac{1 - \exp[\theta(1 - \beta)\psi^*]}{\theta}$$

subject to

$$\frac{\eta(q)\,[\,f(k)\,-\,w\,]}{1\,-\,\beta[1\,-\,\eta(q)\,]}\,=\,k. \tag{14}$$

The terms E, J, and N are defined by (7), (9), and (10); and $\mathbf{U}(A)$ $=\,J(A,\,w^*,\,q^*)$. Conversely, if $\{k^*,\,w^*,\,q^*,\,\psi^*\}$ solves the program above, then there exists an equilibrium $\{\mathcal{K},\,\mathcal{W},\,Q,\,\mathbf{U},\,E,\,J,\,N,\,\mathcal{P}\}$ such that $k^* \in \mathcal{K}$, $w^* \in \mathcal{W}(k^*)$, and $q^* = Q(w)$. (2) Worker $i \in \mathcal{P}$ if and only if $x_i < \psi$. (3) An equilibrium exists. All equilibria have a common, uniquely defined, ψ^*.

As in the static model, we call a $\{k^*,\,w^*,\,q^*,\,\psi^*\}$ that solves this program an equilibrium.

Part 1 of this proposition is the analogue of proposition 1 in the static model (for the case in which $z < \bar{z}$), and its proof is omitted. Unemployed workers choose w and q to maximize $J(A,\,w,\,q)$, subject to firms' making zero profits. From equation (9), this is equivalent to maximizing dissavings while unemployed, ψ, subject to (8).

To understand part 2 of the proposition, notice from lemma 1 that asset levels do not affect the comparison of expected utilities of searching and nonparticipating workers. Therefore, there will exist a cutoff level of outside productivity, $\bar{x}$, such that all workers with higher outside productivity do not participate. This is in turn given by setting the utility of an unemployed participating worker, $\mathbf{U}(A)$, equal to the utility of the marginal nonparticipant, $N(A,\,\bar{x} + z)$, and implies $\bar{x} = \psi$. The proof of part 3 of the proposition is available on request.[14]

PROPOSITION 8. Consider two CARA utility functions with coefficients θ_1 and θ_2 and two unemployment benefits z_1 and z_2. Let $\{k_i,\,w_i,\,q_i,\,\psi_i\}$ be an equilibrium when risk aversion is θ_i and unemployment insurance is z_i, $i \in \{1,\,2\}$. If $z_1 \leq z_2$ and $\theta_1 \geq \theta_2$ and at least one of the inequalities is strict, then $k_1 < k_2$, $w_1 < w_2$, $q_1 < q_2$, and $\psi_1 < \psi_2$.

This proposition is once again proved using revealed preference arguments along the lines of proposition 2 (proof available on request), and the intuition is given by figure 2. An increase in unemployment insurance makes workers' indifference curves everywhere flatter, shifting the point of tangency to the right and raising wages, queue lengths, and capital. Similarly, when workers are more risk-averse, wages are lower, queues are shorter, and firms invest less. The added feature is the participation margin. A higher level of unemployment insurance discourages search since the threshold level

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[14] The uniqueness of ψ^* does not guarantee the uniqueness of equilibrium investments, wages, and queue lengths. It simply states that all equilibria give a common level of utility to participating workers.

of home production ψ is a decreasing function of the level of the unemployment insurance. An increase in risk aversion, on the other hand, encourages nonparticipation since this activity is riskless.

D. Risk Aversion, Unemployment Insurance, and Output

Output is equal to the expected present value of market output produced by a particular worker during her lifetime, net of any investment costs that are incurred on her behalf:[15]

$$Y(q, k, p) \equiv p \frac{\mu(q)f(k) - \beta\mu(q)k - [(1 - \beta)k/q]}{(1 - \beta)\{1 - \beta[1 - \mu(q)]\}}$$

$$= p \frac{\mu(q)w}{(1 - \beta)\{1 - \beta[1 - \mu(q)]\}},$$

(15)

where we have simplified the expression using (14). Market output is an increasing function of the participation rate p. The three terms in the numerator represent the present value of output that the worker produces, the rental cost of capital while she is producing, and the rental cost of capital to maintain $1/q$ vacancies while she is unemployed.

We begin by showing that, in the absence of "conventional" moral hazard (i.e., if all workers always participate), productive efficiency requires an unemployment insurance system.

PROPOSITION 9. Assume that G is dirac at zero, so all workers participate. Let $\{q^e, k^e\}$ be an output-maximizing allocation, and define w^e by (4) and ψ^e by (8). Then $\{z^e, \tau^e\}$ is output-maximizing if $\tau^e = \upsilon z^e$ and $z^e \in [0, w^e)$ satisfies

$$w^e \equiv \frac{1 - \beta[1 - \mu(q^e)]}{1 - \beta} \cdot \frac{\exp[\theta(w^e - z^e - \beta\psi^e)] - 1}{\theta}.$$

(16)

The proof is available on request, and the intuition is exactly the same as for proposition 6. In the risk-neutral limit ($\theta = 0$), $z^e = \tau^e = 0$; but when agents are risk-averse, output-maximizing unemployment insurance is strictly positive.

To get a sense of the importance of the effects highlighted in this model, we undertake a simple calibration exercise. We take a period to be a year and set the discount factor to $\beta = 1/R = 0.94$, and the

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[15] Another definition of output is the cross-sectional output per capita in a given period, which is a decreasing function of the population growth rate, since primarily young workers are unemployed. Our measure of output corresponds more closely to the analysis in Sec. III.

TABLE 1

CALIBRATION RESULTS FOR DIFFERENT COEFFICIENTS OF ABSOLUTE RISK
AVERSION

A.

	COEFFICIENT OF ABSOLUTE RISK AVERSION (θ)				
	0	10	20	40	100
Capital (k)	.216	.197	.167	.100	.028
Wage (w)	.232	.222	.204	.158	.084
Queue (q)	3.85	2.61	1.58	.65	.20
Dissavings (ψ)	.158	.133	.104	.063	.027

B.

	COEFFICIENT OF RELATIVE RISK AVERSION ($\theta\beta\psi$)				
	.00	1.25	1.96	2.37	2.54
Unemployment rate (u) (%)	7.2	6.5	6.0	5.6	5.4
Output (Y)	.158	.156	.147	.116	.063
Output-maximizing unemployment insurance (z^e)	.000	.088	.125	.160	.193

NOTE.—All rows but the last refer to the equilibrium without unemployment insurance. Parameter details are in the text.

birth rate is $\delta = 0.01$. The production function is $f(k) = 10k^{0.5}$. We set $\eta(q) = 1 - \exp(-.15q)$, implying that unemployed workers are hired with probability less than .15 per year. Although such low job-finding probabilities do not fit the data, this is the only way of generating reasonable unemployment rates in a model without any job destruction. Finally, we consider values of the coefficient of absolute risk aversion ranging from zero to 100. They correspond to coefficients of *relative* risk aversion between zero and 2.5 for unemployed workers with no assets and no unemployment insurance.

Table 1 shows the results. An increase in the coefficient of absolute risk aversion from zero to 10 has little effect on output because the equilibrium allocation remains in the neighborhood of the output-maximizing allocation. When risk aversion is higher, the loss in output associated with a lack of insurance is much larger. It increases from 7 percent when $\theta = 20$ to 27 percent when $\theta = 40$, and to 60 percent when $\theta = 100$. The last row gives the output-maximizing level of unemployment insurance, which increases the level of output back to the risk-neutral level, 0.158. In this calibrated economy the introduction of unemployment insurance can increase the level of output by up to 250 percent.

We next consider the effect of unemployment insurance on workers' participation. When G is a diffuse distribution, increasing unemployment insurance discourages search, so the net effect on output is generally ambiguous. If the elasticity of participation with respect to the unemployment benefit is low relative to risk aversion, the conclusions of proposition 9 will be qualitatively unchanged. Nevertheless, the highest level of output (that of the risk-neutral economy) will no longer be attainable because unemployment insurance will reduce participation. Also, output maximization will generally not entail decentralizing $\{q^e, k^e\}$ since, in the neighborhood of this allocation, unemployment insurance achieves only a second-order increase in output, whereas it results in a first-order decrease in participation. Conversely, if the elasticity of participation is high, output might be maximized for negative unemployment insurance, that is, taxing unemployment and nonparticipation.

To see whether this is likely to be the case, we return to calibrations. We first have to choose G. Atkinson and Micklewright (1991) conclude in their survey article that a 10-percentage-point increase in the replacement ratio z/w raises the average unemployment duration by about 6 percent, though some estimates are as large as 10 percent. To be conservative in calculating the benefits of unemployment insurance, we calibrate this effect to be 10 percent. Part of the effect of unemployment insurance on unemployment is captured by market-generated moral hazard since an increase in unemployment insurance leads workers to prefer longer job queues and, hence, longer unemployment durations. The remainder of the effect comes through some workers' decision not to participate.

Table 2 shows the equilibrium of the model in this case. For low levels of risk aversion, the output-maximizing level of unemployment insurance is significantly lower. Indeed, for risk-neutral workers, output-maximizing unemployment insurance is typically *negative*. Similarly, when the coefficient of absolute risk aversion is 10, accounting for conventional moral hazard reduces the output-maximizing level of unemployment insurance by a factor of eight, from 0.088 to 0.011. But for higher levels of risk aversion, moral hazard has little effect on the output-maximizing unemployment insurance, despite the fact that the reduction in search effort raises the unemployment rate significantly. For example, when the coefficient of absolute risk aversion is 40, with no unemployment insurance, the unemployment rate is 5.6 percent. With output-maximizing unemployment insurance and no conventional moral hazard, the unemployment rate rises to 7.2 percent (the risk-neutral level). Conventional moral hazard raises the unemployment rate at the output-maximizing allocation to 10.8 percent, nearly double the

TABLE 2

Output-Maximizing Unemployment Insurance for Different Coefficients
of Absolute Risk Aversion, with Worker Nonparticipation

A.

	Coefficient of Absolute Risk Aversion (θ)				
	0	10	20	40	100
Output-maximizing unemployment insurance (z)	≤0	.011	.098	.153	.193
Capital (k)	$\cdots$	.200	.207	.212	.216
Wage (w)	$\cdots$	.224	.227	.230	.232
Queue (q)	$\cdots$	2.73	3.15	3.55	3.85
Dissavings (ψ)	$\cdots$	.128	.073	.040	.018

B.

	Coefficient of Relative Risk Aversion ($\theta[\beta\psi + z - \tau]$)				
	0	1.31	3.20	7.21	19.6
Unemployment rate (u) (%)	$\cdots$	6.9	9.3	10.8	12.2
Participation rate (p) (%)	$\cdots$	99.7	97.4	95.9	94.6
Output (Y)	$\cdots$	.156	.154	.152	.150

Note.—All rows refer to the equilibrium with output-maximizing unemployment insurance. Parameter details are in the text.

level with no unemployment insurance. Still, output is about 30 percent higher than without any unemployment insurance. Furthermore, since our measure of output excludes the value of home production x_i, this exaggerates the cost of nonparticipation, making our estimates of the benefits of unemployment insurance conservative.

VI. Evidence and Extensions

A. Empirical Evidence

The main contribution of our paper is theoretical: it solves a dynamic general equilibrium search model with risk aversion and precautionary savings and provides analytical solutions for the variables of interest. In the process, we highlight a novel effect: unemployment insurance can improve the composition of jobs and increase output. Calibrations indicate that this effect may be quantitatively important. This section reviews some empirical evidence, suggestive for some of the forces emphasized in this paper.

Acemoglu (1997) analyzes the impact of unemployment benefits

on the composition of jobs and labor productivity across different U.S. states. He classifies jobs into good (high-wage) and bad (low-wage) jobs using interoccupation and interindustry wage differentials after controlling for observable human capital variables. Although part of these differences are certainly explained by compensating wage differentials and unobserved worker heterogeneity, previous research suggests that these wage differentials are also partially related to rents and the desirability of jobs (e.g., Krueger and Summers 1988; see also Murphy and Topel 1987). For example, Holzer, Katz, and Krueger (1991) show that high-wage jobs attract more job applicants, and Katz and Summers (1989) find that high-wage sectors also have higher capital/labor ratios, which is similar to our conclusion that firms offering higher wages also choose higher k.

Acemoglu then investigates how legislated changes in state-level replacement ratios for low-wage workers, between 1983 and 1993, affect the industrial and occupational composition of jobs among non–college graduates. Controlling for state and time effects, he finds that a state that increases its replacement ratio experiences an increase in the unemployment rate but also a relative increase in the number of workers in high-wage occupations and industries, and both higher labor productivity and higher labor productivity growth. In the case of industries, this is caused only by a larger decline in the employment of low-wage industries than of high-wage industries. In the case of occupations, the results are stronger: there is an increase in the number of high-wage jobs, despite the decline in overall employment. For example, a 10-percentage-point increase in the replacement ratio increases the number of high-wage occupations by 1.3 percent, suggesting that unemployment insurance may have an important effect on the types of jobs that are created.

B. Extensions

To explore the robustness of our model, we briefly discuss three variants in which all our main results obtain.

1. It is conceptually easy to allow firms to hire multiple workers. Firms choose capital k ex ante, and output is $y = F(k, l)$, where l is the number of workers that the firm hires. The distribution of the number of hires is an increasing function of the queue length, and the probability that a worker is hired is a decreasing function of the queue. In tighter labor markets, firms attract fewer workers on average and so invest less in capital.

2. Our discussion has concentrated on the risk of not getting a

job. Careers also differ in other dimensions of risk, such as the possibility of layoff, the variability of wage-tenure profiles, and the likelihood of promotion. Many discussions of unemployment insurance emphasize the riskiness of these episodes and of subsequent unemployment spells. Our qualitative results continue to hold in this case. Risk aversion implies that workers require compensation to accept jobs with high rates of separation, and this distorts the balance between different sectors. Unemployment insurance encourages workers to take more risks and firms to create riskier jobs (see Barlevy [1997] for some evidence). Because it restores balance in the production side, moderate unemployment insurance raises output.

3. Our stylized model assumes that when workers find unemployment risk more costly, firms accept additional vacancy risk and therefore reduce their ex ante capital investments. In practice, firms do maintain vacancies for long periods of time (e.g., Myers and Creamer 1967). The reason is typically not that they are unable to locate a worker, but that they are unable to locate a *suitable* worker. For example, using Dutch data, Van Ours and Ridder (1993) estimate that receiving applications takes about three weeks on average, but selecting a qualified applicant from among them takes, on average, 15 weeks.

A firm may have some latitude in choosing the *specificity* of its investment. For example, it can design a secretarial job that most workers with a high school diploma could fill, or it can open a higher-productivity job with more specific requirements, such as familiarity with a range of software, experience in the same line of business, and so on, which would be harder to fill. In this environment, if workers wish to avoid unemployment, firms will accommodate them by making less specific investments.

A slight variant of our model deals with this issue. Each firm chooses ex ante the specificity of the tasks that future employees will perform, a number α. The matching probabilities for the worker and firm, $\mu(q, \alpha)$ and $\eta(q, \alpha)$, are decreasing functions of α, so an increase in specificity raises unemployment and vacancy duration. As an offset to this, more specific jobs are more productive, so the output of a match $\phi(\alpha)$ is increasing in its specificity α. An increase in risk aversion reduces the specificity of jobs offered in equilibrium, whereas an increase in unemployment insurance raises specificity. Once again, an intermediate level of unemployment insurance maximizes output and utility. These effects may become more pronounced if workers also have to make ex ante investments: in the presence of unemployment insurance, workers may choose to specialize their skills and wait for suitable jobs, and firms would create

them. Without unemployment insurance, firms would not demand and workers would not invest in specialized skills (see Grossman and Shapiro [1982] for a partial equilibrium analysis of a similar model).

In practice, the impact of labor market tightness on job specificity may be more important than its impact on physical capital investments. For example, Murnane and Levy (1996) present case study evidence on recruitment practices of firms. In the tight labor markets of the late 1960s, Ford Motor Company recruited basically any able-bodied worker who applied. In contrast, automobile manufacturers in the high-unemployment market of the 1980s attracted a large number of applicants for each position and were extremely selective, using a series of interviews and tests to select the workers who were skilled and suited for the jobs. Consistent with our story, these new manufacturing jobs appear to be of higher quality, require more specific skills, and yield higher productivity.

VII. Conclusion

This paper has constructed a general equilibrium model of search with risk aversion. An increase in risk aversion induces workers to seek lower-wage jobs with less unemployment risk. Firms cater to these preferences by creating jobs with lower wages and lower capital intensity. Unemployment insurance has the reverse effects: insured workers want to seek riskier jobs, and the market once again caters to these preferences by creating the desired jobs, so unemployment insurance increases wages and reduces employment.

Our framework also points to a novel effect of unemployment insurance on productivity. The conventional wisdom emphasizes the trade-off between output and the risk sharing provided by unemployment insurance. We show that general equilibrium interactions in the labor market can reverse this wisdom. Unemployment insurance induces firms to invest more in capital and, in moderation, raises output. Because the distortion in the production side of the economy is due to incomplete insurance and risk aversion, unemployment insurance is the right tool to deal with this market failure. In fact, we establish that, despite the potential nonconvexities in the economy, there exists a level of unemployment insurance that restores output to its maximum level. It is important to reiterate that, despite the beneficial effects of unemployment insurance, our model does not make clear policy recommendations because it does not explain why the private sector cannot offer unemployment insurance. One can argue that adverse selection, as in Rothschild and Stiglitz (1976), might prevent this, but more analysis would be required to reach a definite conclusion.

Our analysis also contains methodological improvements over the existing search literature. We solve a dynamic general equilibrium model with risk aversion, incomplete insurance markets, and endogenous wages, productivity, consumption, and savings. To our knowledge, this is the only paper to accomplish this task. The real advantage of our framework, however, is that all the results can be obtained using a static model, which can be easily applied to other problems in search theory and labor economics.

Appendix

Proofs of the Main Results

Proof of Proposition 1

We prove this proposition in the following order: Step 1 establishes that any equilibrium solves the constrained optimization problem, step 2 shows that any allocation that solves this program is part of an equilibrium, and step 3 proves existence.

Step 1

Let $\{\mathcal{H}, \mathcal{W}, Q, U\}$ be an equilibrium allocation with $k^* \in \mathcal{H}$, $w^* \in \mathcal{W}(k^*)$, and $q^* = Q(w)$. We must prove that $\{k^*, w^*, q^*\}$ solves the constrained optimization problem. First, profit maximization ensures that $\{k^*, w^*, q^*\}$ satisfies constraint (2). Next, optimal application implies

$$U = \mu(q^*)u(A + w^*) + [1 - \mu(q^*)]u(A + z) \geq u(A + z),$$

so $w^* \geq z$, satisfying constraint (3).

Suppose now that another triple $\{k, w, q\}$ satisfies constraint (3) and achieves a higher value of the objective:

$$\mu(q)u(A + w) + [1 - \mu(q)]u(A + z) > U.$$

We shall prove that it must violate constraint (2). Since $\{\mathcal{H}, \mathcal{W}, Q, U\}$ is an equilibrium, optimal application implies

$$\mu(Q(w))u(A + w) + [1 - \mu(Q(w))]u(A + z) \leq U.$$

Since $w \geq z$ by (3), these inequalities imply $\mu(q) > \mu(Q(w))$, so $q < Q(w)$. Thus

$$\eta(q)[f(k) - w] - k < \eta(Q(w))[f(k) - w] - k \leq 0,$$

where the weak inequality exploits profit maximization. Therefore, $\{k, w, q\}$ violates (2) and is not in the constraint set.

Step 2

We now prove by construction that for any solution $\{k^*, w^*, q^*\}$ to the constrained maximization problem, there is an equilibrium $\{\mathcal{K}, \mathcal{W}, Q, U\}$ with $\mathcal{K} = \{k^*\}$, $\mathcal{W}(k^*) = \{w^*\}$, and $Q(w^*) = q^*$. Set

$$U = \mu(q^*)u(A + w^*) + [1 - \mu(q^*)]u(A + z),$$

and let $Q(w)$ satisfy

$$U = \mu(Q(w))u(A + w) + [1 - \mu(Q(w))]u(A + z),$$

or $Q(w) = 0$ if there is no solution to the equation, in particular if $w < z$. It is immediate that $\{\mathcal{K}, \mathcal{W}, Q, U\}$ satisfies optimal application.

We now show that it also satisfies profit maximization. Suppose to the contrary that some triple $\{k', w', Q(w')\}$ violates profit maximization, so

$$\eta(Q(w'))[f(k') - w'] - k' > 0.$$

One implication of this is that $Q(w') > 0$. Since the previous paragraph argued that $Q(w') = 0$ for all $w' < z$, this implies $w' \geq z$. Another implication is that we can choose $q' < Q(w')$ such that

$$\eta(q')[f(k') - w'] - k' = 0.$$

By the construction of $Q(\cdot)$, $q' < Q(w')$ and $w' \geq z$ imply

$$U = \mu(q')u(A + w') + [1 - \mu(q')]u(A + z).$$

We have shown that $\{k', w', q'\}$ satisfies both constraints and yields a higher value of the objective than $\{k^*, w^*, q^*\}$, a contradiction.

Step 3

For existence, we must consider two separate cases. The more interesting possibility is that $z < \bar{z}$. This ensures that the constraint set is nonempty; for example, $\{k, w, q\} = \{\bar{k}, \bar{z}, \infty\}$ satisfies both constraints. The constraint set is also compact and the objective function (1) is continuous. This implies that a solution to the constrained maximization problem exists, and by step 2 above, this solution is an equilibrium.

We also note in passing that if $z < \bar{z}$, there is no equilibrium in which $\mathcal{K} = \overline{\mathcal{W}} = \varnothing$. If there were, a firm could enter at a wage of $w = \frac{1}{2}(z + \bar{z})$, using capital k satisfying $w = f(k) - kf'(k)$. Optimal application implies that it would attract an infinite queue and so would hire a worker with probability one. Its profits would be $k[f'(k) - 1] > 0$ (since $w < \bar{z}$, $k < \bar{k}$, and $f'(k) > 1$), violating profit maximization.

The other possibility is $z > \bar{z}$, which implies that the constraint set of the maximization problem is empty. Step 1 implies that if an equilibrium exists, $\mathcal{K} = \overline{\mathcal{W}} = \varnothing$. We show that one does. Consider a firm that deviates from the proposed equilibrium, buying capital $k > 0$ and posting a wage. If $w < z$, optimal application implies $Q(w) = 0$, so $\eta(Q(w)) = 0$ as well. Thus the deviation is unprofitable, leaving the firm with profit $-k$. On the other hand, if $w > z > \bar{z} = f'(\bar{k}) - \bar{k}$, the firm's profit is

$$\mu(Q(w))[f(k) - w] - k < f(k) - w - k$$
$$< [f(k) - k] - [f(\bar{k}) - \bar{k}] < 0$$

by concavity of f and $f'(\bar{k}) = 1$, so the deviation is again unprofitable. Q.E.D.

Proof of Proposition 2

We begin by introducing a critical property, *revealed preference*: if $\{w_i, q_i, k_i\}$ is the equilibrium with unemployment insurance z_i, preferences u_i, and assets A_i, then

$$\mu(q_1)[u_1(A_1 + w_1) - u_1(A_1 + z_1)]$$
$$\geq \mu(q_2)[u_1(A_1 + w_2) - u(A_1 + z_1)].$$

Because $\{w_2, q_2, k_2\}$ is an equilibrium with z_2, u_2, and A_2, (2) must be satisfied. Then if this inequality were false, $\{w_1, q_1, k_1\}$ would not maximize (1) with z_1, u_1, and A_1: $\{w_2, q_2, k_2\}$ would do better. We exploit revealed preference throughout this Appendix.

1. Take u_1 more risk-averse than u_2. Revealed preference implies

$$\mu(q_1)[u_1(A + w_1) - u_1(A + z)] \geq \mu(q_2)[u_1(A + w_2) - u_1(A + z)],$$
$$\mu(q_2)[u_2(A + w_2) - u_2(A + z)] \geq \mu(q_2)[u_2(A + w_1) - u_2(A + z)].$$

Multiply these inequalities and simplify:

$$[u_1(A + w_1) - u_1(A + z)][u_2(A + w_2) - u_2(A + z)]$$
$$\geq [u_1(A + w_2) - u_1(A + z)][u_2(A + w_1) - u_2(A + z)]. \tag{A1}$$

Suppose $w_1 > w_2 > z$. By the intermediate value theorem, there exists a $\lambda \in (0, 1)$ with

$$u_2(A + w_2) \equiv \lambda u_2(A + w_1) + (1 - \lambda) u_2(A + z). \tag{A2}$$

Since u_1 is a strictly concave transformation of u_2,

$$u_1(A + w_2) > \lambda u_1(A + w_1) + (1 - \lambda) u_1(A + z). \tag{A3}$$

Eliminating $u_2(A + w_2)$ and $u_1(A + w_2)$ from (A1) using (A2) and (A3), we obtain

$$\lambda[u_1(A + w_1) - u_1(A + z)][u_2(A + w_1) - u_2(A + z)]$$
$$> \lambda[u_1(A + w_1) - u_1(A + z)][u_2(A + w_1) - u_2(A + z)],$$

a contradiction, proving $w_2 \geq w_1$. By (4), $k_2 \geq k_1$. Then the optimal capital choice relationship $\eta(q^*)f'(k^*) = 1$ defines an increasing relation between q and k, $q_2 \geq q_1$. And both inequalities are strict if and only if $w_2 > w_1$, which is true because the constraint set and the objective function are smooth, and so all these inequalities are strict (see Edlin and Shannon 1998).

2. Next consider the effect of assets. We show that with DARA, there is a strictly concave transformation v such that $u(A_1 + c) \equiv v(u(A_2 + c))$ for $A_1 < A_2$ and for all c. That v exists and is C^2 follows from the assumptions

on the utility function. To prove that v is strictly concave, twice-differentiate its definition with respect to c:

$$u'(A_1 + c) \equiv v'(u(A_2 + c))u'(A_2 + c),$$

$$u''(A_1 + c) \equiv v'(u(A_2 + c))u''(A_2 + c) + v''(u(A_2 + c))u'(A_2 + c)^2.$$

Use the first relationship to eliminate v' from the second one:

$$-\frac{u''(A_1 + c)}{u'(A_1 + c)} = -\frac{u''(A_2 + c)}{u'(A_2 + c)} - \frac{v''(u(A_2 + c))u'(A_2 + c)^2}{u'(A_1 + c)}.$$

Since $A_1 < A_2$ and u has DARA,

$$-\frac{u''(A_1 + c)}{u'(A_1 + c)} > -\frac{u''(A_2 + c)}{u'(A_2 + c)}.$$

Thus the last term above must be positive, which can be the case only if v'' is negative, proving that v is concave. The arguments with CARA and IARA are analogous. When this transformation has been established, part 1 of this proposition completes the proof.

3. Finally, we establish the effects of unemployment insurance. Analogous with (A1), revealed preference implies

$$[u(A + z_1) - u(A + z_2)][u(A + w_1) - u(A + w_2)] \geq 0.$$

As $z_1 < z_2$, we have $w_1 \leq w_2$. The rest of the argument follows as in part 1 to establish $w_2 > w_1$, $q_2 > q_1$, and $k_2 > k_1$. Q.E.D.

Proof of Proposition 4

Let $u(x) = ax + b$. The objective (1) becomes a linear transformation of $\mu(q)w$. Eliminate w using the constraint, noting that $\eta(q) \equiv q\mu(q)$. The resulting unconstrained maximization problem is identical to the output maximization problem, $\max_{q,k} Y(q, k)$. Q.E.D.

Proof of Proposition 6

We first prove that $\{z^e, \tau^e\}$ exists. The government budget constraint defines τ^e as a continuous function of z, mapping $[0, [1 - \mu(q^e)]w^e]$ into itself. Next, observe that the right-hand side of (5) is continuous and decreasing in z. Also, concavity of u implies $u(A_0 - \tau + w^e) > u(A_0 - \tau) + u'(A_0 - \tau + w^e)w^e$, so the right-hand side of (5) is bigger than w^e when z is zero, and it is equal to $0 < w^e$ when $z = w^e$. Thus (5) defines z^e as a continuous function of τ, mapping $[0, w^e]$ into itself. Finally, existence is guaranteed by Brouwer's fixed-point theorem.

Now we prove that $\{w^e, q^e, k^e\}$ is an equilibrium of the model. From proposition 1, we must show that any alternative allocation $\{w', q', k'\}$ that gives firms zero profits must give workers less utility. Note that $w^e \neq w'$. Otherwise, $k^e = k'$ by (4) and $q^e = q'$ by firms' optimal capital choice $\eta(q)f'(k) = 1$, making the two allocations identical.

We begin with the result from proposition 4. The output-maximizing

allocation is an equilibrium with risk-neutral workers and $z = \tau = 0$, so proposition 1 implies that risk-neutral workers with no unemployment insurance weakly prefer $\{w^e, q^e, k^e\}$ to $\{w', q', k'\}$:

$$\mu(q^e)w^e \geq \mu(q')w'. \tag{A4}$$

Next, since the ratio of a nonnegative, strictly concave function to a nonnegative linear function is a nonnegative, strictly quasi-concave function,

$$\frac{u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)}{w}$$

is strictly quasi-concave in w for $w > z^e$. Moreover, one can confirm that (5) is the first-order condition from choosing w to maximize this function. Therefore,

$$\frac{u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)}{w^e}$$

$$> \frac{u(A_0 - \tau^e + w') - u(A_0 - \tau^e + z^e)}{w'}, \tag{A5}$$

where the strict inequality holds because $w^e \neq w'$. Now multiply (A4) and (A5):

$$\mu(q^e)[u(A_0 - \tau^e + w^e) - u(A_0 - \tau^e + z^e)]$$
$$> \mu(q')[u(A_0 - \tau^e + w') - u(A_0 - \tau^e + z^e)].$$

That is, the worker strictly prefers $\{w^e, q^e, k^e\}$ to $\{w', q', k'\}$. This is an equilibrium, and given the strict inequality, there can be no other equilibrium. Q.E.D.

Proof of Lemma 1

Parts 1 and 3 of this lemma are easily proved. For a worker employed at the wage w in all future periods, the optimal consumption path is constant since $\beta R = 1$. Thus the budget constraint and transversality condition require

$$c_t^e = w + \frac{R-1}{R}A_t - \tau = w + (1 - \beta)A_t - \tau,$$

so $A_{t+1} = A_t$ and $c_{t+1}^e = c_t^e$ for all t. The value of a worker employed at w is then given by (7). The proof of part 3 is identical.

Next consider an unemployed worker. He looks for a job and then chooses his consumption. If he finds a job at time t, we know from above that he will consume $w + (1 - \beta)A_t - \tau$. If he fails to find a job, he earns net benefit $z - \tau$ and consumes c_t^u. Conjecture a linear consumption rule as a function of assets, with marginal propensity to consume out of assets equal to $1 - \beta$. This conjectured consumption function can be written as $c_t^u = \beta\psi + (1 - \beta)A_t + z - \tau$, for some unknown parameter ψ, leaving him with assets $A_{t+1} = A_t - \psi$ next period. To solve for ψ, use the Euler equation:

$$\exp\{-\theta[\beta\psi + (1-\beta)A_t + z - \tau]\} = \mu(q)\exp\{-\theta[w + (1-\beta)A_{t+1} - \tau]\}$$
$$+ [1 - \mu(q)]$$
$$\times \exp\{-\theta[\beta\psi + (1-\beta)A_{t+1} + z - \tau]\}.$$

Multiply both sides of this by $\theta[\beta\psi + (1-\beta)A_t + z - \tau]$ and use $A_{t+1} = A_t - \psi$ to obtain (8), which implicitly defines $\psi \in (0, w - z)$ as a function of w, q, and z.

With this consumption rule, the value function $J(A_t, w', q')$ can be written as

$$J(A_t, w', q') = \mu(q')E(A_t, w') + [1 - \mu(q')]$$

$$\times \left(\frac{1 - \exp\{-\theta[\beta\psi + (1-\beta)A_t + z - \tau]\}}{\theta} + \beta J(A_t - \psi, w', q') \right). \quad (A6)$$

Note that this equation is written for any (w', q'), not necessarily for the same (w, q) that solves (8). We have already solved for $E(A_t, w')$ in equation (7). Now conjecture the form of J as in (9) and substitute this into (A6). This yields

$$J(A_t, w', q') = \mu(q) \frac{1 - \exp\{-\theta[w + (1-\beta)A_t - \tau]\}}{\theta(1-\beta)}$$

$$+ [1 - \mu(q)] \frac{1 - \exp\{-\theta[\beta\psi + (1-\beta)A_t + z - \tau]\}}{\theta(1-\beta)}.$$

Eliminate ψ using its definition in equation (8). The resulting expression reduces to the definition of J in (9), confirming our conjecture. Q.E.D.

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