HOLDUPS AND EFFICIENCY WITH SEARCH FRICTIONS *

By   Daron Acemoglu and Robert Shimer[1]

*Massachusetts Institute of Technology
and Princeton University, U.S.A.*

A natural holdup problem arises in a market with search frictions: Firms have
to make a range of investments before finding their employees, and larger in-
vestments translate into higher wages. In particular, when wages are determined
by ex post bargaining, the equilibrium is always inefficient: Recognizing that
capital-intensive production relations have to pay higher wages, firms reduce
their investments. This can only be prevented by removing all the bargaining
power from the workers, but this, in turn, depresses wages below their social
product and creates excessive entry of firms. In contrast to this benchmark, we
show that efficiency is achieved when firms post wages and workers can direct
their search toward more attractive offers. This efficiency result generalizes to
an environment with imperfect information where workers only observe a few of
the equilibrium wage offers. We show that the underlying reason for efficiency is
not wage posting per se, but the ability of workers to direct their search toward
more capital-intensive jobs.

1.   INTRODUCTION

An investment is *held up* if one party must pay the cost while others share in the
payoff. Williamson (1975) and Grout (1984) show that incomplete contracts are the
underlying cause of holdups: With complete contracts, all those who benefit from
an investment can be forced to pay their share of the cost. Even in the presence of
incomplete contracts, if agents arrange their relationships appropriately, holdups are
often preventable. Before making investments, agents can reallocate property rights
(Williamson, 1975; Grossman and Hart, 1986; Hart and Moore, 1990), impose simple
breach remedies (MacLeod and Malcomson, 1993; Edlin and Reichlestein, 1996), or
enter into long-term relations (Williamson, 1975).

In many situations, however, investments must be sunk before agents meet. For
example, a firm must build a factory before it can hire workers, and similarly, work-
ers must complete their education before finding jobs (Acemoglu, 1996, 1997; Davis,
1995; Masters, 1998). In such cases, contracts and related arrangements are impos-
sible, because agents do not know who their partners will be at the time they invest
(Acemoglu, 1996). This suggests that holdup problems may be much more serious
in the presence of trading frictions. This article analyzes the potential for holdups

in markets with frictions and examines how markets can internalize the resulting ex- ternalities. Our main result is that despite pervasive market incompleteness, the de- centralized equilibrium is efficient under fairly mild conditions. It solves the holdup problem and creates the right incentives for firms to invest in capital and to partici- pate in the production process.

We first show that when firms make ex ante investments before matching with workers and wages are determined by ex post bargaining, the equilibrium is always inefficient. Either wages increase with output, creating a holdup problem for firms' investments, or all the bargaining power is vested in the firm, leading to very low wage levels and excessive entry of firms. We then turn to an economy in which firms post wages and workers direct their search toward different firms. We establish that in this case the equilibrium is efficient. Our results therefore extend those of Moen (1997) and Shimer (1996), who show that wage posting can achieve efficiency in the standard search environment. In these models, where firms do not have ex ante investment decisions, efficiency only demands that the economy creates the right number of jobs. Since Diamond (1982) and Hosios (1990) find that even in the standard search and bargaining model, the equilibrium is efficient for an appropriate bargaining solution, one can interpret Moen's and Shimer's results as showing that wage posting picks the efficient distribution of bargaining power. With ex ante investments, however, no bargaining solution achieves efficiency, and so our efficiency result is much more striking.

Also surprising are our results regarding the role of information: We find that full information, whereby workers observe all wage offers, is not necessary for this effi- ciency result. In particular, it is sufficient for each worker to observe the wage offers of two random firms. When each firm knows that every worker who has observed its wage has also observed at least one other wage, there will effectively be Bertrand competition among firms, and this competition ensures efficiency.

Finally, we examine why ex ante wage offers are so effective in achieving efficiency. We conclude that the key feature is the ability of workers to direct their search toward firms with different levels of capital, not toward those offering higher wages per se. For example, if firms can commit to pay a certain share of the output to their employees (perhaps by committing to play a particular bargaining game) but workers do not observe the ex ante capital choices, efficiency is *not* achieved. This is so because a firm that increases its capital stock is still subject to the holdup problem. In contrast, if firms commit to different sharing rules *and* workers can observe investments before directing their search, the equilibrium is *always* efficient. This finding is useful in understanding the essence of our results: When workers have bargaining power, a firm that invests more does not receive the full benefit of its investment. Nevertheless, when ex ante investments are observed by workers, this attracts more unemployed workers, since workers recognize that a larger investment translates into higher wages. The increase in profits from attracting more workers, and thus filling vacancies faster, can offset the reduction in profits due to higher wages. Moreover, when firms can commit to a bargaining share, they choose a sharing rule that ensures that the decline in profits due to higher wages from investments is *exactly* offset by faster job creation. The holdup problem is therefore avoided. Wage posting solves the holdup problem via the same channel, since it is formally equivalent

to a setup where firms post sharing rules and workers observe investments before their application decisions. In essence, our economy therefore achieves efficiency because it encourages workers to direct their search toward more capital-intensive firms and enables firms to commit to the appropriate sharing rule.

The plan of this article is as follows: Section 2 describes the nature of the trading frictions. Section 3 derives the constrained efficient allocation. Section 4 derives the equilibrium when wages are determined by ex post bargaining and demonstrates the holdup inefficiency. Section 5 proves that when firms post wages in order to attract workers, holdups and search externalities are internalized, even if workers have very limited information about the available wages. Section 6 explores the origins of this efficiency result. Section 7 concludes, and the Appendix contains proofs and technical details.

2. THE ENVIRONMENT

There is a continuum of risk-neutral workers and a larger continuum of risk-neutral firms. All agents live forever in continuous time and discount the future at the common rate r. Firms are inactive until they buy some capital $k > 0$ at marginal cost p, which allows them to attempt to hire an unemployed worker by posting a vacancy. Holdups arise because firms must invest in capital before meeting a worker, and workers may reap some of the benefits from larger investments. If a firm employs one worker and k units of capital, it produces a flow of output $f(k)$ with a price normalized to one. We assume that f is strictly increasing and concave and satisfies the usual Inada conditions. Finally, each piece of capital breaks down with flow probability s, in which case the worker becomes unemployed and the firm becomes inactive. Unemployed workers receive a flow payoff of zero.

Matching is frictional. Suppose in some labor market there are $Q \in [0, \infty]$ unemployed workers seeking each vacancy. When workers do not direct their search toward any particular group of firms, Q is the (market) queue length, or the inverse of the labor market tightness. This is, for example, the case analyzed in the standard bargaining models of Diamond (1982) and Mortensen and Pissarides (1994). Otherwise, different vacancies may be associated with different queue lengths. Matching frictions are modeled using a standard constant returns to scale matching technology. Each worker matches with a firm with flow probability $\mu(Q)$, and each vacancy matches with a worker with flow probability $\eta(Q) \equiv Q\mu(Q)$, and we have $\mu' < 0$ and $\eta' > 0$. We also assume that μ and η map the extended positive real numbers $[0, \infty]$ onto themselves, so $\mu(0) = \eta(\infty) = \infty$ and $\mu(\infty) = \eta(0) = 0$. In words, if there are very few unemployed workers per vacancy, workers find jobs arbitrarily quickly and firms cannot hire workers, and conversely if there are many unemployed workers per vacancy. We also assume that η is concave.[2] These technical conditions guarantee the existence of interior equilibria and efficient allocations. A match is consummated—turned into an employment relation—upon the agreement of both parties.

---

[2] This last assumption is extremely weak. Since $\mu(q) \equiv \eta(q)/q$ is decreasing, $\eta'(q) < \eta(q)/q$. Together with the assumption that $\eta(0) = 0$, this is *almost* a statement of concavity.

3. THE EFFICIENT ALLOCATION

An allocation is (constrained) efficient if it maximizes the net output of the economy subject to search and informational restrictions, the standard definition in this literature. In Appendix A, we use optimal control theory to solve rigorously for the time path of the market queue length Q and capital investment level k that maximize the value of net output. Here we provide a more intuitive derivation of the efficient allocation.

Let λ denote the shadow flow value of an unemployed worker in steady state so that an additional unemployed worker raises steady-state output by λ/r. Then, a recursion defining λ can be written as:

$$(1) \qquad \lambda = \max_{k,Q} \mu(Q)\left[\frac{f(k)-\lambda}{r+s} - pk\right] - \frac{(r+s)pk}{Q}$$

Since workers are homogeneous, it is efficient to turn all matches into employment relations. Thus each unemployed worker is hired at the flow rate $\mu(Q)$, yielding a flow of output $f(k)$ until the match ends. Accounting for both impatience r and match destruction s, the present value of this gross output is $f(k)/(r+s)$. From this, we must net out the flow labor cost, which is by definition λ, and the cost of using k units of capital until it breaks, pk. In addition, while the worker is unemployed, the economy sustains $1/Q$ vacancies for him or her. Since each vacancy uses k units of capital, the flow cost of maintaining these vacancies is $(r+s)pk/Q$. An efficient allocation can now be defined as an allocation that maximizes λ, the shadow value of unemployed workers.

Solving Equation (1) for λ yields a convenient characterization of the efficient allocation:

Proposition 1 *An efficient steady-state allocation exists. It is characterized by a pair* $(k^S, Q^S) \in (0,\infty)^2$ *(i.e., an interior solution) solving*

$$(2) \qquad \max_{k,Q} \frac{\eta(Q)f(k) - [r+s+\eta(Q)](r+s)pk}{(r+s)Q + \eta(Q)}$$

PROOF.   See Appendix.

The maximization problem Equation (2) is not jointly concave in k and Q, and so the first-order conditions are not sufficient for a maximization. Nevertheless, because the efficient allocation is an interior solution to the maximization problem, the first-order conditions are necessary, so they will be useful in recognizing *inefficient* allocations. These conditions are given in the following corollary.

Corollary 1 *The efficient steady-state allocation* (k^S, Q^S) *satisfies*

$$(3) \qquad p = \frac{\eta(Q^S)}{r+s+\eta(Q^S)} \cdot \frac{f'(k^S)}{r+s}$$

$$(4) \qquad p = \frac{\eta(Q^S) - Q^S\eta'(Q^S)}{r+s+\eta(Q^S)+(1-Q^S)\eta'(Q^S)} \cdot \frac{f(k^S)/k^S}{r+s}$$

Equation (3) is easily interpreted. The first fraction on the right-hand side is the current value of a dollar when the vacancy is first filled. This value is discounted because of both impatience and the possibility that the vacancy may be destroyed before it is filled. The second fraction is the discounted marginal product of the job if it is filled. Thus the right-hand side represents the present marginal value of a vacancy using k units of capital. This must be equal to the marginal cost of capital p at an optimum. Equation (4), in turn, requires that the cost of creating another vacancy equals the expected revenue. Namely, it equates the cost of opening one more vacancy pk^S to the additional social value from this vacancy, the right-hand side of Equation (4) (times k^S). The expression for this social value is somewhat complicated, since it takes into account the reduction in the matching probabilities of other vacancies.

4. WAGE BARGAINING

This section examines the search environment of Diamond (1982), Pissarides (1990), and Mortensen and Pissarides (1994). The preferences and production and search technologies are as specified in Section 2. In contrast to Section 3, the economy is decentralized, and wages are determined by bargaining between workers and firms, after the firm has made its investment and contacted the worker. As a result, when the firm makes its investment, it must anticipate how the bargained wage will depend on capital. This contractual incompleteness is the source of holdups in the bargaining model.

Let $\mathcal{K}$ denote the set of capital investments made in equilibrium and $J^V(k)$ denote the expected present value of a firm with a vacancy and k units of capital. An equilibrium must satisfy four conditions: (1) when firms enter the market, they make a profit-maximizing capital investment, so k maximizes $J^V(k') - pk'$ if $k \in \mathcal{K}$; (2) firms entering the market earn zero profits, $J^V(k) - pk = \varnothing$ if $k \in \mathcal{K}$; (3) matches are accepted only if it is in the mutual interest of the worker and firm; and (4) wages are determined by bilateral bargaining between employed workers and firms. To begin, we summarize the result of bargaining by the wage equation $w = w(k)$. This equation is conditional on k, since wages may depend on the size of the firm's irreversible investment. Later we discuss the specification of the bargaining game in more detail.

4.1. *Analysis.* We start by writing the Bellman equations that determine the profit of firms in different states. Since the focus of this article is on steady states, we suppress time dependence.[3]

$$(5) \qquad\qquad rJ^F(k) = f(k) - w(k) - sJ^F(k)$$

---

[3] At this point we assume that workers accept any match, as occurs in efficient allocation. This assumption is not restrictive, because the goal of this section is to show that the equilibrium of the standard search model is always inefficient. If in some equilibria some matches are not turned into employment, the equilibrium is necessarily inefficient. In the next subsection, we verify that with Nash bargaining all matches are accepted in equilibrium.

$J^F(k)$ is the asset value of a filled vacancy with capital k. It generates a flow of output $f(k)$, pays a wage $w(k)$, and gets destroyed at the rate s.

$$(6) \qquad rJ^V(k) = \eta(Q)(J^F(k) - J^V(k)) - sJ^V(k)$$

The value of a vacancy with capital k is due to the possibility of generating a match, which happens at the rate $\eta(Q)$. In the meantime, the equipment breaks down at the rate s.

Equations (5) and (6) imply that

$$(7) \qquad J^V(k) = \frac{\eta(Q)}{r+s+\eta(Q)} \cdot \frac{f(k) - w(k)}{r+s}$$

Again, the first fraction represents the time required to fill a vacancy. The second fraction is the present value of profits for a filled job, as a function of the capital stock.

Using Equation (7), profit maximization implies that any $k \in \mathscr{K}$ solves

$$(8) \qquad \frac{\eta(Q)}{r+s+\eta(Q)} \cdot \frac{f'(k) - w'(k)}{r+s} = p$$

Comparing Equations (3) and (8), a necessary condition for the equilibrium to be optimal is $w'(k^S) = 0$. This formalizes the notion that efficiency requires a solution to the holdup problem. If w is strictly increasing, firms anticipate that investing more amounts to bargaining to a higher wage. Since workers do not share in the cost of ex ante investments, this leads to underinvestment. This holdup is avoided only if w is constant in the neighborhood of the efficient capital stock, k^S.

Again using Equation (7), free entry implies that

$$(9) \qquad \frac{\eta(Q)}{r+s+\eta(Q)} \cdot \frac{f(k) - w(k)}{r+s} = pk$$

Comparing this with Equation (3) yields a second necessary condition for the equilibrium to be optimal, $w(k^S) = f(k^S) - k^S f'(k^S)$. Firms must earn the marginal product of capital, while workers keep the residual. If, for example, workers earned a zero wage, which effectively solves the holdup problem as $w'(k) = 0$, the return on capital would exceed the marginal product, attracting excessive entry. At the root of the excessive entry result is the fact that firms create a negative externality when they enter, since they make it harder for other firms to find workers. At the same time, they create a positive externality on workers because they increase the probability that workers find employment. If wages are very low, the positive externality vanishes, so entry is excessive. In summary, optimality pins down the *level* and *slope* of the wage function in a neighborhood of the efficient level of capital k^S, namely, to achieve efficiency in the bargaining equilibrium, we need $w(k^S) = f(k^S) - k^S f'(k^S)$ *and* $w'(k^S) = 0$.

4.2. *Bargaining Game.* We show that these conditions are never satisfied simultaneously if wages are determined by the Nash bargaining solution, the usual assumption in this literature. In Appendix B we prove the same result for any "regular" bargaining game. Nash bargaining implies that for all k,

$$(10) \qquad \beta[J^F(k) - J^V(k)] = (1 - \beta)[J^E(k) - J^U]$$

where β is the bargaining power of the worker, J^U is the value of an unemployed worker, and $J^E(k)$ is the value of an employed worker in a job with capital k, defined by

$$(11) \qquad rJ^E(k) = w(k) + s[J^U - J^E(k)]$$

His or her flow value equals his or her wage minus the expected loss he or she suffers from job destruction, which occurs at the exogenous rate s.

Solving equations (5) and (6) for $J^F(k) - J^V(k)$ and Equation (11) for $J^E(k) - J^U$ and simplifying Equation (10), we obtain

$$f(k) - w(k) = \frac{(1 - \beta)[r + s + \eta(Q)]}{r + s + (1 - \beta)\eta(Q)}[f(k) - rJ^U]$$

Since the production function f is concave in the firm's investment level k, this implies that net revenue $f - w$ is concave as well. Then Equation (8) implies the investment level in the bargaining equilibrium k^B is the unique solution to

$$(12) \qquad \frac{(1 - \beta)\eta(Q)}{r + s + (1 - \beta)\eta(Q)} \cdot \frac{f'(k)}{r + s} = p$$

Comparing with Equation (3), a necessary condition for efficiency is $\beta = 0$, so the firm has all the bargaining power.

Now we can calculate the value of an unemployed worker.

$$(13) \qquad rJ^U = \mu(Q)[J^E(k^B) - J^U]$$

The value of an unemployed worker is equal to the flow probability that he or she finds a match, times the net present value gain from employment. This equation uses the fact that, in equilibrium, all firms make the same investment k^B, and so all matches are accepted by workers as stated earlier. Then Equation (10) implies that

$$(14) \qquad w(k^B) = \frac{\beta[r + s + \mu(Q)]f(k^B)}{r + s + \beta\mu(Q) + (1 - \beta)\eta(Q)}$$

Plugging this into Equation (9) yields the other necessary and sufficient conditions for a bargaining equilibrium:

$$(15) \qquad \frac{(1 - \beta)\eta(Q)}{r + s + \beta\mu(Q) + (1 - \beta)\eta(Q)} \cdot \frac{f(k^B)/k^B}{r + s} = p$$

This allows us to characterize an equilibrium.

Proposition 2 *A steady-state search-bargaining equilibrium is summarized by a pair* $(k^B, Q^B) \in (0, \infty)^2$ *that solves Equations (12) and (15). An equilibrium exists if and only if* $\beta > 0$. *If the elasticity of the production function* $kf'(k)/f(k)$ *is nonincreasing, the equilibrium is unique.*

An equilibrium never coincides with the efficient allocation (Q^S, k^S). *In particular,*

1. *If* $0 \leq \beta < Q^B \eta'(Q^B)/\eta(Q^B)$, *either* $Q^B > Q^S$ *and* $k^B > k^S$ *or* $Q^B < Q^S$ *and* $k^B < k^S$.
2. *If* $Q^B \eta'(Q^B)/\eta(Q^B) \leq \beta \leq 1$, *either* $Q^B > Q^S$ *or* $k^B < k^S$.

That is, either firms underinvest $(k^B < k^S)$, *or entry is too low* $(Q^B > Q^S)$, *or both.*

PROOF.   See Appendix A.

This proposition extends Hosios' (1990) results, which showed that without a capital choice, the equilibrium is optimal if and only if the worker's bargaining share is equal to the elasticity of the matching function, $\beta = Q^S \eta'(Q^S)/\eta(Q^S)$. We refer to this as the *Hosios condition*. Suppose that capital k is exogenous, as in the standard search and bargaining models. Then the efficient allocation is characterized by the queue length Q^S satisfying Equation (4), and the equilibrium allocation has queue length Q^B given by Equation (15). In this case, $Q^S = Q^B$ if and only if the Hosios condition holds. However, with endogenous capital investment, this bargaining share leads to holdup problems, as shown by Equations (3) and (12).[4] Therefore, even though it is possible for the level of wages to equal the social shadow value of labor, it is impossible to ensure that both the level and the slope of the wage function are equal to the appropriate social values.

In light of Hosios' results, search environments are often viewed as quite "neo-classical": With the right choice of institutional structure to determine the bargaining strengths of labor and capital, the level of wages is equal to the shadow value of labor, and efficient allocation is achieved. Even leaving aside the difficulty of fine-tuning bargaining strengths, Proposition 2 shows that decentralization is impossible when there are ex ante investments. Search frictions prevent ex ante contracting, so wages are forced to accomplish two tasks: encourage investment and discourage entry. With ex post bargaining, these cannot be achieved simultaneously.

5. WAGE POSTING

This section considers a variant of the standard search model. Firms commit to and post wages before meeting workers in an effort to attract applicants. Peters (1991), Montgomery (1991), Shimer (1996), Moen (1997), and Acemoglu and Shimer (1999) also have analyzed such a setup. Following these articles, we assume for now that workers have full information about posted wages. That is, they observe all posted wages and then decide which of these to seek. In this decision, they recognize that

[4] This result is related to Corollary 2 in Acemoglu (1996), which shows that in a two-period model with firm and worker investments, the equilibrium is inefficient: A high bargaining power for workers distorts physical capital investments, while a low bargaining share of workers distorts human capital investments. Here, there are no human capital investments, but a low bargaining power for workers distorts the entry margin, which is unmodeled in Acemoglu (1996).

if the ratio of workers who are seeking vacancies at wage w to firms offering w is $q \equiv q(w)$, then each worker applying to this wage is hired (and hence actually receives the wage) with flow probability $\mu(q)$. Symmetrically, each firm offering this wage expects that it will fill its vacancy at the rate $\eta(q) \equiv q\mu(q)$.

We distinguish between the "market" queue length Q and the queue length associated with a particular wage $q(w)$. With ex post bargaining as in Section 4, capital-intensive jobs yielded higher wages (Equation 14), but workers' application decisions did not respond to these incentives. This was either because workers could not observe firms' capital choices before making their applications or because they were unable to direct their search. As a result in the bargaining model, all jobs necessarily had a common (market) queue length Q. In contrast, this section allows workers to adjust their application decisions in response to wage differentials, so different wages are generally associated with different queue lengths $q(w)$. More precisely, if firms offer different wages in equilibrium, then queue lengths will adjust so that workers are indifferent about which wage to seek. Higher wages will be associated with longer queues.

Search frictions are often interpreted as representing the time required to learn about a job opening. Since we assume here that workers know about all the available wages, they require a different interpretation. One possibility is that firms locate in different geographic or industrial "labor markets." Workers know the wage associated with each labor market. If they attempt to get a job in a labor market offering a wage of w, they recognize that there will be on average $q(w)$ other workers competing for each job opening. Matching frictions exist within individual labor markets, however, so the worker is hired with probability $\mu[q(w)]$. An alternative interpretation follows Peters (1991), Montgomery (1991), and Burdett et al. (1997). Workers use identical mixed strategies in making their applications, and the mixing probabilities are such that they are indifferent about where to apply (so that mixed strategies are optimal). If the realization of the mixed strategy is that there are n other applicants for the job, the worker is employed with probability $1/(n+1)$. Conversely, firms manage to hire workers only if at least one worker actually applies for its job. In this case, the matching technology corresponds to a standard urn-ball process: $\eta(q) \equiv q\mu(q) \propto 1 - e^{-q}$.[5]

An equilibrium of the wage posting game must satisfy four conditions: (1) firms' investments and wage commitments are profit-maximizing, (2) new entrants earn zero profits, (3) workers direct their search toward the wage(s) that maximize(s) their expected wealth, and (4) $q(w)$ is consistent with rational expectations *beginning at any decision node*. More precisely, for wages $w \in \mathcal{W}$, i.e., the set of wages offered in equilibrium, $q(w)$ is the ratio of unemployed workers seeking that wage to firms posting it, and by the third requirement of equilibrium, applying to such a wage w gives the highest possible utility to workers. For any other wage w', $q(w')$ is pinned down by a subgame-perfection requirement with a similar spirit: *If* a firm offered w',

---

[5] This matching technology does not satisfy our requirement $\eta(\infty) = \mu(0) = \infty$. This may lead to a corner solution in the social planner's problem, i.e., an efficient allocation with no active firms. It also may lead to nonexistence of a bargaining equilibrium. However, it does not alter the main conclusion in this section, that the wage-posting equilibrium is efficient.

the queue length would be sufficiently long that workers would not prefer applying to w' instead of $w \in \mathscr{W}$. This final requirement is important for understanding the incentive of a firm to deviate from the prescribed wage set.

5.1. *Analysis.* We once again start by writing the Bellman equations, restricting ourselves to steady states. The value of a vacant firm posting wage w and using capital k is

$$(16) \qquad rJ^V(w, k) = \eta[q(w)][J^F(w, k) - J^V(w, k)] - sJ^V(w, k)$$

This is identical to Equation (6), except that it allows queue lengths to depend on wages so that $q(w) \neq Q$.[6] Similarly,

$$(17) \qquad rJ^F(w, k) = f(k) - w - sJ^F(w, k)$$

which has exactly the same reasoning as Equation (5) in the preceding section.

For workers, the value of being employed at wage w is

$$(18) \qquad rJ^E(w) = w + s[J^U - J^E(w)]$$

Note that the value of an employed worker only depends on his or her wage, not on the firm's investment k. In fact, workers do not need to observe firms' investment decisions. In the bargaining equilibrium of Section 4, the value of employment depended on k, because the bargaining solution split the surplus in the match. In contrast, here the wage is determined solely by the firm's ex ante decision.

Next, the value of an unemployed worker applying to a job with wage w is

$$(19) \qquad rJ^U(w) = \mu[q(w)][J^E(w) - J^U]$$

Again, the construction of these equations parallels Equations (11) and (13), with the value of an unemployed worker defined by the highest value that he or she can attain while unemployed:

$$(20) \qquad J^U = \sup_{w \in \mathscr{W}} J^U(w)$$

where $\mathscr{W}$ is the set of wages offered in equilibrium.

Finally, we formalize the requirement that $q(\cdot)$ satisfies rational expectations as a pair of conditions that apply for all w, including $w \notin \mathscr{W}$:

$$(21) \qquad \begin{aligned} &\text{(i)} \quad q(w) = 0 \qquad \text{if } J^U > J^U(w) \\ &\text{(ii)} \quad J^U \geq J^U(w) \end{aligned}$$

where J^U is defined by Equation (20). The first condition ensures that workers do not apply for a wage w (even off the equilibrium path) unless it gives them utility at least

---

[6] We once again impose that workers accept any match. This is not a restriction, since our definition of equilibrium ensures that $q(w) = 0$ if workers prefer unemployment to employment at a wage of w. Thus any wage offered in equilibrium will be accepted by workers.

equal to the value of unemployment J^U. Manipulating Equations (18) and (19), this is equivalent to requiring $q(w) = 0$ for all $w \leq [r + s + \mu(0)]/[\mu(0)]rJ^U$. The second condition ensures that unemployed workers can never expect to earn more than the value of unemployment. Together, they determine $q(w)$ for all w and in particular ensure that $J^U = J^U(w)$ for all $w \geq [r + s + \mu(0)]/[\mu(0)]rJ^U$.[7]

An equilibrium of this economy can now be defined more succinctly as the appropriate Bellman equations, $J^V(w, k)$, $J^F(w, k)$, $J^E(w)$, $J^U(w)$, and J^U, as described earlier; a queue length function q that satisfies Equation (21); and a nonempty joint support of the distribution of wages and capital investments $\mathcal{X}$ maximizing firms' profits, i.e., $\mathcal{X} \subseteq \arg\max_{w,k} J^V(w, k) - pk$, and satisfying the free-entry condition, $\max_{w,k} J^V(w, k) - pk = 0$. It is simpler to characterize an equilibrium as the solution to a constrained optimization problem:

Lemma 1 $(q, \mathcal{X})$ *with* $(w^p, k^p) \in \mathcal{X}$ *and* $q^p = q(w)$ *is a steady-state wage-posting equilibrium if and only if* (w^p, k^p, q^p) *solves*

$$(22) \qquad \max_{w,k,q} \frac{\mu(q)}{r + s + \mu(q)} w$$

$$subject\ to \quad \frac{\eta(q)}{r + s + \eta(q)} \cdot \frac{f(k) - w}{r + s} \geq pk$$

Any wage, capital, and queue combination observed in equilibrium must maximize the utility of the representative worker, subject to new vacancies earning zero profits. Acemoglu and Shimer (1999) prove this lemma, and we do not repeat the proof here. Intuitively, if another triple (w', k', q') gives workers more utility and satisfies the free-entry condition, then a firm could offer a slightly lower wage than w', still attract workers, and make strictly positive profits.

Lemma 1 allows us to characterize an equilibrium of the wage-posting game. Generically, there is a unique solution to this constrained optimization problem, and so generically, all firms make the same investment and offer the same wage. This result is intuitive in this complete information environment: If the wage and/or capital distributions are not degenerate, there must exist two different wage, investment, and queue triples that yield firms the same profit *and* workers the same utility.

More important for the focus of this article, the equilibrium and efficient allocations coincide. Intuitively, Equations (2) and (22) are equivalent optimization problems. Formally:

Proposition 3 *If* (q^S, k^S) *is an efficient allocation as characterized in Proposition 1, and*

$$(23) \qquad\qquad w^S = f(k^S) - k^S f'(k^S)$$

then there is a wage-posting equilibrium $(q, \mathcal{X})$ *with* $q^S = q(w^S)$ *and* $(w^S, k^S) \in \mathcal{X}$. *Conversely, if* $(q, \mathcal{X})$ *is a wage-posting equilibrium with* $q^P = q(w)$ *and* $(w^P, k^P) \in \mathcal{X}$, *then* (q^P, k^P) *is an efficient allocation, as characterized in Proposition 1.*

---

[7] In imposing these conditions, we are implicitly assuming that the value of unemployment is unaffected by a single firm's decision. This is natural in our atomless economy and is formally equivalent to the limit of a finite-agent economy (Burdett et al., 1997).

PROOF.   See Appendix B.

Equation (23) shows that there is an increasing equilibrium relationship between wages and productivity. Firms that undertake larger investments pay higher wages. Thus the absence of a holdup problem (i.e., the fact that the equilibrium is efficient) appears to contradict the intuition from the bargaining equilibrium. The difference is that firms here are not *compelled* to offer higher wages when they invest more but instead can conceive of investing more while keeping their wage constant. They offer higher wages precisely because they want to attract more workers. Thus, at the margin, the higher wage that a capital-intensive firm offers is *exactly* offset by the faster rate of job creation. In fact, the wage is always equal to the marginal product of capital (Equation 23) because at a given wage, firms adjust their capital investment until this condition obtains.

The second part of the efficiency result is that entry decisions are optimal. We know from the definition of the optimal allocation that jobs (or labor markets) with capital investment k^S and queue lengths q^S produce more net output than any other possible combination. Since in any equilibrium all profits are driven to zero, the expected present value of wages must be higher in jobs offering k^S and q^S than in any alternative. Thus, if these jobs are offered, all workers will be drawn to these jobs giving them the highest expected wages. Therefore, no other allocation can be an equilibrium, and there are no profitable deviations from the efficient allocation with k^S and q^S. The essence of this result is that wage posting induces unfettered competition among firms, and as Lemma 1 shows, this ensures that in equilibrium worker utility is maximized.

5.2. *Imperfect Information*.   The assumption that unemployed workers know about all the available wages is strong and, fortunately, not necessary for most of our results. Suppose that each worker only observes two posted wages, independently chosen from the set of vacancies.[8] The rest of the setup is unchanged. This will not affect the efficient equilibrium described in Proposition 3. Here, we summarize the main result of this subsection:

Proposition 4 *Suppose that each worker only observes two independently drawn wages from the wage distribution. Then there exists an equilibrium in which all firms choose capital k^S, attract queue length Q^S, and offer wage $w^S = f(k^S) - k^S f'(k^S)$.*

PROOF.   See Appendix B.

The intuition for this result can be seen as follows: Starting from the equilibrium (q^S, k^S, w^S), a reduction in workers' information reduces the profitability of some deviations but does not raise the profitability of any others. Because all other firms are offering wage w^S with associated queue q^S, any firm that posts a different wage realizes that it will only attract workers by giving them the efficient level of utility $J^U = [\mu(q^S)]/[r + s + \mu(q^S)] w^S$. However, we know from the perfect information benchmark that this is incompatible with the firm making positive profits. More

---

[8] This analysis borrows from our earlier paper (Acemoglu and Shimer, 1997), where we endogenized the amount of information gathering by workers but did not investigate the efficiency issues discussed here.

specifically, if a firm offers a wage $w' < w^S$, it will obtain a shorter queue length, as described by the perfect information-indifference condition Equation (21). And if it offers a wage $w' > w^S$, it will obtain a longer queue length—but only up to a point. If the unemployment-vacancy ratio is Q (i.e., the market queue length), only $2Q$ workers observe a given firm's wage, so a deviating firm's queue length cannot exceed $2Q$. As a result, the second part of condition (21) may be violated for sufficiently high wages. This change in the extensive form of the game reduces the profitability of some deviations from prescribed equilibrium without raising the profitability of any others. Therefore, it does not change the fact that there is no profitable deviation when all firms make investment k^S and offer the wage w^S. This remains an equilibrium.[9]

By reducing the profitability of some deviations, however, we may introduce other equilibria. In particular, take a local maximum of the constrained optimization problem Equation (22), (w^*, k^*, q^*). If there is no (w, k, q) with $q \leq 2q^*$ that yields a higher value to this problem, then this will be an equilibrium. This is unlikely to be a real issue, however, because simulations suggest that the constrained optimization problem has a unique local maximum, (w^S, q^S, k^S), the efficient allocation. Moreover, as the number of wage observations of each worker increases, inefficient equilibria become progressively less likely.

Another way to obtain the intuition of this result is by comparing our imperfect information environment with Bertrand competition. With Bertrand competition, each firm has one opponent, but price competition forces them to set price equal to marginal cost. The efficient allocation is similar, since firms choose wages to maximize workers' utility subject to nonnegative profits. Differently from standard Bertrand competition, the firm does not know who its rival is. This does not matter, however, because it knows that the rival also maximizes workers' utility subject to zero profits.

The result in this subsection is also related to Burdett and Judd (1983), who show in a model of price search that when each consumer observes two random prices, Bertrand competition is obtained. However, our result is stronger because in Burdett and Judd's article, firms have infinite capacity, so each firm knows that the other firm observed by the consumer can supply him or her with the required good. In contrast, in our setting, each firm can only hire one worker, so the other firm whose wage is observed by the worker may not hire the worker. Nevertheless, the expectation of competition with this "other" firm is sufficient to take us to Bertrand competition.

6. UNDERSTANDING EFFICIENCY

With the traditional bargaining setup, the equilibrium is always inefficient (see Section 4), while when firms post wages and workers direct their search, efficiency is guaranteed (see Section 5). It is important to understand what underlies these

---

[9] This result would change substantially if each worker only observed the wage of one firm. Then a firm that lowers its wage slightly would not suffer a reduction in queue length, since workers have no alternative. It is easy to show that the unique equilibrium of such a model has firms paying a zero (monopoly) wage. This is the Diamond (1971) paradox.

results. We can begin by noting that there are two important differences between the environments of Sections 4 and 5: In Section 5, firms are able to post wages and workers can direct their search. Although there is a natural association between these features, it is possible to inquire which one is the source of the strong efficiency results we obtained in Section 5.

At a trivial level, the possibility of directing search is important. If firms could commit to wages but workers applied to firms randomly, firms obviously would commit to a zero wage, and we would obtain the equilibrium of Section 4 with $\beta = 0$. To investigate the role of directed search and wage posting more seriously, we consider three hybrids of the environments analyzed in Sections 4 and 5.

1. *Wages are determined by ex post Nash bargaining, as in Section 4, but workers are able to observe firms' capital investment and direct their search appropriately, as in Section 5.* The fundamental condition of Section 5 that determines applications decisions will once again be an equilibrium condition: Workers must have the same expected utility at all jobs (with positive queue length). As a result, the equilibrium is characterized by the constrained optimization problem (Equation 22), with one additional constraint: Wages are not a choice variable but instead are set ex post by Nash bargaining. Thus, in this hybrid environment, wages must satisfy the additional constraint (Equation 14). It is straightforward to see that constraint (14) does not bind at the equilibrium (efficient) values of k^S, w^S, and q^S of Sections 3 and 5 and (i.e., at the efficient allocation) *if and only if* workers' bargaining power β is equal to the elasticity of the matching function η. In other words, when the Hosios "bargaining power equals elasticity" condition holds, we obtain the efficient allocation as the equilibrium of this hybrid environment. Therefore, when workers can direct their search toward firms with more capital, holdups are avoided if and only if the Hosios condition is satisfied.

2. *Firms commit to and advertise a bargaining rule β before the matching stage. Workers observe each firm's bargaining rule but not its capital investment, and they direct their search accordingly.* The equilibrium coincides with the bargaining equilibrium, with $\beta = Q^B \eta'(Q^B)/\eta(Q^B)$, where Q^B is the market queue length. As in Section 4, there are now holdup problems. More specifically, capital investments are given by Equation (12), so there is underinvestment. Firms compete efficiently along the dimension that workers can observe, which ensures the Hosios condition. Larger investments translate into higher wages but do not attract a longer queue because workers do not observe investments, so there is a holdup inefficiency: Firms are not fully rewarded for their investments.

3. *Firms can commit to and advertise a bargaining rule β before the matching stage. Workers can observe each firm's bargaining rule and capital investment, and they direct their search accordingly.* The equilibria of this economy coincide with the efficient equilibria characterized in Sections 3 and 5. A choice of β conditional on the capital investment is formally equivalent to a wage commitment.

These hybrid models clarify the role of different ingredients of our economy. When workers observe the capital choices of firms and direct their search accordingly, the Hosios condition solves the holdup problem. The reason is that even though workers take some of the returns from investments in the form of higher wages, the higher wage bill is exactly offset by the increase in profits due to the longer queues that are attracted. *Holdups do not arise because workers' application decisions internalize the dependence of wages on investments.* Although the assumption that workers observe all wages and can perfectly direct their search is extreme, it seems plausible that workers have some idea about prevailing wages and can direct their search to some degree. They can choose, for example, between jobs in the manufacturing and service sectors. The forces emphasized in this article therefore should reduce the scope of holdups in practice.

Complementing the role of directed search, which solves the holdup problem when the Hosios condition is satisfied, wage or bargaining rule commitments (posting) ensure that all firms offer a division of output satisfying the Hosios condition. The combination of wage commitments and directed search therefore leads to an efficient allocation.

Note conversely that holdup problems arise if the value of a firm's wage commitment depends on its capital investment, and this investment is unobserved. This is considerably weaker than a more naive conjecture that holdups arise whenever wages increase with investment. Remarkably, in the natural search environment that we have considered, whenever workers can direct their search between two randomly chosen jobs with known capital intensity, the Hosios condition is all we need for workers to internalize the dependence of wages on investments. In particular, when the Hosios condition holds, a firm that chooses a larger investment pays higher wages but also attracts sufficiently longer queue lengths that its investment incentives are aligned with efficient incentives. We do not have a very good intuition for why exactly the Hosios condition is required to balance the two opposing effects on firm profits, although a more intuitive understanding can be obtained by thinking of each investment level as a separate "island" economy and workers' job applications as decisions to enter one of these islands. We know from Hosios (1990) that when the elasticity condition is satisfied, net output and expected wages are maximized within each island, so workers will choose to enter the island with the highest expected wages, which will be the one with the greatest net output, so efficiency will be achieved.

To conclude, with wage posting and directed search, the market achieves efficiency both in the entry and investment margins, despite the large number of missing markets and possible widespread imperfect information about available wages. This is so because the wage posting environment of Section 5 enables two distinct but related phenomena. First, it allows workers to direct their search toward more capital-intensive firms. Even though in the environment of Section 5 workers do not care about firms' investment decisions, they observe wages, and firms that make larger investments offer higher wages. Second, wage posting ensures that wages satisfy the Hosios condition, the other requirement of efficiency. This is related to but stronger than the results of Moen (1997) and Shimer (1996) discussed in the introduction, since it obtains in a model with ex ante investments, where bargaining equilibria are always inefficient.

7.   CONCLUDING COMMENTS

A natural conjecture is that when there are ex ante investments and trading fric-tions, holdups and inefficiencies are unavoidable. In this article we formalized this conjecture and showed why it is not always true. In fact, our main result is in strik-ing contrast to this conjecture. With standard assumptions on the form of trading frictions, an economy in which workers can direct their search toward more capital-intensive jobs achieves efficiency. We showed that this result arises naturally when firms post wages and workers direct their search toward higher wage firms.

The results presented here may suggest the opposite conjecture to the one we started with: Perhaps with a sufficient commitment technology, trading frictions do not lead to inefficiencies. We believe that this conjecture also would be incorrect be-cause there are other, harder to avoid inefficiencies, once again arising from frictional trading and the informational problems underlying search. For example:

1. We have assumed that there is only one-sided investment. The inefficien-cies emphasized in Acemoglu (1996) and Masters (1998) rely on two-sided investments. It is unclear whether the ability of firms to post complex con-tracts can prevent inefficiencies in this type of environment. This is left for future work.

2. An important cause of inefficiencies in search models is random matching: With two-sided heterogeneity, the equilibrium matching configurations may not coincide with those preferred by a social planner. This problem may be avoided if workers have complete information about the available jobs, but this assumption seems too strong in an economy with an atomless distri-bution of jobs. With imperfect information, skilled workers will sometimes have to match with low-capital firms, and mismatch will be present. In this case, the skill distribution of workers affects firms' capital investments and, via this channel, also wages [as in Acemoglu (1996) in the context of bar-gaining]. Therefore, the return to worker and firm investments will depend on the actions of other firms and workers, introducing externalities.

3. We treated the information structure as exogenous. In practice, workers de-cide *how much to search* both in the sense of determining reservation wages (as we have here) and how much information to gather. When the amount of information that workers have is endogenous, the equilibrium will be in-efficient, because of an informational externality. In particular, workers do not take into account the impact of their information on the wage distribu-tion and thus on the profits of firms and the wages of other workers (see Acemoglu and Shimer, 1997).

APPENDIX A: PROOFS OF MAIN RESULTS

PROOF OF PROPOSITION 1.   We first characterize the efficient allocation and then prove existence. We are looking for a steady-state solution and therefore suppress time dependence. A "social planner" chooses the time path of the market queue length Q, firms' capital investments k, and the unemployment rate u to maximize the

value of net output. That is:

(A.1) $\displaystyle\max_{Q,k,u} \int_0^\infty \left(u(t)\mu[Q(t)]\left\{ \frac{f[k(t)]}{r+s} - pk(t) \right\} - \frac{u(t)}{Q(t)}(r+s)pk(t) \right) e^{-rt}\, dt$

subject to

(A.2) $\dot{u}(t) = s[1 - u(t)] - \mu[Q(t)]u(t)$

The constraint (A.2) is a standard equation describing the evolution of the unemployment rate: The increase in unemployment is equal to the flow into unemployment minus new hires. Equation (A.1) is a bit more complex. Think of vacant firms renting capital at a cost $(r+s)p$, which accounts both for the interest rate (equal to workers' rate of time preference under risk neutrality) and depreciation. The first term in parentheses represents the payoff from newly created jobs. The number of new jobs is the number of unemployed workers times the probability that each is hired, $u(t)\mu[Q(t)]$. A newly created job produces $f[k(t)]$ units of output until the capital is destroyed. However, in the process, the job must continue to rent $(r+s)pk(t)$ units of capital. Therefore, the net expected present value of a newly created job is $f[k(t)]/(r+s) - pk(t)$. The second term represents the cost of maintaining open vacancies, i.e., the rental cost of a vacancy, times the number of unfilled vacancies $u(t)/Q(t)$, times the capital used by each vacancy $k(t)$. All payoffs are discounted back to an initial time.

The maximization of Equation (A.1) subject to Equation (A.2) gives the planner an extra degree of freedom not afforded by the model. He or she may costlessly adjust the capital investment of existing vacancies after they are created but before they are filled. In steady state, the planner would never take advantage of this option. Thus the solution to the maximization problem described here is the solution to the social planner's problem. More subtly, we restrict the planner to choose the same investment for all jobs. The optimality of this follows from concavity of the objective in k for arbitrary Q.

Write the current valued Hamiltonian associated with this dynamic optimization problem.

$$H(k, Q, u, \lambda) = u\left\{ \mu(Q)\left[\frac{f(k)}{r+s} - pk \right] - \frac{(r+s)pk}{Q} \right\} + \lambda[s(1-u) - \mu(Q)u]$$

Let $\{k(\lambda), Q(\lambda)\}$ maximize $H(k, Q, u, \lambda)$ for a given value of λ. Crucially, these are independent of $u > 0$, since u enters the maximization problem linearly:

(A.3) $\displaystyle\{k(\lambda), Q(\lambda)\} \in \arg\max_{k,Q} \mu(Q)\left[\frac{f(k)}{r+s} - pk - \lambda \right] - \frac{(r+s)pk}{Q}$

Define the maximized current value Hamiltonian as

$$\mathscr{H}(u, \lambda) \equiv H[k(\lambda), Q(\lambda), u, \lambda]$$

Arrow's generalization of Mangasarian's sufficiency theorem (Kamien and Schwartz, 1991: 222) states that the following conditions are necessary *and* sufficient for

$\{k, Q, u\}$ to be a steady-state solution to the dynamic optimization problem: $\mathcal{H}$ is a concave function of u (in fact, it is a linear function), $k = k(\lambda)$, $Q = Q(\lambda)$,

$$(A.4) \quad r\lambda = \frac{\partial \mathcal{H}(u, \lambda)}{\partial u} = -\frac{(r+s)pk}{Q} + \mu(Q)\left[\frac{f(k)}{r+s} - pk\right] - \lambda[s + \mu(Q)]$$

and u satisfies a steady-state version of the state equation (A.2):

$$(A.5) \qquad\qquad\qquad s(1 - u) = \mu(Q)u$$

Solving Equation (A.4) for λ,

$$\lambda = \frac{\eta(Q)\frac{f(k)}{r+s} - [r + s + \eta(Q)]pk}{(r+s)Q + \eta(Q)}$$

Substituting this into Equation (A.3) and simplifying, we obtain Equation (2). Thus the solution to the static maximization problem (Equation 1) in the text is the efficient capital and queue length.

Now we can establish the existence of an interior solution to this maximization problem. First, observe that the maximization problem is continuous in (k, Q) defined on the compact set $[0, \infty]^2$, implying existence of a maximum. Next, extremal values of k or Q yield zero or negative value to the objective Equation (2). The value of the objective is zero when $k = 0$, since $f(k) = 0$. It is negative when k is sufficiently large, since $f(k)/k \to 0$ by the Inada conditions. It is negative when Q is sufficiently small, since $\eta(0) = 0$. And it is zero when $Q = \infty$, since $\mu(\infty) = 0$.

Finally, we prove that there exist values of k and Q that yield positive payoff, implying that the maximum must be obtained at an interior value. Fix Q with $\eta(Q)$ positive and finite. Choose $k > 0$ to satisfy $f'(k) = [r + s + \eta(q)]/\eta(q)](r+s)p$, which exists by the Inada conditions. Then, by the fundamental theorem of calculus, the value of the objective at (k, Q) is

$$\int_0^k \frac{\eta(Q)f'(\kappa) - [r + s + \eta(Q)](r+s)p}{(r+s)Q + \eta(Q)} d\kappa$$

$$> \int_0^k \frac{\eta(Q)f'(k) - [r + s + \eta(Q)](r+s)p}{(r+s)Q + \eta(Q)} d\kappa = 0$$

where the inequality exploits concavity of f: $f'(\kappa) > f'(k)$ for all $\kappa < k$, and the equality uses the definition of k. $\quad\square$

PROOF OF PROPOSITION 2. EXISTENCE. Consider the graph of Equation (12) in (k, Q) space, the nonnegative quadrant of the plane. Since η is increasing and f is concave, the graph is upward-sloping. By the Inada conditions on f, it implicitly defines a continuously differentiable function $\mathbf{K}(Q)$. Moreover, $\mathbf{K}(0) = 0$, since $\eta(0) = 0$ and $f'(0) = \infty$; and $\mathbf{K}(\infty) = k_1$, defined implicitly by $f'(k_1)/(r+s) \equiv p$.

Next, divide Equation (12) by Equation (15) and simplify to obtain:

$$(A.6) \qquad\qquad \frac{\beta\mu(Q)}{r + s + (1 - \beta)\eta(Q)} = \frac{f(k)}{kf'(k)} - 1$$

Given the assumptions on μ, η, and β, the left-hand side is a decreasing function of Q, mapping $(0, \infty)$ onto itself. Thus, for all k, a unique Q solves this equation, $\mathbf{Q}(k)$. Since $\mathbf{Q}(0) > 0$, Equation (A.6) lies above Equation (12) when $k = 0$. Since $\mathbf{Q}(k_1) < \infty$, the two curves cross at some $k < k_1$. Such an intersection is an equilibrium, establishing existence.

Note that if $\beta = 0$, the curves intersect only at $(Q, k) = (0, 0)$. However, this is not an equilibrium, since we have assumed that $k \in (0, \infty)$. More precisely, as $\beta \to 0$, the equilibrium capital stock sequence converges to zero too; i.e., $k^B \to 0$. However, since $k = 0$ is not allowed, this equilibrium sequence is not lower hemicontinuous.

UNIQUENESS. If the elasticity of f is nonincreasing, Equation (A.6) describes a nonincreasing relationship between Q and k. Thus there can be at most one intersection between Equation (12) and Equation (A.6) and hence at most one equilibrium.

EFFICIENCY. Take any $\beta > 0$. Equations (3) and (12) imply that:

$$\frac{\eta(Q^S)f'(k^S)}{(r+s)[r+s+\eta(Q^S)]} = p < \frac{\eta(Q^B)f'(k^B)}{(r+s)[r+s+\eta(Q^B)]}$$

Then either $\eta(Q^S) < \eta(Q^B)$ or $f'(k^S) < f'(k^B)$. This implies that either $Q^S < Q^B$ or $k^S > k^B$, since η is increasing and f is concave. This is the desired result for $\beta \geq Q^B \eta'(Q^B)/\eta(Q^B) > 0$.

Next, consider $0 < \beta < Q^B \eta'(Q^B)/\eta(Q^B)$. Equations (4) and (15) imply that:

$$\frac{[\eta(Q^S) - Q^S\eta'(Q^S)]f(k^S)/k^S}{r+s+\eta(Q^S)+(1-Q^S)\eta'(Q^S)} = \frac{(1-\beta)\eta(Q^B)f(k^B)/k^B}{r+s+(1-\beta)\eta(Q^B)+\beta\mu(Q^B)}$$

The right-hand side is decreasing in β. Therefore, substituting for β with $Q^B \eta'(Q^B)/\eta(Q^B) < \beta$ implies that

$$\frac{[\eta(Q^S) - Q^S\eta'(Q^S)]f(k^S)/k^S}{r+s+\eta(Q^S)+(1-Q^S)\eta'(Q^S)} > \frac{[\eta(Q^B) - Q^B\eta'(Q^B)]f(k^B)/k^B}{r+s+\eta(Q^B)+(1-Q^B)\eta'(Q^B)}$$

Hence either $f(k^S)/k^S > f(k^B)/k^B$ or

$$\frac{\eta(Q^S) - Q^S\eta'(Q^S)}{r+s+\eta(Q^S)+(1-Q^S)\eta'(Q^S)} > \frac{\eta(Q^B) - Q^B\eta'(Q^B)}{r+s+\eta(Q^B)+(1-Q^B)\eta'(Q^B)}$$

Since f satisfies the Inada conditions, the former possibility implies that $k^S < k^B$. Alternatively,

$$\frac{d}{dQ}\left[\frac{\eta(Q) - Q\eta'(Q)}{r+s+\eta(Q)+(1-Q)\eta'(Q)}\right] \propto -\eta''(Q)[(r+s)Q + \eta(Q)] > 0$$

where the inequality exploits concavity of η. Thus the latter possibility implies that $Q^S > Q^B$.

Combining the results for $\beta > 0$ and $\beta < Q^B \eta'(Q^B)/\eta(Q^B)$ implies that either $k^S > k^B$ and $Q^S > Q^B$ or both inequalities are reversed.   ∎

PROOF OF PROPOSITION 3.   At a solution, the constraint in Equation (22) is binding. Otherwise, it would be possible to reduce w and raise the value of the objective without violating the constraint. Eliminating w from the objective through the binding constraint yields Equation (2). Thus a (k, q) pair appears in equilibrium only if they are efficient.

To obtain the wage equation, observe that the objective in Equation (22) does not depend on k; only the constraint does. Since the constraint is binding, optimality dictates that a small change in k must lead to a violation of the constraint, or at least keep the constraint binding. Equivalently,

$$\frac{d}{dk}\left\{\frac{\eta(q)(f(k) - w)}{(r + s)[r + s + \eta(q)]} - pk\right\} = 0$$

Simplifying the derivative with the binding constraint yields Equation (23).   ∎

PROOF OF PROPOSITION 4.   To formalize the issues related to imperfect information, we introduce some additional notation. Let $p(w, w')$ be the probability that a worker who observes wages w and w' applies to w in preference of w'. Since applying for w and w' are mutually exclusive, $p(w, w') + p(w', w) \equiv 1$. Applications are optimal if $p(w, w') = 1$ whenever $J^U(w) > J^U(w')$, implicitly taking into account the application decisions of other workers. The expected value of an unemployed worker is

$$(A.7) \qquad J^U = \int \int [p(w, w')J^U(w) + p(w', w)J^U(w')] \, dG(w)dG(w')$$

where G is the equilibrium wage distribution. The term in brackets is the maximal utility of an unemployed worker conditional on observing two wages w and w', and the integrals take expectations over all possible realizations of these two observations.

Finally, we need to define $q(w)$. With full information, it was pinned down to ensure indifference across wages. With imperfect information, such indifference is not guaranteed. In any subgame, a worker may not observe some wages and, for this reason, cannot apply to them, even though he or she would like to. Instead, the equilibrium queue function satisfies

$$(A.8) \qquad\qquad q(w) = 2Q \cdot \int p(w, w')dG(w')$$

where Q is the equilibrium number of unemployed workers per firm. The equilibrium queue length for a wage w is equal to the number of workers who observe this wage, which is $2Q$ because there are Q workers per firm and each observes two wages, times the probability that each of those will apply for this wage instead of the alternative w', a wage randomly drawn from G.

An equilibrium is once again defined as the appropriate Bellman equations, a queue length function q satisfying Equation (A.8), a joint support of the wage and capital distribution $\mathcal{X}$, each element of which maximizes firms' profits and satisfies the free-entry condition, a wage distribution G, and a preference function p that satisfies the optimality condition $p(w, w') = 1$ whenever $J^U(w) > J^U(w')$. Now we can give the proof of Proposition 4.

Proposition 3 implies that (w^S, k^S, Q^S) solves Equation (22). It trivially solves the same maximization problem with the additional constraint $q \in [0, 2Q^S]$. Letting J^{U*} be the maximized value of this program, (w^S, k^S, Q^S) must similarly solve the dual program

$$\max_{w,k,q} \quad \frac{\eta(q)(f(k) - w)}{(r + s)[r + s + \eta(q)]} - pk$$

$$\text{subject to} \quad \frac{\mu(q)w}{r + s + \mu(q)} \geq J^{U*}$$

$$\text{and} \quad q \in [0, 2Q^S]$$

We prove that this dual program is in fact the problem that a firm faces when it considers deviating from a proposed equilibrium with all other firms offering wage w^S.

First, profit maximization and free entry drive the other firms to use capital k^S and drive the market queue length to Q^S. Thus the value to a worker of applying for a job at wage w^S is J^{U*}. The deviating firm realizes that all workers who observe its wage also will observe a wage of w^S. Thus it will be unable to attract any applicants unless it promises them at least J^{U*}. On the other hand, since only $2Q^S$ workers will observe its wage in expectation, it cannot have a queue length longer than this. Thus the firm attempts to maximize the expected value of its profits, subject to these two constraints. Since the solution is (w^S, k^S, Q^S), this is an equilibrium.   ∎

APPENDIX B: INEFFICIENCY WITH ALTERNATIVE BARGAINING RULES

In Section 4 we showed that no equilibrium with Nash bargaining achieves efficiency. Here we extend that result to arbitrary bargaining rules that satisfy weak regularity conditions. For this purpose, consider a bargaining game with transferable utility between two players, 1 and 2, where conditional on agreement they obtain joint surplus y. If there is disagreement, 1 obtains d_1 and 2 obtains d_2, where we assume that $d_1 + d_2 < y$ so that agreement is mutually beneficial. We denote the bargaining payoff of player i to be u_i. A bargaining rule A, with payoffs $u_1^A(y, d_1, d_2)$ and $u_2^A = y - u_1^A$ is *regular* if the following conditions are satisfied:

1. Individual rationality: $\forall y \geq d_1 + d_2$, $u_1^A(y, d_1, d_2) \geq d_1$ and $u_1^A(y, d_1, d_2) \leq y - d_2$
2. Weak monotonicity: $\forall y' > y > d_1 + d_2$, if $d_1 < u_1^A(y, d_1, d_2) < y - d_2$, then $u_1^A(y', d_1, d_2) > u_1^A(y, d_1, d_2)$

The first condition states that both players obtain at least as much as their disagreement points. The second condition requires that if at some level of surplus each player receives more than his or her outside option and the level of surplus is increased, then each payoff increases.

As examples, consider two well-known bargaining rules: Nash bargaining imposes that $u_1^N = \beta(y - d_1 - d_2) + d_1$ for some $\beta \in [0, 1]$. Rubinstein-Shaked and Sutton bargaining imposes $u_1^R = d_1$ if $\beta y \leq d_1$, $u_1^R = y - d_2$ if $\beta y \geq y - d_2$, and $u_1^R = \beta y$ otherwise, for some $\beta \in [0, 1]$. It is straightforward to verify that both surplus divisions satisfy both conditions for all values of $\beta \in [0, 1]$.

We next state

Proposition 5 *If $w(k)$ is determined by a regular bargaining solution, the equilibrium is always inefficient.*

PROOF.   Recall that efficiency requires $w(k^S) = f(k^S) - k^S f'(k^S)$ and $w'(k^S) = 0$. Also, $d_1 = J^V(k)$ and $d_2 = J^U$ in this case. Weak monotonicity then implies that $w'(k^S) = 0$ is only possible if

1. Either $w(k) = rJ^U$
2. Or $J^F(k^S) = J^V(k^S)$

The second condition is not possible as long as $\eta(Q) < \infty$. The first implies that $w(k^S) < f(k^S) - k^S f'(k^S)$, leading to suboptimal entry.   ■

REFERENCES

ACEMOGLU, D., "A Microfoundation for Social Increasing Returns in Human Capital," *Quarterly Journal of Economics* 111 (1996), 779–804.

———, "Training and Innovation in an Imperfect Labor Market," *Review of Economic Studies* 64 (1997), 445–64.

——— AND R. SHIMER, "Wage and Technology Dispersion," MIT and Princeton mimeo, 1997.

——— AND ———, "Efficient Unemployment Insurance," *Journal of Political Economy,* 107 (1999), 893–928.

BURDETT, K., AND K. JUDD, "Equilibrium Price Dispersion," *Econometrica* 51 (1983), 955–69.

——— AND D. MORTENSEN, "Equilibrium Wage Differentials and Employer Size," Northwestern University mimeo, 1989.

———, S. Shi, and R. Wright, "Pricing with Frictions," University of Pennsylvania mimeo, 1997.

DAVIS, S., "The Quality Distribution of Jobs in Search Equilibrium," University of Chicago mimeo, 1995.

DIAMOND, P., "A Model of Price Adjustment," *Journal of Economic Theory* 3 (1971), 156–68.

———, "Wage Determination and Efficiency in Search Equilibrium," *Review of Economic Studies* 49(2) (1982), 217–27.

EDLIN, A., AND S. REICHLESTEIN, "Holdups, Standard Breach Remedies, and Optimal Investment," *American Economic Review* 86 (1996), 478–501.

GROSSMAN, S., AND O. HART "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy* 94 (1986), 691–719.

GROUT, P., "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach," *Econometrica* 52 (1984), 449–60.

HART, O., AND J. H. MOORE, "Property Rights and the Nature of the Firms," *Journal of Political Economy* 98 (1990), 1119–1158

HOSIOS, A., "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies* 57 (1990), 279–98.

KAMIEN, M., AND N. SCHWARTZ, *Dynamic Optimization*, 2d ed. (New York: Elsevier Science Publishing, 1991).

MACLEOD, B., AND J. MALCOMSON, "Investments, Holdup and the Form of Market Contracts," *American Economic Review* 83 (1993), 811–37.

MASTERS, A., "Efficiency of Physical and Human Capital Investments in a Model of Search and Bargaining," *International Economic Review* (1998).

MOEN, E., "Competitive Search Equilibrium," *Journal of Political Economy* (April 1997).

MONTGOMERY, J., "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics* 106 (1991), 163–79.

MORTENSEN, D., AND C. PISSARIDES "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61 (1994), 397–416

PETERS, M., "Ex Ante Price Offers in Matching Games: Non-Steady States," *Econometrica* 59 (1991), 1425–54.

PISSARIDES, C., *Equilibrium Unemployment Theory* (Oxford: Basil Blackwell, 1990).

SHIMER, R., "Essays in Search Theory," PhD dissertation, Massachusetts Institute of Technology, 1996.

WILLIAMSON, O., *Markets and Hierarchies: Analysis and Antitrust Implications* (New York: Free Press, 1975).