A MICROFOUNDATION FOR SOCIAL INCREASING RETURNS IN HUMAN CAPITAL ACCUMULATION*

DARON ACEMOGLU

This paper proposes a microfoundation for social increasing returns in human capital accumulation. The underlying mechanism is a pecuniary externality due to the interaction of ex ante investments and costly bilateral search in the labor market. It is shown that the equilibrium rate of return on the human capital of a worker is increasing in the average human capital of the workforce even though all the production functions in the economy exhibit constant returns to scale, there are no technological externalities, and all workers are competing for the same jobs.

I. INTRODUCTION

Human capital externalities arise when the investment of an individual in his skills creates benefits for other agents in the economy. Social increasing returns are a strong form of these externalities whereby the rate of return on human capital is increasing, at least in some region, in the stock of human capital of the economy. Despite much evidence indicating that such externalities and social increasing returns may be important, and their prominence in policy debates, little effort has been spent in investigating what underlies these phenomena. Quite often these externalities are simply built into an aggregate production function in the form of *technological increasing returns*. This paper shows that when the labor market is characterized by costly search, social increasing returns[1] arise naturally but they are *pecuniary* rather than technological. Even though all the production functions of the economy exhibit constant returns to scale, the interaction of ex ante investments and bilateral search in the

1. To refer to the case in which the increasing returns are built into the aggregate production function, I use the term technological increasing returns or technological externalities. The analysis will establish that an economy with a constant returns to scale production function can have the equilibrium rate of return of an individual increasing in the average human capital of the workforce. I refer to this feature as (*pecuniary*) social increasing returns. The assumption of constant returns is a simplifying one, and a decreasing returns aggregate production function would lead to the same results so long as the degree of decreasing returns is not too strong. However, note that the pecuniary externalities derived in this paper will never introduce "endogenous" growth as long as the returns to all variable factors are decreasing.

labor market will make the rate of return on human capital increasing in the average of human capital of the workforce. This result contrasts starkly with the standard conclusion: in the Walrasian (frictionless) analog of the model, the equilibrium rate of return on human capital would be a decreasing (or at least, nonincreasing) function of other workers' skills.

The basic idea is simple. Workers have to make a large part of their human capital investments (e.g., schooling, acquisition of general knowledge) before they know for whom they will work. The privately optimal amount of schooling depends on what type of jobs will be available, and what kind of equipment and machinery they expect to use. On the other hand, firms' choices of jobs and physical capital depend on the education and skills of the workforce. If a group of workers increases its education, the firms that expect to employ these workers would want to invest more. And yet, in a non-Walrasian labor market, it is not known in advance who these firms will be nor what exact characteristics they will possess. As a result, many firms hoping to match with the more skilled workers will invest more. Therefore, even some of the workers who have not increased their human capital—and who are competing for the same jobs—end up working with more physical capital and earning an increased rate of return on their human capital. In other words, as recently emphasized by Lucas [1988], the rate of return on human capital of a worker is increasing in the human capital stock of the workforce.[2] However, in contrast to Lucas' contribution, this feature is not assumed to be part of the aggregate technology; instead, it is derived from market interactions. Moreover, our argument immediately implies that there will also exist *social increasing returns in physical capital accumulation:* that is, if a group of firms invests more, education decisions of workers will be affected, and as a result, the rate of return for other firms will increase.

Are social increasing returns in human capital accumulation empirically important? A wealth of evidence suggests that the answer is yes. Although each piece of evidence is potentially open to a different interpretation, together they paint a picture supportive of social increasing returns in human capital. For instance,

2. More precisely, the mechanism proposed here suggests a link between the rate of return on human capital and the human capital investments of other agents. However, this link arises because firms, expecting more skilled workers—i.e., a higher "stock"—invest more. It is straightforward to generalize the model to a dynamic setting whereby the past stock of human capital contributes to the skills of the current generation, and thus increases the current rate of return.

Rauch [1993] finds that a one-year increase in the average educa-
tion in a metropolitan area is associated with a 3 percent increase
in wages even in regressions that control for observed character-
istics.[3] Further, high-skilled workers tend to migrate to areas
where such skills are abundant, which are also the areas with
the higher skill premium (see, for instance, Borjas, Bronars, and
Trejo [1992]). Such emigration of high-skilled workers—*the brain
drain*—is often argued to be an important barrier to the develop-
ment of many countries, e.g., Bhagwati and Rodriguez [1975].
Underlying this belief is the view that the low stock of human
capital in many underdeveloped countries creates a vicious
cycle—through social increasing returns—whereby the rates of
return on both human and physical capital are low because the
stock of human capital is limited. As Lucas [1990] notes, this view
provides the natural explanation for the lack of physical capital
flows to poor countries. Physical capital wants to go, as skilled
workers do, to areas where human capital is already abundant.

   Cross-country regressions also reveal an important role for
human capital variables in explaining international variations in
investment and growth rates (see among others Barro [1991] and
Benhabib and Spiegel [1994]). These studies lend support to Lu-
cas' [1990] hypothesis that differences in the stock of human capi-
tal restrict capital inflow and growth of poor countries. Finally,
following Krugman [1991], many economists believe that most
manufacturing industries are geographically concentrated, as
would be suggested by the common examples of the Silicon Valley
and Route 128, e.g., Ellison and Glaeser [1994]. Although such
geographical concentration can be explained by other models of
agglomeration, when combined with the migration flows and the
willingness of skilled workers to also concentrate in one area,
they are supportive of increasing returns in human capital.[4]

   This discussion suggests that social increasing returns in the
human (and perhaps also the physical) capital accumulation pro-
cess are important. But why do we need to look for the micro-
foundations of such increasing returns? Although part of the

   3. Maré [1994] finds that much of this premium disappears once one controls
for metropolitan area fixed effects.
   4. Note that this last piece of evidence can also be interpreted as supportive
of social increasing returns in physical capital, that is, higher return on invest-
ment in areas with high investment. Interestingly, the mechanism here leads to
social increasing returns not only in human capital but also in physical capital,
therefore, it accords with this different interpretation of some of the evidence (see
subsection V.A).

human capital externalities is undoubtedly technological, assuming that this is the only form of interaction is unsatisfactory. First, with a black box interpretation of these externalities, the policy recommendations and the testable economic hypotheses are limited. Where will these technological externalities apply? (For instance, according to Lucas [1988], human capital externalities can apply within any unit consisting of different decision makers.) Should we expect to find them in industries, cities, countries, or all over the world? Also, is it the total stock of human capital or the average human capital per worker that matters? (Suppose that we add a high school graduate to an economy of university graduates so that average human capital falls but the total increases: what happens to returns?) When the interactions arise from aggregate technological externalities, the answers are not very clear. In contrast, the approach taken in this paper, by providing microfoundations, makes predictions about where such externalities are expected to arise and what form they should take.

The second problem for the purely technological view is that excluding education and R&D, major human capital interactions happen among employees within a firm: for example, young workers learn from their more experienced colleagues. But these interactions should be internalized within the firm, and no economywide human capital externalities should be observed.

Are the microfoundations that this paper is proposing plausible? Unfortunately, there is no direct evidence for this (since these issues are so far unexplored). Nevertheless, empirical support for the three main foundations of this mechanism can be found: first, the allocation of workers to jobs is a costly activity; second, human and physical capital are complementary; and third, costs of changing partners in the labor market create a surplus (quasi-rent) that is shared between workers and firms. Considerable evidence suggests that job search is an important and costly activity. Sicherman [1991] and Topel and Ward [1992] show how workers are only slowly allocated to the jobs most suited to them. Barron, Bishop, and Dunkelberg [1989], for example, show that employers spend considerable resources in recruiting activities. Capital-skill complementarity also appears quite plausible, especially for technologies in use today (for instance, most truck drivers working in delivery need to use hand-computers). Further, econometric studies find strong evidence in favor of such complementarities (see, among others, Griliches [1969] and Bar-

tel and Lichtenberg [1987]). Finally, the findings of high correlation between profitability of firms and wages in both union and nonunion sectors (e.g., Dickens and Katz [1987], Estevao and Tevlin [1995], and Katz and Summers [1989]) support the view that there is rent-sharing.

It is also useful at this stage to relate the main mechanism of this paper to two strands of earlier research. The first is the search literature, such as Diamond [1982] and Mortensen [1982], which stresses that in environments characterized by search there will be important external effects. As more workers start looking for a job, it will be easier for firms to find the right worker, and their profits will increase. Although such externalities are theoretically appealing, their importance is limited. With a more crowded market, some other agents will also find it more difficult to find partners, and this second effect will tend to counteract the first. In fact, econometric studies have found that the matching technologies in the labor market exhibit constant returns to scale so that when the number of participants on both sides of the market doubles, the meeting probabilities are unchanged (e.g., Pissarides [1986] and Blanchard and Diamond [1989]). As a result, there appear to be no aggregate externalities on this front. The second literature follows Grout [1984] and discusses potential underinvestment in situations where investors cannot capture the whole of the surplus they create because of ex post holdup problems. To derive these results, a strong form of incompleteness of contracts and credit market imperfections are assumed, and further because there are no general equilibrium interactions, no aggregate human capital externalities are generated.[5] In contrast to these two literatures, this paper shows that in economies with ex ante investments and costly search, there exist external effects that work through changes in the *value of future matches,* and it demonstrates that these externalities take the form of social increasing returns because agents, even when they have increased their investments, will stop searching before finding the best possible partner.

The plan of the paper is as follows. Section II describes the basic environment and the Walrasian allocation. Section III derives the equilibrium with random matching, and demonstrates

---

5. For instance Caballero and Hammour [1996] and Davis [1994] discuss the implications of Grout's holdup problem for job creation and destruction dynamics and composition of jobs, respectively.

the presence of social increasing returns in human capital accumulation. Section IV analyzes the model with an efficient matching technology. Section V considers some extensions; in particular, it shows how multiplicity of equilibria can easily arise in this setting. The Appendix includes the proofs, and provides a game-theoretic justification for the wage determination rule used in the text.

II. THE BASIC ENVIRONMENT AND THE WALRASIAN ALLOCATION

The economy lasts for two periods, $t = 0$ and $t = 1$, and consists of two types of risk-neutral agents: firms (entrepreneurs) and workers. There is a continuum of equal measure of both types of agents, normalized to 1. At $t = 0$ workers choose their education level, and firms choose their capital stock.[6] At $t = 1$ productive relations are formed in one-worker–one-firm partnerships, production takes place, and output is divided between the firm and the worker. For the purposes of the analysis here whether firms are owned by the workers or are owned by entrepreneurs who consume the proceeds does not make a difference. The assumption that production takes place in a worker-firm pair is a simplifying one. It does not change the main conclusions, and since the measures of firms and workers are equal, issues related to unemployment are avoided until subsection V.C.

The production function for pair (i,j) is

$$(1) \qquad y_{ij} = A h_i^\alpha k_j^{1-\alpha},$$

where h_i is the human capital of the worker i, k_j is the physical capital of firm j, and $0 < \alpha < 1$. Firms maximize expected profits, and the cost of capital is constant and equal to μ.

Worker i maximizes his utility,

$$(2) \qquad V_i(c_i, h_i) = c_i - \frac{1}{\delta_i} \frac{h_i^{1+\Gamma}}{1 + \Gamma},$$

with $\Gamma > 0$. c_i is the consumption of worker i, and since the economy comes to an end in period $t = 1$, it is equal to his total income W_i. In what follows, I will define $\gamma \equiv 1/\Gamma$ for notational convenience. The parameter $1/\delta_i$ measures the relative disutility of hu-

---

6. The investment decisions of firms are irreversible. These can be thought of as decisions regarding the type of jobs they open (e.g., a secretary or an administrator) or the quality of the fixed equipment they purchase.

man capital acquisition for the worker. I assume that at the time of investment, the distribution of δ across workers is common knowledge.

A complete description of behavior in this economy consists of (i) an investment decision for each worker and firm; (ii) given the distribution of investments of workers and firms, an allocation of workers to firms; and (iii) a wage level for each level of human capital (given the level of physical capital that the worker is matched with), $w(h,k)$, and a rate of return for each level of physical capital (given the level of human capital it is matched with), $r(h,k)$.

I start with the frictionless Walrasian system. At $t = 0$, the auctioneer calls out wage and rate of returns schedules, and trade stops when all markets clear.[7] The economy will be in *Walrasian equilibrium,* if and only if (a) given the distribution of investments, the allocation of workers to firms and the wage and rate of return functions are in equilibrium, and (b) given the final rewards, the ex ante investment decisions are privately optimal.

With Walrasian markets, a worker is allocated to the firm where his marginal product is highest, and since human and physical capital are complements, the most skilled worker will be working for the most productive firm (e.g., Sattinger [1992]). In other words, imagine a ranking of all firms, and of all workers, in descending order. Worker i and firm j will be matched together when they have the same ranks in their respective orders (the Appendix gives a formal definition of this equilibrium allocation). This allocation is a Walrasian equilibrium if and only if all agents are paid their marginal products in their pairings. Therefore,

(3a) $$w(h_i,k_j) = \alpha A h_i^{\alpha-1} k_j^{1-\alpha}$$

(3b) $$r(h_i,k_j) = (1 - \alpha)A h_i^{\alpha} k_j^{-\alpha},$$

---

7. Note that with the Walrasian auctioneer all trades are taking place at time $t = 0$. An alternative scenario would be one where agents choose their investments at $t = 0$, and then trade in a competitive labor market at $t = 1$. With this scenario the Walrasian allocation is still an equilibrium, but in the absence of some heterogeneous agents who choose different levels of human capital, there may also exist other equilibria. This is because with sequential competitive trading, it matters whether all markets are open: when all workers choose the same level of human capital, the markets that correspond to the other levels of education would be closed. However, even when these other equilibria exist, they do not satisfy some appropriate game-theoretic refinement such as stability or trembling hand perfection. Trading through the Walrasian auctioneer at $t = 0$ avoids this problem and enables me to establish uniqueness without additional technical details.

for all equilibrium pairs (h_i, k_j). The total equilibrium income of the worker will therefore be given as $W_i = w(h_i, k_j)h_i$. The total income of the firm, R_j, is similarly defined.

In the Walrasian allocation it must be the case that $r(h_i, k_j) = \mu$ (or equivalently $R(h_i, k_j) = \mu k_j$) for all equilibrium pairs (h_i, k_j) which, therefore, must have the same physical to human capital ratio:

$$(4) \qquad \frac{h_i}{k_j} = \left(\frac{\mu}{(1 - \alpha)A}\right)^{1/\alpha}.$$

The optimal human capital of worker i is given by maximizing (2):

$$(5) \qquad \alpha A h_i^{\alpha-1} k_j^{1-\alpha} = h_i^{\Gamma}/\delta_i.$$

Substituting from (3) and (4), the optimal investment for worker i can be obtained as

$$(6) \qquad h_i = \left(\alpha(1 - \alpha)^{(1-\alpha)/\alpha} A^{1/\alpha} \mu^{-(1-\alpha)/\alpha} \delta_i\right)^{\gamma}.$$

PROPOSITION 1.
   (i)   There exists a unique Walrasian equilibrium where the human capital investments are described by (6), and all firms have a constant human to physical capital ratio given by (4).
   (ii) The equilibrium allocation is Pareto optimal.
   (iii) The rate of return on human capital is independent of the distribution of δ_i.

As with all the other results in the main text (unless otherwise stated), Proposition 1 is proved in Appendix 2. It establishes that the above wage and investment functions characterize the unique equilibrium allocation. The last part of the proposition states a result important for later comparison; a worker is indifferent about the human capital choices of other workers. This result is partly due to the exogenously given cost of capital. Suppose instead that μ, the required rate of return on capital, is an increasing function of the aggregate demand for capital, K. Then:[8]

COROLLARY 1. Suppose that $\mu(K)$ is a strictly increasing function of K and $\delta_i = \delta_1$. Now consider an increase in δ_i from δ_1 to δ_2

8. Proof is straightforward and thus is omitted. This corollary also holds for an arbitrary rightward shift of the distribution of δ. Details are available from the author upon request.

$> \delta_1$ for a proportion ρ of the workers, where $1 > \rho > 0$. Then the rate of return on human capital and the wage of δ_1-workers fall.

This is the conventional story: an increase in the stock of human capital (or the average human capital of the workforce) reduces the rate of return on human capital—or, in the limiting case of constant μ, leaves it unchanged. As the cost of education falls for a group of workers, the stock of human capital in the economy increases, and firms want to increase their physical capital. This pushes the cost of capital up, and the remaining workers end up working with lower physical to human capital ratios and receive lower wages.

III. DECENTRALIZED EQUILIBRIUM WITH SEARCH AND RANDOM MATCHING

I now turn to the model of costly search and start with the case where $\delta_i = \delta_1$ for all workers. As before, firms and workers make their investments at $t = 0$, and then they take part in the labor market at $t = 1$. I also assume that workers and firms are allocated to each other via a *random matching technology;* that is, at $t = 1$ each worker has an equal probability of meeting each firm, irrespective of his human and the firms' physical capital. Again, there is no unemployment and matches are one-to-one. I also assume that once a partnership is formed, breaking up the match and finding a new partner is costly. As in the standard search models, wages will be determined by bargaining (e.g., Pissarides [1990]). In the main text I make the simplifying assumption that the total wage bill of worker i (matched with firm j) is equal to $W_i = \beta y_{ij}$. This means that the worker receives a proportion β of the total surplus (output). All the qualitative results hold with other sharing rules so long as workers do not always receive their marginal product. Further, Appendix 1 derives this wage determination rule as the unique equilibrium of a general equilibrium wage bargaining game where switching partners has an arbitrary cost $\varepsilon > 0$.

Two important features are introduced in this section, both as the result of the replacement of the Walrasian auctioneer by costly search. (i) Wages are no longer equal to marginal product. Thus, the Walrasian mechanism can no longer be relied upon to provide the right contingent prices and to induce efficient investments. (ii) *Anonymity*—workers do not know who their employ-

ers will be when they acquire their education, and so, they cannot write contracts to improve investment incentives. Hence, there is a natural incompleteness of contracts.[9]

With these features in place, the expected returns for workers and firms can be written as

$$(7a) \qquad W(h_i, \{k_j\}) = \beta A h_i^\alpha (\textstyle\int k_j^{1-\alpha} dj),$$

$$(7b) \qquad R(k_j, \{h_i\}) = (1 - \beta) A (\textstyle\int h_i^\alpha di) k_j^{1-\alpha}.$$

Due to random matching, wages depend on the whole distribution of physical capital across firms, $\{k_j\}$. Because each worker has an equal probability of being matched with each firm, his expected output is $A h_i^\alpha (\int k_j^{1-\alpha} dj)$, and his expected return is a proportion β of this. Similarly, the return to firms depends on the whole distribution of human capital in the economy, $\{h_i\}$.

It can be seen that for all $\{h_i\}$, the return to the firm is a strictly concave function of its own investment. Therefore, all firms will choose the same level of investment k. Similarly, given $\delta_i = \delta_1$, all workers will choose the same level of human capital, given by (8a) and (8b):

$$(8a) \qquad (1 - \beta)(1 - \alpha) A k^{-\alpha} h^\alpha = \mu$$

$$(8b) \qquad \beta \alpha A k^{1-\alpha} h^{\alpha-1} = h^\Gamma / \delta_1.$$

Thus,

$$(9) \quad \forall i,\ h_i = h_R \equiv \left(\alpha\beta[(1 - \alpha)(1 - \beta)]^{(1-\alpha)/\alpha} A^{1/\alpha} \mu^{-(1-\alpha)/\alpha} \delta_1 \right)^\gamma.$$

PROPOSITION 2. Suppose that matching is random, $W_i = \beta y_{ij}$, and $\delta_i = \delta_1$. Then,
  (i) There exists a unique equilibrium in which $h_i = h_R$ as given by (9).
  (ii) A small increase in the investments of all agents would make everyone better off.

The first observation is that (9) differs from (6), and thus this economy is inefficient. Further, if all firms *and* workers simultaneously increased their physical and human capital investments, the welfare of all agents would rise. This is the first sense in which there exist social increasing returns. The terms β and

9. Incompleteness of contracts means that employers and workers do not write contracts conditional on the education level of the worker and the type of job before the worker finishes his education process.

$(1 - \beta)$ in (9), which were absent in (6), are responsible for these results, and these are the terms introduced by the incompleteness of contracts. Note that a similar result could be obtained in a reduced-form model with the incompleteness of contracts exogenously imposed, but here this incompleteness is a direct consequence of search which creates anonymity and therefore prevents bilateral contracts between workers and firms at the investment stage.

It is also clear that no possible value of β can make the expressions (6) and (9) identical. Therefore, in contrast to Mortensen [1982] or Hosios [1990], there is no sharing rule that could restore efficiency in this economy. The externalities that the previous literature concentrated upon related to the changes in the matching probabilities, and with constant returns to scale in the matching technology (which is what the present model also has since the total number of matches is linear in the number of firms and workers), an optimal sharing rule could balance these externalities exactly and restore constrained efficiency. However, in the current economy achieving constrained efficiency is impossible since externalities work through the *value of future matches* and are always positive. Consequently, as β approaches 1, correct investment incentives are restored for the worker, but the firms' investment decisions are increasingly distorted. The converse applies when β is reduced toward zero. Therefore (proof omitted),

COROLLARY 2. The decentralized equilibrium is always constrained Pareto inefficient, and the level of aggregate output is too low. Aggregate output is maximized when $\beta = \gamma\alpha/(1 - \alpha + \gamma)$.

I now show that there is a stronger form of social increasing returns in this economy. Let us repeat the thought experiment of the last section and increase δ from δ_1 to δ_2 for a proportion ρ of the workers and assume that wages are still determined by $W_i = \beta y_{ij}$; thus, (7) applies.[10] Evaluating the integral in (7),

10. If instead of assuming that wages are a proportion β of total output, I use the wage bargaining formulation in the Appendix, I need to impose that δ_2/δ_1 and ρ are small relative to the mobility cost ε in order to ensure that a firm that meets with a worker of lower skill does not turn him down, and thus the wage rule $W_i = \beta y_{ij}$ can be derived as part of the equilibrium. This reasoning also suggests that the results here would not hold when δ_2/δ_1 is large, and thus less skilled workers will not benefit from having more skilled competitors around when the gap between these two groups is large (details are available upon request; also see Acemoglu [1995] where this case is analyzed).

(10) $R(k_j,\{h_i\}) = (1 - \beta)Ak_j^{1-\alpha}[(1 - \rho)h_1^\alpha + \rho h_2^\alpha],$

where h_1 is the investment of workers with $\delta = \delta_1$, and h_2 corresponds to workers with $\delta = \delta_2$. Once again, $R(k_j,.)$ is strictly concave in k_j, and all firms will choose the same level of investment, $k_j = k$. Therefore, the optimal human capital investments are given by

(11a) $\beta\alpha Ak^{1-\alpha}h_1^{\alpha-1} = h_1^\Gamma/\delta_1$

(11b) $\beta\alpha Ak^{1-\alpha}h_2^{\alpha-1} = h_2^\Gamma/\delta_2.$

Let us divide (11a) by (11b) and substitute into the first-order condition derived from (10). Then

(12) $\dfrac{k}{h_1} = \left((1 - \alpha)(1 - \beta)A\left[(1 - \rho) + \rho\left(\dfrac{\delta_2}{\delta_1}\right)^{\alpha\theta}\right]\mu^{-1}\right)^{1/\alpha},$

where I have defined $\theta \equiv \gamma/(\gamma(1 - \alpha) + 1) \equiv 1/(\Gamma + (1 - \alpha))$. Therefore, the physical to human capital ratio for workers who have not experienced a change in their cost of education is increasing in ρ.

PROPOSITION 3. Suppose that matching is random, $W_i = \beta y_{ij}$, and that there is an increase in δ from δ_1 to δ_2 for a proportion ρ of workers. Then the equilibrium rate of return on the human capital of all other workers increases.

Intuitively, all firms will try to take advantage of the increased level of human capital by investing more in their physical capital, but not all of those firms will match with the high human capital workers. Therefore, the workers who did not increase their human capital investments will benefit by being able to also work with more physical capital. In the real world this can be thought of as a firm that invested in a new technology hiring workers who require more training rather than waiting for other workers who would need less training, or employers giving the benefit of the doubt to interviewees at the screening stage. As a consequence, the rate of return on human capital is increasing in the average skill of the workforce, even though in the absence of matching frictions, workers would be pure substitutes. Interestingly, this result also implies that if the education of one group of workers were subsidized, other workers could also benefit.

It is worth emphasizing that the presence of two different types of social increasing returns have been demonstrated, and

that these two forms of increasing returns are conceptually differ-ent (and do not always hold together—see the next section). The first is established in Proposition 2. When all agents increase their investments by a small amount, everyone is made better off, and this was shown to be driven by the *incompleteness of con-tracts* (which was due to anonymity). The second is in Proposition 3. When a group of workers increase their investments and firms respond to it, the rate of return on the human capital of re-maining workers increases. This second aspect is directly related to *imperfect matching.* Even with the terms β and $(1 - \beta)$ re-moved from equation (12), exactly the same result would be ob-tained. This second type of increasing returns is entirely driven by the fact that in the absence of the Walrasian auctioneer, all workers have a positive probability of working with firms that have invested more.

A number of additional features have to be noted. *First,* this model implies not only social increasing returns in human capital but also in physical capital accumulation. When a group of firms increases its investment, workers will choose more education in response, and the rate of return to firms that have not invested more will increase as a result. *Second,* if the increase in δ had happened after the physical capital investments of the firms, or if it had been unknown to them, there would have been no change in the physical capital investments and thus no change in the wages of the other workers. This is due to the constant returns property of the production function. In this economy *increasing returns* are derived not from the production function but from *pecuniary* interactions, and they *always* work through the invest-ment response of the other side of the market. *Third,* what mat-ters is not the total human capital in the economy, but the *average human capital* of workers because firms make their in-vestment based on the expected human capital (or more gener-ally, *quality*) of the workers they will meet. For instance, a doubling of the number of workers and firms would not change the equilibrium rates of return because the matching technology is constant returns, and so, the matching probabilities will re-main unaltered.

Finally, as an application, consider two separate geographi-cal locations A and B such that A has a higher ratio of δ_2-workers. Now because the rate of return on physical capital is higher in A than B, more capital will go to A, and as a result there will be a large wedge between the returns on human capital in these two

labor markets. Next, provided that costs of migration are less than proportional to human capital (e.g., fixed costs of migration per person), δ_2-workers from area B will be the ones to migrate, thus creating a high degree of (often inefficient) concentration of economic activity as it is often observed in practice.

IV. DECENTRALIZED EQUILIBRIUM WITH "EFFICIENT" MATCHING

The assumption of completely random matching is extreme. In this section I look at the polar case: efficient matching. By this I mean a matching technology that replicates the allocation of the Walrasian economy (the most talented worker is allocated to the firms with the highest physical capital—see the Appendix for a formal definition). However, this is not the same as a Walrasian labor market, because after the initial matching, there is still the cost of breaking up the match, ε. I first characterize the equilibrium with $\delta_i = \delta_1$, $\forall i$.

PROPOSITION 4. Suppose that the matching technology is efficient, $W_i = \beta y_{ij}$, and $\forall i \ \delta_i = \delta_1$, then the allocation characterized in Proposition 2 is the unique equilibrium.

The proof of this proposition follows from Lemma 2 and Proposition 7 in Appendix 1. Intuitively, all workers have the same cost of education, and they will all choose the same level of investment. Whether matching is efficient or random does not make a difference to the investment incentives. As in the case of random matching technology, there are inefficiently low physical and human capital investments, and an increase in these investments will make everyone better off. Therefore, the incompleteness of contracts and the social increasing returns of the first type are still present. However,

PROPOSITION 5. Suppose that the matching technology is efficient. Consider a change in δ from δ_1 to δ_2 for a proportion ρ of workers. Then in equilibrium, a proportion ρ of the firms increase their physical investment from k_1 to k_2, the rest leave it unaltered at k_1, and the rate of return on the human capital of workers with δ_1 remains unchanged.

Although the equilibrium with homogeneous workers is exactly the same with efficient matching as it was with random matching, when the costs of education for a group of workers

change, wages for the remaining workers are not affected. Thus, the first form of social increasing returns remains, but the second one disappears. The intuition is simple: with the efficient matching technology, a firm that has increased its investment never meets a worker who has not invested more in his skills. Therefore, in this economy the employers never face problems in contacting the right workers, and the second type of increasing returns is absent.

Nevertheless, because an efficient matching technology is an extreme case, the result of Proposition 5 is only useful as a benchmark. The real world matching technology should be thought of as a combination of random and efficient matching,[11] and as a result, social increasing returns will be present in general (though their importance will be limited if we are close to an efficient technology). A number of empirical observations indeed suggest a limited degree of efficiency in the matching technology. First, recall that workers eventually finding the "right" employer by matching and then breaking up with unsuitable firms is *not* efficient matching. The efficient technology refers to an "invisible hand" creating the right matches within the pool of unmatched firms and workers, and thus ensuring that workers who are less skilled do not benefit at all from the increase in the physical capital of firms. Although certain labor market institutions, such as job advertising and interviewing, play this role, there is still a high degree of randomness. In practice, workers spend a long time in unemployment and in jobs that are not right for them. Topel and Ward [1992] document the loss of earnings in the process of workers finding the jobs that are best suited for them. Heckman [1993, p. 108] reports that even in Germany, where firms have better internal labor markets, there is a high degree of reallocation at some point in workers' careers. A large literature (e.g., Rumberger [1981]) suggests that there can be substantial "mismatch" at any given point in time between the education level of workers and the required human capital of the jobs they perform. Although the concept of required human capital is somehow difficult to interpret, Sicherman [1991] shows, using PSID data, that this mismatch has a significant impact on wages, and that a substantial part of such mismatch is transitory because

---

11. See Acemoglu [1995] for a matching technology that is a hybrid of random and efficient. Social increasing returns in human capital are present in this more general case so long as the gap between high and low skill workers is not too large.

over time workers move to more appropriate jobs. But he also finds this process of transition to be long. Overall, the evidence suggests a process of search far from the fully efficient scenario.

V. Extensions

V.A. *Multiplicity of Equilibria and Social Increasing Returns in Physical Capital*

The analysis so far has provided a microfoundation for social increasing returns to scale. However, the equilibrium has been unique despite these increasing returns. This feature is driven by the linearity of the production technology. With some degree of nonconvexity, the social increasing returns proposed in this paper will lead to multiple Pareto-ranked equilibria. To illustrate the main ideas, I maintain the wage determination rule[12] as $W_i = \beta y_i$. But instead of continuous choices, I suppose that each firm has a choice between two levels of physical capital, k_1 and $k_2 > k_1$ (e.g., two different technologies). The rest of the model is unchanged; in particular, I assume that $\delta_i = \delta$ for all workers.

PROPOSITION 6. Suppose that $(1 - \beta)Ak_2^{1-\alpha}(\alpha\beta\delta Ak_2^{1-\alpha})^{\alpha\theta} - \mu k_2 > (1 - \beta)Ak_1^{1-\alpha}(\alpha\beta\delta Ak_2^{1-\alpha})^{\alpha\theta} - \mu k_1$, and $(1 - \beta)Ak_2^{1-\alpha}(\alpha\beta\delta Ak_1^{1-\alpha})^{\alpha\theta} - \mu k_2 < (1 - \beta)Ak_1^{1-\alpha}(\alpha\beta\delta Ak_1^{1-\alpha})^{\alpha\theta} - \mu k_1$, where θ was defined in Section III. Then there exist two symmetric pure strategy equilibria. In one, firms choose k_2, and workers choose $h_2 = (\alpha\beta\delta Ak_2^{1-\alpha})^\theta$, and in the other, firms choose k_1, and workers have $h_1 = (\alpha\beta\delta Ak_1^{1-\alpha})^\theta$. The first Pareto dominates the second.

When all firms choose k_2, the rate of return on human capital increases, and workers are willing to invest more (h_2 instead of h_1). But since human capital is not paid its marginal product, this raises the rate of return on physical capital; thus, in the terminology of Cooper and John [1988], there are strategic complementarities. However, in contrast to Section III, where the same forces were also present, the indivisibilities imply that the social increasing returns can lead to multiple equilibria.

Expressing this intuition differently is also useful: by investing more, a firm is creating a positive externality on other firms because it encourages workers to accumulate more human

---

12. In this case, for this wage determination rule to be derived from microfoundations as in Appendix 1, a lower bound for the mobility cost ε is once again required.

capital. Therefore, as well as the social increasing returns in human capital, *social increasing returns in physical capital accumulation* are present. Such increasing returns are often used in recent work (e.g., Romer [1986]), but again they are assumed to be technological, whereas in this economy they have been derived as pecuniary externalities resulting from interactions in the labor market.

V.B. Directed Search and Wage Posting by Firms

Random matching implies that search is *undirected,* and therefore there is no point for firms to announce wages or wage contracts associated with their vacancies. This is a strong assumption. A previous version of the paper showed that if firms can post and fully commit to wage contracts that map from the general and firm-specific human capital of workers to a wage level, the Pareto optimum can be achieved. However, full commitment to wage contracts requires that a sufficient number of agents who are capable of punishing a deviant firm should be able to observe the exact characteristics of the worker and his wage—also an extreme assumption. As soon as full observability is abandoned, the optimality result collapses, and we are back to social increasing returns. The intuition is similar to Diamond [1971]. Suppose that all workers have chosen the surplus maximizing level of human capital, and all firms are offering wage contracts that reward workers appropriately. With full observability, firms will not be able to deviate to cut their workers' wage because they will fear punishment by other agents in the economy. However, if observability is imperfect, a firm will be able to reduce its worker's payment by a small amount without getting detected. And if one firm can do this, all other firms can too, and this will reduce the "outside option" of the worker, and therefore increase the power of the firm over the worker. By applying this argument recursively, it can be seen that without full observability, investment incentives will be distorted, and the economy will not be able to get close to a Pareto-efficient outcome.

V.C. Unemployment and Human Capital Accumulation

Finally, it is straightforward to endogenize job creation and unemployment in this setting. The main result of this exercise is that in a high unemployment environment, workers often expect their human capital not to be used and thus may end up investing less in their education (a similar point is made by Robinson

[1995] in the context of a model with technological externalities). However, with a low human capital workforce, the profitability of jobs is also reduced, and this depresses job creation. These interactions open the way for an additional multiplicity of equilibria: one equilibrium with high unemployment and low human capital, and another with high investments in human capital and low unemployment. In the Pareto-dominated equilibrium (the former), workers do not invest because they expect that, even with high education, they will not be able to get jobs. This finding may be part of an explanation for high dropout rates from high school in ghettos and poor neighborhoods. Also, note that in contrast to Diamond [1982], the multiplicity here does not rely on increasing returns in the matching technology, but on the social increasing returns introduced by ex ante investments and search.

VI. CONCLUDING REMARKS

This paper has demonstrated how matching imperfections in the labor market together with investment decisions on the part of firms and workers lead to social increasing returns in human (and physical) capital accumulation. This result explains quite naturally the importance of human capital externalities in the development context, geographical concentration of skill-intensive industries, higher rates of return on human capital in high average education urban areas, the lack of capital flow to low-skill areas, and other similar findings without resorting to technological externalities.

APPENDIX 1: WAGE DETERMINATION

A. Wage Determination with Random Matching and Homogeneity

I now analyze wage determination explicitly rather than assume that $W_i = \beta y_{ij}$. Suppose that once a pair is formed, both parties must incur a cost ε to change partners. This can be interpreted as a monetary or nonmonetary mobility cost, or a flow loss because finding a new partner takes time. I also assume that a randomly drawn proportion v of the agents looking for a match remains unmatched at the end of every round. I will analyze this economy as $v \to 0$. This modeling assumption enables me to avoid the problem that when an agent wants to switch partners, there
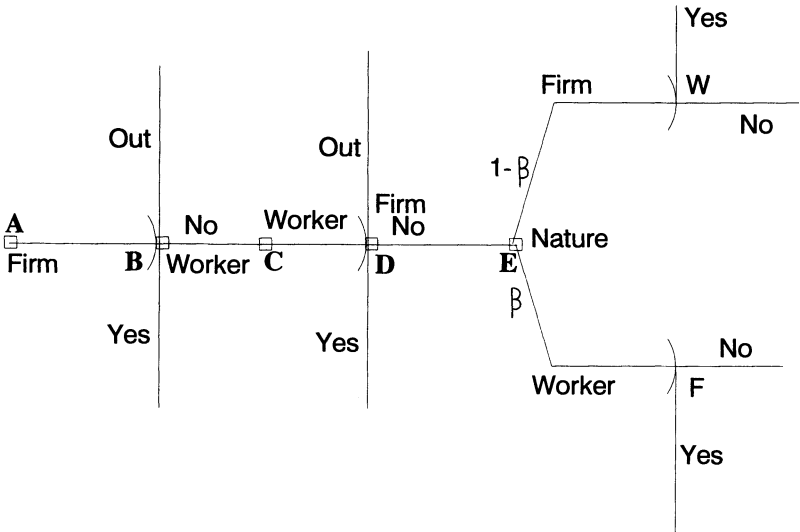
FIGURE I

may be no one to match with. It is also a convenient substitute for the more rigorous strategy of characterizing the steady state of a bargaining market with entry and exit (e.g., Gale [1987], and Osborne and Rubinstein [1990]). Bargaining after a match takes the form of the firm, F [she], and the worker, W [he], playing a continuation game $\Gamma(y)$ (Figure I), where y is the total output produced if there is agreement in this round. First, the firm makes a wage demand (node A) which can be refused by the worker (node B). If the worker refuses, he can at no cost make a counteroffer (node C) or decide to leave and find a new partner at cost ε (to both parties). If he makes an offer, the firm can refuse this (node D), and quit at cost ε. If she decides to continue with bargaining, Nature decides whether the firm or the worker will make the last offer (node E), and at this stage quitting is no longer permitted.[13] The probability that the worker will make the offer is denoted by β. If β is high, the worker has a strong bargaining position. In the case where one of the parties terminates bargaining, both find new partners (incurring the cost ε), and

---

13. I chose Nature to move at node E to simplify the game tree. The alternative game is one in which the worker and the firm asymmetrically alternate in making offers.

play a similar continuation game $\Gamma(y')$ (where y' is not necessarily the same as y if there is heterogeneity among the agents). Note that as ε becomes smaller, the amount of frictions is diminished.

At any point in time this game will have a complicated history that describes the way agents have matched and bargained up to that point. I am only interested in equilibria that do *not* depend on this history in a complicated way. Since I also want the equilibrium to be subgame perfect, I am only interested in Markov Perfect Equilibria, where all agents will play the same strategies in each completely identical continuation game $\Gamma(y)$ (see, for instance, Fudenberg and Tirole [1991] or Osborne and Rubinstein [1990]).

LEMMA 1. With homogeneous agents, random matching, and the game $\Gamma(y)$ as described above, $W_i = \beta y$ is the unique equilibrium for all $\varepsilon > 0$.

Proof. Consider the extensive form in Figure I, and let us denote the supremum expected return of the firm by V_F^S. If the game reaches node E, the expected return of the worker is βy and that for the firm is $(1 - \beta)y$. Next consider node D: since all pairs are playing the same continuation game, in a Markovian equilibrium the maximum the firm can expect from changing partners is $V_F^S - \varepsilon$. On the other hand, she can choose to go to node E and obtain $(1 - \beta)y$. The supremum of her equilibrium payoff is therefore $\max\{V_F^S - \varepsilon, (1 - \beta)y\}$. Now at node B, the worker can obtain $y - \max\{V_F^S - \varepsilon, (1 - \beta)y\}$ by proceeding to node C. Alternatively, he can leave and obtain his outside option. Since V_F^S is the supremum payoff for the firm, the worker needs to expect his infimum, denoted by V_W^I. Then at node A, the supremum of the payoff set of the firm is

(A1) $V_F^S = y - \max\{V_W^I - \varepsilon, y - \max\{V_F^S - \varepsilon, (1 - \beta)y\}\}.$

Now noting that $V_F^S + V_W^I = y$, this equation has a unique solution which is $V_F^S = (1 - \beta)y$.

Repeating the above argument with the infimum of the payoff gives $V_F^I = (1 - \beta)y$. Thus, $\forall \varepsilon > 0$, $W_i = \beta y$, $\forall i$ is the unique Markov Perfect Equilibrium.

QED

Lemma 1 establishes that even with very small search frictions, there may be a large wedge between the marginal product

of factors of production and their rates of return. The intuition is simple. When the worker leaves, he will meet another firm and enter a very similar bargaining situation. In fact, if the firm he meets is exactly the same as the one he is bargaining with, he cannot expect anything better, and hence his outside option will not be binding (i.e., *the threat to quit is not credible*), and bargaining will take place over the whole of the pie (see Diamond [1971] or Shaked and Sutton [1984] for a similar intuition). It follows immediately from this result that if the model of the text is augmented with the wage determination game outlined here, the allocation of Proposition 2 would be the unique symmetric equilibrium.

The limitations of this result should also be noted. First, it does not always hold in the presence of heterogeneous agents because the worker (or the firm) may be moving to a better partner (but see Lemma 3). Second, I am not allowing Bertrand type competition so that a worker is never able to bargain with two firms simultaneously. If a firm were allowed to meet two workers (or vice versa) with a certain probability (e.g., Burdett and Judd [1983]), then there would be a closer relationship between the rates of return and the marginal products, but the Walrasian outcome will not be achieved unless the mobility costs were completely removed.[14] Third, it should be noted that if Nature decides when bargaining will end and when parties will be forced to take their outside options, bargaining markets converge to the Walrasian equilibrium as $\varepsilon \to 0$. This is not surprising since we know from Binmore, Rubinstein, and Wolinsky [1986] that in these cases, the Nash solution to bargaining applies, and with this solution, as the frictions disappear, the allocation must converge to the Walrasian one.

B. Robustness of Wage Determination: Heterogeneity and Efficient Matching

As in the text, if the matching technology is *efficient,* initial matches are organized so that the highest skilled worker is matched to the firm with the highest capital. In other words, the initial match will be the same as the Walrasian allocation of Sec-

14. It is natural in this context that two firms would never simultaneously meet a worker and vice versa. If a worker is bargaining simultaneously with two firms, both firms will get zero surplus. Thus, ex ante a firm will have no incentive to contact a worker who is already in negotiation with, or employed by, another firm.

tion III. Formally, let us define for every worker i, $\Omega_w(i) = \int_{s \in S_W : h_s > h_i} ds$ and similarly for each firm j, $\Omega_F(j) = \int_{s \in S_F : k_s > k_j} ds$, where S_F and S_W are the sets of firms and workers who are looking for a match. Then $\Omega_F(j)$ *and* $\Omega_W(i)$ are the ranks of firm j and worker i in the set of firms and workers looking for a match. With the efficient matching technology, i and j will be matched if and only if $\Omega_W(i) = \Omega_F(j)$; or (ii) $\Omega_W(i) < \Omega_F(j)$, but all j^* that have $\Omega_F(j^*) < \Omega_F(j)$ are matched with i^* such that $\Omega_W(i^*) \leq \Omega_W(i)$; or (iii) $\Omega_w(i) > \Omega_F(j)$ but all i^*'s that have $\Omega_W(i^*) > \Omega_w(i)$ are matched with j^* such that $\Omega_F(j^*) \leq \Omega_F(j)$. Lemma 2 in Acemoglu [1995] establishes that with ex ante investments, out of these three possibilities, the economy must always be in (i). That is, $\Omega_W(i) = \Omega_F(j)$. The intuition for this result is quite simple: if this were not the case, an agent could improve his or her payoff by investing more by an infinitesimal amount. (I refer the reader to Acemoglu [1995] for a formal proof.) Then:

LEMMA 2. Let us assume that matching is efficient and agents are potentially heterogeneous at the wage determination stage. Then in the unique equilibrium we have $W_i = \beta y_{ij}$, where $\Omega_W(i) = \Omega_F(j)$.

Proof. Suppose that worker i leaves firm j. Matching at the second stage is efficient and $\Omega_W(i) = \Omega_F(j)$. Thus, he will meet a firm exactly at the same physical capital as firm j. Hence the proof of Lemma 1 applies.

<div align="right">QED</div>

Thus, Lemma 2 demonstrates that the divergence between rates of returns and marginal products is not due to the random matching assumption, but is driven by the presence of transaction costs of decentralized trading (which combined with anonymity introduces the incompleteness of contracts). Next a similar result for random matching with heterogeneous agents can be stated, but only when ε is arbitrarily small.[15]

LEMMA 3. Let us assume that matching is random and agents are potentially heterogeneous at the wage determination stage. Then as $\varepsilon \to 0$, the unique equilibrium is $W_i = \beta y_{ij}$, where $\Omega_W(i) = \Omega_F(j)$.

---

15. Note that $\Omega_W(i) = \Omega_F(j)$ is important for this result. To see this, suppose the case in which there are one type of firm and two types of workers. Then even as $\varepsilon \to 0$, the wage determination rule $W_i = \beta y_{ij}$ would not be an equilibrium.

Proof (sketch). Take an allocation where worker i who has the lowest (best) rank is matched with firm j who does not $(\Omega_W(i) = 0, \Omega_F(j) > 0)$. Then for ε small enough, either i or j will want to switch partners. Therefore, for ε small enough we cannot have an allocation that is not "efficient" so that we must have $\Omega_W(i) = \Omega_F(j)$.

Now suppose that worker i prefers to separate and matches with firm j^*. By definition it cannot match with a better partner. Thus, $k_{j^*} \leq k_j$. But since i prefers to match with j^*, $\beta y_{ij} < W_{ij^*} - \varepsilon$. Then, j^* receives less than $y_{ij^*} - W_{ij^*} - \varepsilon$. If j^* deviates and finds i^* with the same rank, then she can get (after the mobility cost) $y_{i^*j^*} - \varepsilon$. For j^* not to prefer this, it must be the case that $y_{ij^*} > (1 - \beta)y_{i^*j^*} + \beta y_{ij}$. This is not possible given complementarity of skills and capital. Hence a contradiction. Now, I can apply this argument recursively to the next lowest rank worker and firm. This establishes Lemma 3.

<div align="right">QED</div>

PROPOSITION 7. With efficient matching or with random matching and as $\varepsilon \to 0$, the equilibrium of Proposition 2 is the unique equilibrium of the fully specified economy even when agents are allowed to use nonsymmetric strategies.

Proof (sketch). Consider two workers of human capital h_1 and h_2 who have different utility levels $u_1 > u_2$. Suppose that $h_1 > h_2$. Then the second worker can increase his investment to slightly above h_1, get a better rank, and thus obtain the job of worker 2, and the same utility, u_2 (or arbitrarily close to it), since $\varepsilon \to 0$. If $h_1 < h_2$, this time he can reduce his investment to slightly above h_1, and the same argument applies. Hence a contradiction, and all workers must obtain the same level of utility. The same argument applies to firms. Since the problem is strictly convex, this can only be possible if they all choose the same level of ex ante investment, and this is the allocation of Proposition 2.

<div align="right">QED</div>

APPENDIX 2: PROOFS OF PROPOSITIONS

Proof of Proposition 1. (i) All competitive equilibria have to satisfy (4). Thus, all competitive equilibria have constant human to physical capital ratios. Then (6) defines the optimal human capital decision of workers. Since (6) has a unique solution, there is a unique competitive equilibrium. (ii) As all markets are com-

plete, the equilibrium is Pareto efficient. (iii) follows from the fact that the human to physical capital ratio is given by (4).

<div align="right">QED</div>

Proof of Proposition 2. (i) In the case of $W_i = \beta y_i$ and $\delta_i = \delta_1$, (9) uniquely describes equilibrium. (ii) Take the return of the firm given by (7), and evaluate these small changes at the equilibrium values (h_R, k_R). This gives the change in the return of the firm:

(A2) $\alpha(1 - \beta)Ak_R^{1-\alpha}h_R^{\alpha-1}dh + \{(1 - \alpha)(1 - \beta)Ak_R^{-\alpha}h_R^{\alpha} - \mu\}dk.$

The second term that multiplies dk is zero by the first-order condition (8a); the first term is positive; and thus, (A2) > 0. A similar reasoning applies to workers.

<div align="right">QED</div>

Proof of Proposition 3. In equilibrium, the physical to human capital ratio for the low human capital workers is given by (12) which is strictly increasing in ρ.

<div align="right">QED</div>

Proof of Proposition 5. With efficient matching technology, (8) defines the human to physical capital ratio for all pairs irrespective of the value of δ. Therefore, the distribution of δ does not matter, and the rate of return on human capital is independent of this distribution.

<div align="right">QED</div>

Proof of Proposition 6. When all firms choose k_2, workers will choose h_2 as defined in the proposition. Then, firm profit when k_2 is chosen is given as

(A3) $\Pi_2(k_2) = (1 - \beta)Ak_2^{1-\alpha}(\alpha\beta\delta Ak_2^{1-\alpha})^{\alpha\theta} - \mu k_2.$

In contrast, if a firm deviates and chooses k_1, I have

(A4) $\Pi_2(k_1) = (1 - \beta)Ak_1^{1-\alpha}(\alpha\beta\delta Ak_2^{1-\alpha})^{\alpha\theta} - \mu k_1.$

From the first condition in Proposition 6, (A3) is greater than (A4), and thus, when all other firms choose k_2, it is more profitable to do so.

Repeating this exercise for the case where all firms choose k_1, we get the profit for choosing k_2 as

(A5) $\Pi_1(k_2) = (1 - \beta)Ak_2^{1-\alpha}(\alpha\beta\delta Ak_1^{1-\alpha})^{\alpha\theta} - \mu k_2.$

And in contrast with k_1, the profit is

(A6) $\Pi_1(k_1) = (1 - \beta)Ak_1^{1-\alpha}(\alpha\beta\delta Ak_1^{1-\alpha})^{\alpha\theta} - \mu k_1.$

Provided that (A6) is larger than (A5), it is too costly to deviate to k_2, when other firms choose k_1. Thus, with both conditions in Proposition 6 satisfied, there is a multiplicity of equilibria. Also in the equilibrium with k_2, firms are making higher profits, and workers have higher utility, hence the Pareto ranking.

QED

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

REFERENCES

Acemoglu, Daron, "Matching, Heterogeneity and the Evolution of Income Distribution," MIT Working Paper No. 95-25, 1995.

Barro, Robert, "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics,* CVI (1991), 363–94.

Barron, John M., John Bishop, and William C. Dunkelberg, "Employer Search: The Interviewing and Hiring of New Employees," *Review of Economics and Statistics,* LXVII (1989), 43–52.

Bartel, Ann, and Frank Lichtenberg, "The Comparative Advantage of Educated Workers in Implementing New Technologies," *Review of Economics and Statistics,* LXIX (1987), 1–11.

Benhabib, Jess, and Mark M. Spiegel, "The Role of Human Capital in Economic Development: Evidence from Aggregate Cross-Country Data," *Journal of Monetary Economics,* XXVIII (1994), 143–73.

Bhagwati, Jagdish, and Carlos Alfredo Rodriguez, "Welfare-Theoretical Analyses of the Brain Drain," *Journal of Development Economics,* II (1975), 195–221.

Binmore Ken, Ariel Rubinstein, and Asher Wolinsky, "The Nash Bargaining Solution in Economic Modelling," *Rand Journal of Economics,* XVII (1986), 176–88.

Blanchard, Olivier, and Peter Diamond, "The Beveridge Curve," *Brookings Papers on Economic Activity* (1989:1), 1–60.

Borjas, George J., Stephen G. Bronars, and Stephen J. Trejo, "Self-Selection and Internal Migration in the United States," NBER Working Paper, No. 4002, 1992.

Burdett Kenneth, and Kenneth Judd, "Equilibrium Price Dispersion," *Econometrica,* LI (1983), 955–69.

Caballero, Ricardo, and Mohamad Hammour, "The Timing and Efficiency of Job Creation and Destruction," *Quarterly Journal of Economics,* CXI (1996), 805–52.

Cooper, Russell, and Andrew John, "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics,* CIII (1988), 441–65.

Davis, Steve, "The Quality Distribution of Jobs in Search Equilibrium," University of Chicago, 1994.

Diamond, Peter, "A Model of Price Adjustment," *Journal of Economic Theory,* III (1971), 156–68.

——, "Aggregate Demand Management in a Search Equilibrium," *Journal of Political Economy,* XC (1982), 881–94.

Dickens, William, and Lawrence Katz, "Inter-Industry Wage Differences and Industry Characteristics," in Kevin Land and Jonathan S. Leonard, eds., *Unemployment and the Structure of the Labor Markets* (New York: Basil Blackwell, 1987), pp. 48–89.

Ellison, Glenn, and Edward Glaeser, "Geographical Concentration in U. S. Manufacturing Industries: A Dartboard Approach," MIT and Harvard University, 1994.

Estevao, Marcello, and Stacey Tevlin, "The Role of Profits in Wage Determination: Evidence from U. S. Manufacturing," MIT, 1995.

Fudenberg, Drew, and Jean Tirole, *Game Theory* (Cambridge MA: MIT Press, 1991).

Gale, Douglas, "Limit Theorems for Markets with Sequential Bargaining," *Journal of Economic Theory*, XLIII (1987), 20–54.

Griliches, Zvi, "Capital-Skill Complementarity," *Review of Economics and Statistics*, LI (1969), 465–68.

Grout, Paul, "Investment and Wages in the Absence of Binding Contracts: A Nash Bargaining Approach," *Econometrica*, LII (1984), 449–60.

Heckman, James J., "Is Job Training Oversold?" *Public Interest*, CXV (1993), 91–115.

Hosios, Arthur, "On the Efficiency of Matching and Related Models of Search and Employment," *Review of Economic Studies*, LVII (1990), 279–98.

Katz, Lawrence, and Lawrence Summers, "Industry Rents: Evidence and Implications," *Brookings Papers on Economic Activity: Microeconomics* (1989), 209–75.

Krugman, Paul, *Geography and Trade* (Cambridge, MA: MIT Press, 1991).

Lucas, Robert E., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, XXII (1988), 3–42.

——, "Why Doesn't Capital Flow from Rich to Poor Countries?" *American Economic Review Papers and Proceedings*, LXXX (1990), 92–96.

Maré, David, "Living with Graduates: Human Capital Interactions in Metropolitan Areas," Harvard University, 1994.

Mortensen, Dale, "Property Rights and Efficiency in Mating, Racing and Related Games," *American Economic Review*, LXXII (1982), 968–79.

Osborne, Martin, and Ariel Rubinstein, *Bargaining and Markets* (New York: Academic Press: 1990).

Pissarides, Christopher A., "Unemployment and Vacancies in Britain," *Economic Policy*, III (1986), 499–559.

——, *Equilibrium Unemployment Theory* (Oxford: Basil Blackwell, 1990).

Rauch, James, "Productivity Gains from Geographic Concentration," *Journal of Urban Economics*, XXXIV (1993), 380–400.

Robinson, James A., "Unemployment and Human Capital Accumulation," University of Southern California, 1995.

Romer, Paul, "Increasing Returns and Long-run Growth," *Journal of Political Economy*, XCIV (1986), 1002–37.

Rumberger, Russell W., *Overeducation in the U. S. Labor Market* (New York: Praeger, 1981).

Sattinger, Michael, "Assignment Models of Distribution of Earnings," *Journal of Economic Literature*, XXXI (1992), 831–80.

Sicherman, Nachum, "Overeducation in the Labor Market," *Journal of Labor Economics*, IX (1991), 101–22.

Shaked, Avner, and John Sutton, "Involuntary Unemployment as a Perfect Equilibrium in a Bargaining Model," *Econometrica*, LII (1984), 1341–64.

Topel, Robert, H., and Michael P. Ward, "Job Mobility and the Careers of Young Men," *Quarterly Journal of Economics*, CVII (1992), 439–80.